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Firefly Algorithm for Uncapacitated Facility Location Problem and Number of Fireflies

Kohei Tsuya, Mayumi Takaya, Szilárd Zsolt Fazekas, Akihiro Yamamura

Akita University

Abstract

We apply the firefly algorithm to the uncapacitated facility location problem which is one of optimization problems and investigate the optimum number of the fireflies. The light absorption coefficient parameter γ of the firefly algorithm is examined to obtain better performance and suitable values of γ are explored for the uncapacitated facility location problem. Effectiveness of local search in the firefly algorithm is also investigated. In addition, we investigate the optimum number of fireflies for the firefly algorithm.

1 Introduction

The method of simulating swarm intelligence is inspired by the movement and foraging behavior in herd animals [1, 11, 21]. As typical examples, particle swarm optimization was inspired by the behavior of groups of birds and fish, ant colony optimization was inspired by the foraging behavior of ants. Numerous swarm intelligence algorithms have been proposed and investigated: particle swarm optimization [10, 11], ant colony optimization [6], artificial bee colony algorithm [9], bat algorithm [22], cuckoo search [23], genetic algorithm [5, 8] and so on. These have been applied to a wide range of computational problems like data mining and image processing [1] in addition to numerous optimization problems like the traveling salesman problem and the flow shop scheduling problem.

Firefly algorithm (FA for short) is one of metaheuristic algorithms and has been studied by many researchers. Recently, effectiveness of FA and suitable values of parameters of FA are investigated for some optimization problem in [18]. We introduce the results in [18] and add some results on effectiveness of local search.

Efficient supply chain management has led to increased profit, increased market share, reduced operating cost, and improved customer satisfaction for many businesses [13, 16, 17]. For this purpose, it is getting more and more important in information and communications technologies to solve optimization problem such as the *uncapacitated facility location problem* (UFLP) which is a combinatorial optimization problem. The objective of the UFLP is to optimize the cost of transport to each customer and the cost associated to facility opening, when the set of potential locations of facilities and the customers are given, whereas UFLP is known to be NP-hard (see [13]). In the context of performing economic activities efficiently, various objects have been considered as facilities, such as manufacturing plants, storage facilities, warehouses, libraries, fire stations, hospitals or wireless service stations. Several techniques have been applied to the UFLP such as the swarm intelligence algorithms or a meta-heuristics algorithm; *particle swarm optimization* (PSO) [7], *ant colony optimization* (ACO) [12], *artificial bee colony algorithm* (ABC) [19], and *genetic algorithm* [14].

UFLP is described as follows. Given a set F of facilities and a set C of customers, a fixed opening cost $f_i \in \mathbb{R}_+$ for each facility $i \in F$, and a transport cost $c_{ij} \in \mathbb{R}_+$ from each facility $i \in F$ to each customer $j \in C$, where $\mathbb{R}_+$ stands for the set of positive real numbers, UFLP asks to find a combination of opening facilities to minimize cost of transportation and opening facilities.

Each customer j is expected to select a facility i from the opened facilities so that the cost c_{ij} is lowest. It is asked to find a subset X of facilities to be opened and the assignment $\sigma : C \to X$ of each customer to an appropriate facility so that these minimize the sum of the opening costs of the facilities and the transport costs given by the formula (1).

$$\sum_{i \in X} f_i + \sum_{j \in C} c_{\sigma(j)j} \tag{1}$$

Several swarm intelligence algorithms have been applied to UFLP until now. For example, particle swarm optimization, ant colony optimization, artificial bee colony algorithm (ABC for short), and genetic algorithm are studied in [7, 12, 14, 19, 20]. In [18], FA is applied to UFLP and explore suitable value of parameters of FA. Effectiveness of equipping a local search mechanism to FA is also discussed to minimize cost (1). FA is also compared with ABC algorithm to find a solution of UFLP. Experiments are carried out to answer these issues. In this paper, we investigate the optimum number of fireflies for FA in addition to the results in [18].

2 Firefly algorithm for the uncapacitated facility location problem

2.1 Applying to UFLP

Each *firefly* k is given an *open facility vector* $Y_k(= [y_{k1}, y_{k2}, y_{k3}, \ldots, y_{kn}])$ representing a potential solution of opening facilities, where n is the number of facilities. If the i-th facility is open, $y_{ki} = 1$. For the open facility vector Y_k of a firefly k, $X(k)$ is defined to be the set of facilities i in F such that $y_{ki} = 1$, that is, $X(k)$ is the set of the facilities that are opened. For the open facility vector Y_k, assignment function $\sigma : C \to X(k)$ from C to $X(k)$ is defined by the transport costs c_{ij} as follows. For each j in C, $\sigma(j)$ is defined to be $i \in X(k)$ such that c_{ij} is the smallest among $\{c_{hj} \mid h \in X(k)\}$. If there are several candidates h, one of them is selected randomly. The *total cost* $T(Y_k)$ for each firefly k is computed as the sum of the cost of opening facilities determined by the open facility vector Y_k and the transport cost determined by the assignment function $\sigma : C \to X(k)$ for Y_k. The cost $T(Y_k)$ for the open facility vector Y_k is defined by the formula (1) and is rewritten as follows.

$$T(Y_k) = \sum_{i \in X(k)} f_i + \sum_{j \in C} c_{\sigma(j)j} \tag{2}$$

The notations used in this paper is summarized in Table 1.

Table 1: Notations

F	Set of facilities, $i \in F$,
C	Set of customers, $j \in C$,
f_i	Opening cost of facility $i \in F$
c_{ij}	Transport cost from $i \in F$ to $j \in C$
m	Number of customers
n	Number of facilities
σ	Assignment function
K	Number of fireflies
$k(k = 1, 2, \ldots, K)$	Firefly
$Y_k = [y_{k1}, y_{k2}, y_{k3}, \ldots, y_{kn}]$	Open facility vector of firefly k
$y_{ki} \in \{0, 1\}$	1 if the facility i is open. 0, otherwise.
r_{st}	Hamming distance between fireflies s and t
$T(Y_k)$	Total cost of firefly k
$I(Y_k)$	Light intensity of firefly k
γ	Light absorption coefficient
β	Probability that y_{ki} changes

An example of a problem instance of UFLP is given in Table 2. Suppose that A, B, C, D, E are facilities, a, b, c, d are customers, and $[1, 0, 1, 0, 1]$ is the open facility vector Y_s given to a firefly s. Note that $f_{\mathrm{A}} = 3$, $f_{\mathrm{B}} = 4$, $f_{\mathrm{C}} = 6$, $f_{\mathrm{D}} = 7$, and $f_{\mathrm{E}} = 2$ and $\sigma(\mathrm{a}) = \mathrm{A}$, $\sigma(\mathrm{b}) = \mathrm{B}$ (it may be E as well), $\sigma(\mathrm{c}) = \mathrm{C}$, and $\sigma(\mathrm{d}) = \mathrm{D}$ for this open facility vector Y_s. Then the total cost $T(Y_s)$ for the open facility vector Y_s is computed following the equation (2). Similarly, the open facility vector $Y_t = [0, 1, 1, 1, 1]$ of firefly t is as follows.

$$\begin{aligned} T(Y_s) &= \text{Opening costs} + \text{Transport cost} \\ &= (3+6+2)+\min(1,2,9)+\min(8,5,2)\min(4,3,6)+\min(9,4,3) \\ &= 20. \\ T(Y_t) &= (4+6+7+2)+(2+2+3+2) \\ &= 28. \end{aligned}$$

Table 2: Example of open facility vector for a firefly k

Facility		A	B	C	D	E
Opening cost		3	4	6	7	2
Open facility vector Y_k		1	0	1	0	1
Customer (Transportation costs)	a	1	3	2	6	9
	b	8	2	5	3	2
	c	4	8	3	5	6
	d	9	5	4	2	3

Attractiveness of a firefly is represented by the *light intensity*. From the assumption 3, the light intensity of the firefly is given by the objective function, that is, the total cost $T(Y_k)$. *Light intensity* $I(Y_k)$ of a firefly k is represented by the equation (3).

$$I(Y_k) = \frac{1}{T(Y_k)}. \tag{3}$$

Note that $I(Y_k)$ gets larger as $T(Y_k)$ gets smaller. It can be determined whether the firefly has a good solution by comparing $I(Y_k)$. The *Light intensity* of firefly s and t is as follows.

$$I(Y_s) = \frac{1}{20}, I(Y_t) = \frac{1}{28}. \tag{4}$$

Distance between any two fireflies is represented by the hamming distance of their open facility vectors. Suppose that the firefly s and the firefly t have open facility vectors below. Then the hamming distance r_{st} between s and t is 3.

$$\begin{aligned} Y_s &= [1,0,1,0,1] \\ Y_t &= [0,1,1,1,1] \end{aligned}$$

If $I(Y_s) > I(Y_t)$ holds for two fireflies s and t, then t moves towards s. *Movement* of the firefly t toward the firefly s is the conversion of the open facility vector of t. Common components of Y_s and Y_t are carried over to the new open facility vector Y_t.

$$\text{new } Y_t = [?,?,1,?,1]$$

Each of components of Y_t different from Y_s is replaced with a probability β. The probability β is given by the following formula (5).

$$\beta = \frac{\beta_0}{1 + \gamma r_{st}^2} \tag{5}$$

where β_0 is the probability at $r_{st} = 0$ and β_0 is set 1, and γ is the *light absorption coefficient*. In this paper, suitable values of γ is explored.

Let firefly t be close to firefly s by probability β as follows. The total cost of new Y_t at this time is 24, which shows that the cost is decreasing.

$$\begin{aligned} \text{new } Y_t &= [1, 1, 1, 0, 1] \\ \text{new } T(Y_t) &= (3 + 4 + 6 + 2) + (1 + 2 + 3 + 3) \\ &= 24. \end{aligned}$$

2.2 Pseudocode of FA

FA program generates fireflies and set the parameters. Fireflies compare the intensity each other and then change their positions, that is, the open facility vectors according to their distance. Repeating this process, FA program updates fireflies' intensity and their open facility vectors, and then it outputs a solution. A pseudocode of FA program is illustrated below.

```
begin
    Objective function minT(Y),
    Initialize positions of fireflies Y_k(k = 1, 2, ..., K)
    I(Y_k) is defined by the reciprocal of T(Y_k),
    Define parameter γ and β
    while (r < Repeat count)
        for t = 1 to K
            for s = 1 to K
                if I(Y_s) > I(Y_t)
                Move firefly t towards firefly s;
                else
                Move firefly t randomly;
                end if
                Update of intensity and total cost;
            end for s
        end for t
        Rank the fireflies and find the current best.
        Local Search.
        r++;
    end while
    Post-process results and visualization
end
```

3 Experiments and Results

3.1 Objectives

A computer program of FA to solve UFLP is implemented and experiments are carried out. Suitable values of the light absorption coefficient parameter γ is explored to accomplish better performance. A program is implemented in C# language using Visual Studio and run on an Intel Core i5 2.67GHz Desktop with 4.0GB memory. Several test data sets of benchmark problems are borrowed from OR-Library [4], which is a collection of test data sets for numerous operations research problems and originally described in [2]. UFLP is called an *uncapacitated warehouse location problem* in OR-Library. There are 15 data files provided; the test data sets VII, X, XIII and A to C from [3]. The test data sets VII (cap71, 72, 73, 74), X (cap 101, 102, 103, 104) and XIII (cap 131, 132, 133, 134) are employed for experiments. These test data sets are summarized in Table 3, in which m stands for the number of customers and n stands for the number of facilities. The optimal solutions for these test data sets are taken from OR-Library.

Table 3: Test data from OR-Library

Data set	m	n	Optimum solution
Problem set VII (cap71)	16	50	932615.750
Problem set VII (cap72)	16	50	977799.400
Problem set VII (cap73)	16	50	1010641.450
Problem set VII (cap74)	16	50	1034976.975
Problem set X (cap101)	25	50	796648.437
Problem set X (cap102)	25	50	854704.200
Problem set X (cap103)	25	50	893782.112
Problem set X (cap104)	25	50	928941.750
Problem set XIII (cap131)	50	50	793439.562
Problem set XIII (cap132)	50	50	851495.325
Problem set XIII (cap133)	50	50	893076.712
Problem set XIII (cap134)	50	50	928941.750
Problem set A (cap a)	100	1000	17156454.478
Problem set B (cap b)	100	1000	12979071.582
Problem set C (cap c)	100	1000	11505594.329

3.2 Number of fireflies

Find the optimum number of fireflies for FA. In this paper, *average relative percent error* (ARPE for short) and *hit to optimum rate* (HR for short) is used for performance evaluation of the algorithm. ARPE is the average of the difference from the optimum expressed in percentages. If ARPE is lower, then we have more chance to obtain better solutions. ARPE is given by the formula (6).

$$ARPE = \sum_{i=1}^{R} \left(\frac{H_i - U}{U} \right) \times \frac{100}{R} \tag{6}$$

where H_i denotes the i-th replication solution value, R is the number of replications, and U is the optimal value provided by [4]. HR represents the number of times that the algorithm finds the optimal solutions over all repetitions. If HR is higher, then the probability to obtain a better solution is higher. γ is set to be 0.01, the repeat count 50, and the number of fireflies is one of the values 10, 15, 20, 25, 30 or 35 in the experiments. The result is the average value when the algorithm is executed 100 times. The table 4 and table 5 is summary of ARPE or HR at repeat count is 50.

Table 4: ARPE for each firefly number (number of firefly, ARPE)

Porblem	10	15	20	25	30	35
cap71	0.01	0.00	0.00	0.00	0.00	0.00
cap72	0.01	0.01	0.00	0.00	0.00	0.00
cap73	0.05	0.05	0.01	0.01	0.00	0.01
cap74	0.03	0.03	0.01	0.01	0.01	0.01
cap101	0.38	0.26	0.15	0.12	0.09	008
cap102	0.21	0.17	0.11	0.14	0.11	0.13
cap103	0.17	0.14	0.12	0.14	0.09	0.08
cap104	0.59	0.45	0.35	0.37	0.28	0.20

Table 5: HR for each firefly number (number of firefly, HR)

Problem	10	15	20	25	30	35
cap71	0.94	0.97	1.00	1.00	1.00	1.00
cap72	0.94	0.98	1.00	0.99	1.00	1.00
cap73	0.63	0.79	0.93	0.92	0.97	0.92
cap74	0.85	0.87	0.93	0.97	0.97	0.97
cap101	0.18	0.20	0.40	0.39	0.52	0.53
cap102	0.13	0.15	0.36	0.25	0.30	0.21
cap103	0.12	0.13	0.15	0.20	0.20	0.25
cap104	0.32	0.40	0.49	0.42	0.57	0.62

4 Summary

We discussed several aspects of the firefly algorithm for UFLP and obtained two results in this paper.

First, optimal values of the light absorption coefficient γ of FA in UFLP are obtained. It is found that FA works better when γ is 0.001, 0.005 or 0.01. These values work well regardless of size of problem instance of UFLP. Suitable values for particular test data are obtained by experiments, however, suitable values of γ have not been completely classified for a general case. Therefore, it is necessary to seek suitable parameters when FA is applied to an optimization problem.

Second, it is verified that local search improves performance of FA and so the combination of FA and local search algorithm is effective. FA is compared with FA with local search and ABC algorithm with respect to average relative percent error and hit to optimum rate. No superiority of FA over ABC algorithm is recognized, whereas FA with local search works as well as ABC algorithm for UFLP.

It seems necessary to compare FA and ABC algorithm for more various sizes of test data and other type of optimization problems in order to comprehend the strong points of FA. It is also desirable to explore suitable parameters makes FA with local search to work better.

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