Characteristics of viscoelastic crustal deformation following a megathrust earthquake: discrepancy between the apparent and intrinsic relaxation time constants

Yukitoshi Fukahata[1*] and Mitsuhiro Matsu'ura[2]

[1]Disaster Prevention Research Institute, Kyoto University, Uji, Kyoto 611-0011, Japan

[2]Institute of Statistical Mathematics, Tachikawa, Tokyo 190-8562, Japan

Abbreviated title: Viscoelastic Crustal Deformation

(Submitted to Pure and Applied Geophysics on May 31, 2017)

Corresponding Author: Yukitoshi Fukahata

Disaster Prevention Research Institute, Kyoto University

Gokasho, Uji, Kyoto, 611-0011, Japan

Phone: +81-774-384226

Fax: +81-774-384239

E-mail: fukahata@rcep.dpri.kyoto-u.ac.jp

Abstract

The viscoelastic deformation of an elastic-viscoelastic composite system is significantly different from that of a simple viscoelastic medium. Here, we show that complicated transient deformation due to viscoelastic stress relaxation after a megathrust earthquake can occur even in a very simple situation, in which an elastic surface layer (lithosphere) is underlain by a viscoelastic substratum (asthenosphere) under gravity. Although the overall decay rate of the system is controlled by the intrinsic relaxation time constant of the asthenosphere, the apparent decay time constant at each observation point is significantly different from place to place and generally much longer than the intrinsic relaxation time constant of the asthenosphere. It is also not rare that the sense of displacement rate is reversed during the viscoelastic relaxation. If we do not bear these points in mind, we may draw false conclusions from observed deformation data. Such complicated transient behavior can be explained mathematically from the characteristics of viscoelastic solution: for an elastic-viscoelastic layered half-space, the viscoelastic solution is expressed as superposition of three decaying components with different relaxation time constants that depend on wavelength.

Key words: Viscoelastic relaxation, Maxwell time, megathrust earthquake, postseismic deformation

1. Introduction

Since the start of 21st century, megathrust earthquakes in plate subduction zones, such as Sumatra, Maule (Chile), and Tohoku (Japan), with the magnitude of around 9 have occurred one after another. For postseismic transient deformation following such large earthquakes, we cannot neglect the effect of viscoelastic behavior of the Earth (e.g., Nur and Mavko 1974; Fukahata *et al*. 2004; Johnson and Segall 2004; Wang *et al*. 2012; Watanabe *et al*. 2014; Noda *et al*. 2017). As is well known, when a strain is suddenly given for a homogeneous Maxwell body, the stress of it decays exponentially due to viscoelastic relaxation. Because of that, we often see that observed geodetic data after a large earthquake were simply fitted by an exponential function (e.g., Paul *et al*. 2007; de Linage *et al*. 2009; Reddy *et al*. 2010; Tanaka and Heki 2014).

However, the lithosphere is basically elastic for a timescale of earthquake cycles, even though the asthenosphere behaves viscoelastically after a large earthquake (e.g., Matsu'ura and Iwasaki 1983; Thatcher and Rundle 1984; Watts *et al*. 2013). In other words, the uppermost part of the solid Earth is not a homogeneous Maxwell body, but a composite system of the elastic lithosphere and the viscoelastic asthenosphere. For such a composite system, the viscoelastic stress relaxation does not follow a simple exponential function, but shows much more complicated behavior (e.g., Rundle 1978; Matsu'ura *et al*. 1981). If we do not understand this point well, we may draw a false conclusion from observed deformation data. In this study, we show that complicated viscoelastic relaxation can occur even in a very simple situation. The complicated viscoelastic behavior results in clear discrepancy between the apparent and intrinsic relaxation time constants: the former is obtained from fitting to observed data, while the latter is the time constant of the asthenosphere itself. We also give mathematical explanation for such characteristics.

2. Setting of Numerical Simulation

We compute crustal deformation due to viscoelastic stress relaxation in one of the simplest situations: a layered half-space under gravity, consisting of an elastic surface layer (lithosphere) and a viscoelastic substratum (asthenosphere). The asthenosphere is assumed to be a Maxwell fluid. Recently, we often see results of numerical simulation for a more realistic structure and/or rheology model (e.g., Sun *et al*. 2014; Ichimura *et al*. 2016; Lambert and Barbot 2016), but it is not easy to understand the characteristics of viscoelastic behavior in such complicated computations, which often shows unintuitive deformation. This study aims to contribute to having better insight into the deformation of an elastic-viscoelastic composite system.

The values of the structural parameters used in this study are summarized in Table 1. The viscosity of the asthenosphere is assumed to be 1.0×10^{19} Pa s. Then, the Maxwell relaxation time τ of the asthenosphere, defined as the ratio of viscosity to rigidity in the asthenosphere, is about 4.6 years for realistic values of seismic velocities and density (Table. 1). This is the nominal (intrinsic) value of the relaxation time of the asthenosphere.

The plate interface, on which a megathrust earthquake occurs, is represented by a curved fault plane that divides the elastic-viscoelastic half-space into two parts, continental and oceanic blocks, and that extends infinitely along the strike of an island arc. In other words, computations are carried out as a two-dimensional problem, for simplicity. In fact, even a megathrust earthquake has a finite fault length along the strike. So, it should be noted that horizontal transient displacement at far distance from the plate interface, computed in the following section, is not a good approximation of a real system.

We take the x-axis from the trench (x = 0) to the direction of the island arc and the z-axis downward. In order to represent a megathrust earthquake, we give spatially uniform displacement discontinuity (fault slip) of 5 m along the plate interface within the lithosphere at t = 0. When the plate convergence rate is 50 mm/yr, this is equivalent to the amount of slip deficit for 100 years. Because the displacement response (Green's function) of a layered half-space due to a unit step slip has already been obtained analytically (Fukahata and Matsu'ura 2005, 2006), we can easily compute coseismic displacement and subsequent viscoelastic transient motion caused by the megathrust earthquake (see Appendix). The reference point to measure the horizontal displacement is taken at x = 500 km (nearly stable point on the hanging wall) throughout this study.

3. Results

3.1 Viscoelastic displacements following a megathrust earthquake

In Fig. 1, we show the coseismic and postseismic displacements caused by the megathrust earthquake and the subsequent viscoelastic stress relaxation in the asthenosphere. Figure 1 shows that viscoelastic stress relaxation proceeds for a few hundred years, which is much longer than the nominal relaxation time, τ = 4.6 years (Sato and Matsu'ura 1988; Fukahata and Matsu'ura 2006). In the computation, the viscosity scales the rate of viscoelastic relaxation of this system. This means that when the viscosity of the asthenosphere becomes half (5.0×10^{18}), then the time needed for viscoelastic relaxation becomes half (e.g., 5 yr becomes 2.5 yr). In Fig. 1 we can also see that simple block-like motion is realized after the completion of viscoelastic stress relaxation in the asthenosphere (Fukahata and Matsu'ura 2016); there is almost no motion in the hanging wall, while the footwall approaches from a distance to the hanging wall and descends along the plate interface.

In Fig. 2, we show the profiles of horizontal (a) and vertical (b) surface displacements due to viscoelastic stress relaxation in the asthenosphere. These profiles show the cumulative postseismic displacements at different times, and do not include the coseismic displacement. The profile at t = 500 yr is nearly identical to that at $t = \infty$. As can be seen from Fig. 2, the cumulative horizontal displacement almost cancels out the coseismic displacement (broken line) on the hanging wall side. Here, the sign of coseismic displacement is reversed for comparison. In contrast, on the footwall side, the sum of the coseismic displacement and the cumulative postseismic displacement almost amounts to the plate convergence. This is consistent with the simple block-like motion observed in the bottom diagram of Fig. 1. The postseismic vertical displacement also proceeds so as to cancel the coseismic displacement, but the cumulative subsidence around the trench and uplift in the island arc (around x = 120 km) at t = 500 yr clearly exceed the coseismic displacement (broken line). These residual displacements accumulate for a long time that includes many earthquake cycles, and contribute to form characteristic topography and gravity anomalies of island arc-trench systems (Matsu'ura and Sato 1989; Hashimoto *et al*. 2004; Fukahata and Matsu'ura 2016).

Figure 2 also shows that postseismic displacements caused by viscoelastic stress relaxation in the asthenosphere do not proceed monotonously in time. For example, the direction of horizontal displacement changes from the ocean-ward (negative horizontal displacement) to the inland-ward (positive horizontal displacement) at a distance from the trench, and the uplift peak moves inland-ward with time. In order to see such complicated transient behavior in detail, we investigate displacement-rate profiles due to viscoelastic stress relaxation in the next section.

3.2 Spatial and temporal changes of displacement rate

Figure 3 shows horizontal (a) and vertical (b) surface displacement-rate profiles after the megathrust earthquake up to t = 50 yr. The deformation is solely caused by the viscoelastic stress relaxation in the asthenosphere. Therefore, after a long time, the displacement rate profile converges to zero. As shown in Fig. 3(a), the inland region (x > 100 km) moves to the trench at first, but the direction of motion is reversed later. The region that moves inland-ward gradually expands with time. We may feel strange for such change in the direction of motion, but it is necessary to realize the block-like motion after the completion of viscoelastic relaxation.

As for the vertical displacement (Fig. 3b), similar reversal of motion occurs in the inland region (x > 150 km), where subsidence motion changes to uplift. We can also notice that the subsidence with its peak around x = 60 km is steadily decelerated, while the uplift with its peak around x = 120 km is slightly accelerated at first and rapidly decelerated later. We can observe an even more complicated phenomenon near the trench (inset of Fig. 3b), where subsidence occurs at first (t = 1 yr), but it is followed by uplift (t = 10 - 25 yr) and changes to subsidence again (t = 50 yr).

In Fig. 4, we show the time histories of the horizontal (a) and vertical (b) surface displacement rates at x = 60 km (red), 120 km (blue), and 180 km (green). To make a comparison easily, the signs of some displacement-rate curves are reversed. These curves basically show the decay of displacement rates with time, but the decay rates are significantly different among these curves. In addition to this, some curves cross the zero line, not simply converging to zero. It should be noted that all the displacement rates become zero eventually.

In order to characterize the change of displacement rates, we manually fitted an exponential function, $a\exp(-t/b\tau) + c$, to each time-history curve, by adjusting the parameters a, b and c, where τ represents the intrinsic relaxation time of the asthenosphere. The offset term c is added to avoid poor fitting. The fitting (black line) to each time-history curve is excellent in this time range except the early stage (t < 10 yr) vertical displacement rate at x = 120 km and the later stage (t > 25 yr) one at x = 60 km. On the other hand, the parameter b that controls the decay time constant, shown by the number in a colored rectangle for each fitting, is significantly different. If the behavior of the system was simply controlled by the Maxwell relaxation time of the asthenosphere, then the parameter b would be equal to 1. But actually b is larger than 1 in every case. For some time-history curves, b is around 1.5, while it can be as large as 10. No correlation is also recognized between the values of b for the horizontal and vertical displacement rates at the same location. It should be noted that the apparent good fitting of the exponential function to the displacement-rate profiles in Fig. 4 is valid only for a limited time range, because the displacement rate at each point converges to zero at very large t.

4. Discussion

Figures 3 and 4 show very complicated behavior of postseismic displacements due to viscoelastic relaxation. It is difficult to estimate the relaxation time constant τ of the asthenosphere from time series of displacement data, without computing the response of the elastic-viscoelastic composite system. This is because the viscoelastic solution of an elastic-viscoelastic layered half-space is expressed as the superposition of a finite number of decaying components with different relaxation time constants, as described in Appendix (Matsu'ura et al. 1981; Fukahata and Matsu'ura 2006). In the case of this study, where an elastic surface layer is underlain by a viscoelastic substratum, the number of decaying components is three. More complicatedly, as shown in Fig. 5, the relaxation time constant of each decaying component depends on wavenumber (the integration variable, see Appendix). Larger wavenumber corresponds to smaller scale deformation and smaller wavenumber corresponds to larger scale deformation. Figure 5 shows that two of the three relaxation time constants (solid curves) significantly change with wavenumber, although each relaxation time constant converges to a certain value at a large wavenumber. All relaxation time constants are longer than the nominal relaxation time constant τ (4.6 years), shown by the dotted line, in the whole range of wavenumber.

The reason why this elastic-viscoelastic composite system has three decaying components with different time constants is evident mathematically (Matsu'ura et al. 1981; Fukahata and Matsu'ura 2006), but not easy to explain physically. According to the mathematical derivation, one decaying component comes from the viscoelastic layer itself, and the other two come from the continuity condition of displacement and stress components on the layer interface between the elastic layer and the viscoelastic substratum. In this study, we solved only the so-called *P-SV* problem, because only pure dip slip is given as the source. In a case with strike slip components, we also have to solve the so-called *SH* problem. For the *SH* problem, the number of the decaying component is only one; this one component comes from the continuity condition. The dependence of this component on wavenumber is also shown by the broken curve in Fig. 5.

In the computation of the displacement field, we take the summation of the mode solutions (Eqs. A1-A3 in Appendix). In Fig. 6, we show the weight of each mode $a_{ikm}(z, \xi; d)$ (see Appendix) by a color scale. The cases of i = 1 and 2 correspond to the components of horizontal and vertical displacements, respectively. Surprisingly, the weight of each mode, which is a function of wavenumber, changes several orders of magnitude. So, the portion of larger weight (indicated by warm colors) has more significant effect on viscoelastic deformation, though we should take the effect of integral interval into account (note that the abscissa is drawn in a logarithmic scale). The diagrams of (i, k) = (1, 0) and (1, 2) in Fig. 6 show that horizontal displacement of very long wavelengths (small wavenumbers) has extremely long effective relaxation time constants (more than several hundred years). The diagram of (i, k) = (2, 0) shows that vertical displacement also has very long effective relaxation time constants (about 100 years) for the wavelength of about 50 - 100 km. These are important properties of viscoelastic stress relaxation in this kind of problems.

In our formulation (Matsu'ura et al. 1981; Fukahata and Matsu'ura 2006), we can obtain the exact values of relaxation time constants as functions of wavenumber. This is an advantage of the analytical method in comparison to numerical methods like a finite element method. Even in analytical methods, Rundle (1978), who first obtained general solution for layered elastic-viscoelastic problems, applied some approximation in the inverse Laplace transform to avoid numerical instability due to the use of the up-going propagator matrix (Fukahata and Matsu'ura 2005). Wang et al. (2006), whose FORTRAN programs (PSGRN and PSCMP) are widely distributed, also numerically calculate the inverse Fourier transform with some approximation. So, from these studies we were unable to obtain the exact values of relaxation time constants.

As expressed in Eq. (A2), the relaxation time constants (Fig. 5) are determined only by the structural parameters (Table. 1) and do not depend on the locations of source and observation points, although the weight of each mode (Fig. 6) depends on them. The scale of the horizontal axis (wavenumber) of Fig. 5 is approximately inversely proportional to the thickness of the elastic layer. For a thinner elastic layer, the horizontal scale of Fig. 5 is enlarged and the convergence of relaxation time constants to a certain value occurs at a larger wavenumber. This means that viscoelastic behavior can be more complicated for a thinner elastic layer. On the other hand, the dependence of viscoelastic behavior on wavenumber becomes less important for a thicker elastic layer. This would be consistent with that the viscoelastic response approaches to an elastic response as the thickness of the elastic surface layer increases.

As shown in Fig. 4, the exponential function with an offset term has great flexibility. Therefore, time series of displacement data can be well fitted by an exponential function quite often. Because observed data are contaminated by various sources including seasonal variation, the fitting to actual observed data is easier than the fitting to synthetic data. However, as shown in this study, the relaxation time constant obtained by such fitting is generally quite different from the intrinsic relaxation time constant of the asthenosphere.

An offset term is usually needed in the fitting (Fig. 4), but the offset term must become zero after the completion of viscoelastic relaxation. This means that the fitting by an exponential function with an offset term is applicable only to a data set in a limited time range, even if the time range could be quite long. In other words, when observed displacement-rate data systematically deviates from an exponential fitting at a time after a megathrust earthquake, this does not always mean that something different happens. Such change in deformation trend can solely occur due to viscoelastic relaxation in the asthenosphere. In geodetic data analyses, transient displacement rate is often separated from time series by subtracting a linear trend (e.g., Tanaka and Heki 2014; Ochi and Kato 2013). However, we should note that not only the transient displacement rate that decays exponentially but also the displacement rate that proceeds in an almost constant rate for a few or several tens of years (e.g., the blue and green lines of Fig. 4a and the green line of Fig. 4b) can be a part of the total response of an elastic-viscoelastic composite system to a megathrust earthquake.

Recently, Tobita (2016) has shown that observed GNSS time series data after the 2011 Tohoku-oki earthquake can be well fitted by a combination of exponential and logarithmic functions. According to his analysis, the relaxation time constants of these functions are common for different stations in eastern Japan. The fitting is good both for horizontal and vertical displacement components. However, this result apparently contradicts with our theoretical result; the time history of displacement (Figs. 3 and 4) shows spatially quite complicated pattern. For a more realistic structure, the behavior would be more complicated. We consider that this contradiction is mainly ascribed to the dominant cause of crustal deformation after the 2011 Tohoku-oki earthquake. That is to say, not the viscoelastic stress relaxation in the asthenosphere but the afterslip at the downward extension of the main rupture would be the dominant cause in the early stage of postseismic period, particularly for displacements on land (Yamagiwa *et al*. 2015; Noda *et al*. 2017). Because crustal deformation due to afterslip can be essentially regarded as elastic, its temporal decay is independent of locations, as far as the spatial distribution of afterslip does not change in time, and directly reflects the decay of afterslip itself. After the early stage, however, viscoelastic stress relaxation in the asthenosphere should become the dominant cause of crustal deformation (Noda *et al*. 2017).

Appendix

In this study, we consider deformation caused by a megathrust earthquake within an elastic surface layer overlying a viscoelastic substratum under gravity (Fig. 1). We solve the coupled equations of the definition of strain, the constitutive equation, and the equation of motion. For a pure elastic problem, these equations can be summed up to Navier's equation. The boundary conditions to be satisfied are stress free at the surface of the elastic-viscoelastic half-space, the continuity of displacement and stress components on the layer interface, a certain amount of tangential displacement discontinuity on the megathrust fault, and the finiteness of displacement and stress components in the depths of the elastic half-space. These conditions result in no stress and strain at a far distance from the fault, but spatially uniform displacement there is allowed as a solution.

By solving Navier's equation for the given boundary conditions, we can analytically obtain the solution of elastic problems (Fukahata and Matsu'ura 2005). In order to obtain the viscoelastic solution analytically, the correspondence principle of linear viscoelasticity (Lee 1959; Radok 1957) is commonly used. We first derive the associated elastic solution of the viscoelastic problem, where the viscoelastic layer is replaced by a perfectly elastic layer with the same elastic constants. According to the correspondence principle, the Laplace transform of the viscoelastic solution is directly obtained from the associated elastic solution by replacing the source time function with its Laplace transform and the elastic constants of the viscoelastic layer with the corresponding s-dependent moduli. By applying the inverse Laplace transform to the viscoelastic solution in the Laplace domain, we obtain the viscoelastic solution in the time domain (Matsu'ura et al., 1981; Fukahata and Matsu'ura 2006).

We take the x and z axes to be horizontal and vertical, respectively. The viscoelastic solution in the time domain due to a dislocation source of a pure dip slip at a time t = 0 and a point $(x,z)=(0,d)$ in the elastic lithosphere (d < 35 km) is expressed by a semi-infinite integral with respect to wavenumber ξ (Fukahata and Matsu'ura 2006):

$$\begin{cases} u_x(x,z,t;d) = \dfrac{\Delta u}{4\pi}\displaystyle\int_0^\infty e^{-|z-d|\xi}\sum_{k=0}^{2} Y_{1k}(z,t,\xi;d)f_{1k}(x,\xi)d\xi = \dfrac{\Delta u}{4\pi}\int_0^\infty e^{-|z-d|\xi}\mathbf{Y}_1(z,t,\xi;d)\mathbf{f}_1(x,\xi)d\xi \\ u_z(x,z,t;d) = \dfrac{\Delta u}{4\pi}\displaystyle\int_0^\infty e^{-|z-d|\xi}\sum_{k=0}^{2} Y_{2k}(z,t,\xi;d)f_{2k}(x,\xi)d\xi = \dfrac{\Delta u}{4\pi}\int_0^\infty e^{-|z-d|\xi}\mathbf{Y}_2(z,t,\xi;d)\mathbf{f}_2(x,\xi)d\xi \end{cases} \tag{A1}$$

with

$$Y_{ik}(z,t,\xi;d) = Y_{ik}^{\infty}(z,\xi;d) + \sum_{m=1}^{M} a_{ikm}(z,\xi;d)\exp(-t/b_m(\xi)\tau) \qquad (i=1,\ 2;\ k=0,\ 1,\ 2) \tag{A2}$$

$$\begin{cases} \mathbf{f}_1(x,\xi) = \begin{pmatrix} -\frac{1}{2}\sin 2\theta \sin\xi x \\ -2\cos 2\theta \cos\xi x \\ -\frac{1}{2}\sin 2\theta \sin\xi x \end{pmatrix} \\ \mathbf{f}_2(x,\xi) = \begin{pmatrix} \frac{1}{2}\sin 2\theta \cos\xi x \\ -2\cos 2\theta \sin\xi x \\ \frac{1}{2}\sin 2\theta \cos\xi x \end{pmatrix} \end{cases} \tag{A3}$$

where u_x and u_z represent horizontal and vertical displacements, respectively. Y_{ik}^{∞} corresponds to the response after the completion of viscoelastic relaxation. θ is a dip angle of the source and Δu is the amount of dislocation (displacement discontinuity along a fault). As shown in Eq. (A2), $b_m(\xi)$ modifies the intrinsic relaxation time constant τ of the asthenosphere. The explicit values of $b_m(\xi)\tau$ with dependence on ξ are shown in Fig. 5. The subscript m of b in Eq. (A2) specifies a decaying component. For the case of this study, the number M of decaying components is 3. The coefficient $a_{ikm}(z,\xi;d)$ in Eq. (A2) controls the weight of each mode specified by a modified relaxation time constant on each wavenumber. It should be noted that $b_m(\xi)$ does not depend on *x*, *z* and *d*, while $a_{ikm}(z,\xi;d)$ depends on *z* and *d* as well as on *i* and *k*. $Y_{ik}^{\infty}(z,\xi;d)$ and $a_{ikm}(z,\xi;d)$ have very complicated forms, but they can be analytically calculated. The values of $a_{ikm}(z,\xi;d)$ for the surface displacement (z = 0) with the source depth d = 10 km are shown by the color scale in Fig. 6 for each pair of *i* and *k*. Note that the weight of each mode changes several orders of magnitude. On the other hand, the dependence of $a_{ikm}(z,\xi;d)$ on d is not significant. The cases of i = 1 and 2 correspond to the horizontal and vertical displacements, respectively (Eq. A1); *k* originally corresponds to the order of the Bessel function, but it reduces to trigonometric functions in the two-dimensional problem (Fukahata and Matsu'ura 2005).

By taking the summation of the response for each dislocation source, we can obtain the displacement field due to a megathrust earthquake.

Acknowledgements

We thank anonymous reviewers for useful comments, which contributed to improving the manuscript. This study was partly supported by the Grant-in-Aid for Scientific Research on Innovative Areas (Kakenhi No. 26109003) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) to YF.

REFERENCES

de Linage, C. *et al.* (2009), *Separation of coseismic and postseismic gravity changes for the 2004 Sumatra-Andaman earthquake from 4.6 yr of GRACE observations and modelling of the coseismic change by normal-modes summation*, Geophys. J. Int. *176*, 695-714.

Fukahata, Y. and Matsu'ura, M. (2005), *General expressions for internal deformation fields due to a dislocation source in a multilayered elastic half-space*, Geophys. J. Int. *161*, 507-521.

Fukahata, Y. and Matsu'ura, M. (2006), *Quasi-static internal deformation due to a dislocation source in a multilayered elastic/viscoelastic half-space and an equivalence theorem*, Geophys. J. Int. *166*, 418-434.

Fukahata, Y. and Matsu'ura, M. (2016), *Deformation of island-arc lithosphere due to steady plate subduction*, Geophys. J. Int. *204*, 825-840.

Fukahata, Y., Nishitani, A., and Matsu'ura, M. (2004), *Geodetic data inversion using ABIC to estimate slip history during one earthquake cycle with viscoelastic slip-response functions*, Geophys. J. Int. *156*, 140-153.

Hashimoto, C., Fukui, K., and Matsu'ura, M. (2004), *3-D modelling of plate interfaces and numerical simulation of long-term crustal deformation in and around Japan*, Pure Appl. Geophys. *161*, 2053-2068.

Ichimura, T., Agata, R., Hori, T., Hirahara, K., Hashimoto, C., Hori, M., and Fukahata, Y. (2016), *An elastic/viscoelastic finite element analysis method for crustal deformation using a 3D island-scale high-fidelity model*, Geophys. J. Int., *206*, 114-129.

Johnson, K. M., and Segall, P. (2004), *Viscoelastic earthquake cycle models with deep stress-driven creep along the San Andreas fault system*, J. Geophys. Res., *109*, B10403.

Lee, E. H. (1955), *Stress analysis in visco-elastic bodies*, Q. Appl. Math. *13*, 183-190.

Lambert, V. and Barbot, S. (2016), *Contribution of viscoelastic flow in earthquake cycles within the lithosphere-asthenosphere system*, Geophys. Res. Let. *43*, 10142-10154.

Matsu'ura, M. and Iwasaki, T. (1983), *Study on coseismic and postseismic crustal movements associated with the 1923 Kanto earthquake*, Tectonophysics *97*, 201-215.

Matsu'ura, M. and Sato, T. (1989), *A dislocation model for the earthquake cycle at convergent plate boundaries*, Geophys. J. Int. *96*, 23-32.

Matsu'ura, M., Tanimoto, T., and Iwasaki, T. (1981), *Quasi-static displacements due to faulting in a layered half-space with an intervenient viscoelastic layer*, J. Phys. Earth *29*, 23-54.

Noda, A., Takahama, T., Kawasato, T., and Matsu’ura, M. (2017), *Interpretation of offshore crustal movements following the 2011 Tohoku-oki earthquake by the combined effect of afterslip and viscoelastic stress relaxation*, Pure Appl. Geophys. doi:10.1007/s00024-017-1682-z.

Nur, A. and Mavko, G. (1974), *Postseismic viscoelastic rebound*, Science *183*, 204-206.

Ochi, T. and Kato, T. (2013), *Depth extent of the long-term slow slip event in the Tokai district, central Japan: A new insight*, J. Geophys. Res. *118*, 4847-4860.

Paul, J., Lowry, A. R., Bilham, R., Sen, S., and Smalley, R. (2007), *Postseismic deformation of the Andaman Islands following the 26 December, 2004 Great Sumatra–Andaman earthquake*, Geophys. Res. Let. *34*, doi:10.1029/2007gl031024.

Radok, J. R. M. (1957), *Visco-elastic stress analysis*, Q. Appl. Math. *15*, 198-202.

Reddy, C. D., Prajapati, S. K., and Sunil, P. S. (2010), *Co and postseismic characteristics of Indian sub-continent in response to the 2004 Sumatra earthquake*, Journal of Asian Earth Sciences *39*, 620-626.

Rundle, J. B. (1978), *Viscoelastic crustal deformation by finite quasi-static sources*, J. Geophys. Res. *83*, 5937-5945.

Sato, T. and Matsu'ura, M. (1988), *A kinematic model for deformation of the lithosphere at subduction zones*, J. Geophys. Res. *93*, 6410-6418.

Sun T. *et al*. (2014), *Prevalence of viscoelastic relaxation after the 2011 Tohoku-oki earthquake*, Nature *514*, 84-87.

Tanaka, Y. and Heki, K. (2014), *Long‐and short‐term postseismic gravity changes of megathrust earthquakes from satellite gravimetry*, Geophys. Res. Let. *41*, 5451-5456.

Thatcher, W. and Rundle, J. B. (1984), *A viscoelastic coupling model for the cyclic deformation due to periodically repeated earthquakes at subduction zones*, J. Geophys. Res. *89*, 7631-7640.

Tobita, M. (2016), *Combined logarithmic and exponential function model for fitting postseismic GNSS time series after 2011 Tohoku‑ Oki earthquake*, Earth Planets Space, *68*, 41, doi:10.1186/s40623-016-0422-4.

Wang, K., Hu, Y., and He, J. (2012), *Deformation cycles of subduction earthquakes in a viscoelastic Earth*, Nature *484*, 327-332.

Wang, R., Lorenzo-Martín, F., and Roth, F. (2006), *PSGRN/PSCMP–a new code for calculating co- and post-seismic deformation, geoid and gravity changes based on the viscoelastic-gravitational dislocation theory*, Computers & Geosciences *32*, 527-541.

Watanabe, S., Sato, M., Fujita, M., Ishikawa, T., Yokota, Y., Ujihata, N., *et al.* (2014), *Evidence of viscoelastic deformation following the 2011 Tohoku-Oki earthquake revealed from seafloor geodetic observation*, Geophys. Res. Let. *41*, 5789-5796.

Watts, A. B., Zhong, S. J., and Hunter, J. (2013), *The behavior of the lithosphere on seismic to geologic timescales*, Ann. Rev. Earth Plan. Science *41*, 443-468.

Yamagiwa, S., Miyazaki, S., Hirahara, K., and Fukahata, Y. (2015). *Afterslip and viscoelastic relaxation following the 2011 Tohoku-oki earthquake (Mw9.0) inferred from inland GPS and seafloor GPS/Acoustic data*, Geophys. Res. Lett. *42*, 66-73.

Table

Table 1. Two-layered structure model. V_p, V_s, ρ, η, and τ represent the *P*- and *S*-wave velocities, density, viscosity, and Maxwell relaxation time, respectively.

No.	V_p (km/s)	V_s (km/s)	ρ (kg/m^3)	η (Pa s)	τ (yr)	Thickness (km)
1	7.0	4.0	3.0×10^3	∞	∞	35
2	8.0	4.5	3.4×10^3	1.0×10^{19}	4.6	∞

Figure Captions

Figure 1: Coseismic displacement (t = 0) and its viscoelastic relaxation (t > 0) associated with a megathrust earthquake. The earthquake ruptures the whole plate interface within the lithosphere at t = 0. The plate interface is represented by the solid curve, which extends infinitely along the strike of an island arc, i.e., computations are carried out as a two-dimensional problem. The displacement discontinuity (fault slip) of the earthquake is 5 m uniformly along the plate interface. The sum of the coseismic and postseismic displacements is shown. The reference point to measure the horizontal displacement is taken at x = 500 km (x-axis is taken from the trench (x = 0) to the left hand side). Note that viscoelastic stress relaxation proceeds much longer than the nominal relaxation time τ (4.6 years).

Figure 2: Temporal evolution of cumulative horizontal (a) and vertical (b) surface displacements due to viscoelastic stress relaxation in the asthenosphere. Cumulative postseismic displacements at t = 5 years (green), 20 years (light blue), 50 years (dark blue), 150 years (purple), and 500 years (black) are shown. These profiles do not include the coseismic displacement. The positive direction of horizontal displacement is taken from the trench to the island arc. The broken lines are the profiles of coseismic displacement, but the sign of them is reversed for comparison. The profile at t = 500 year is nearly identical to that at $t = \infty$.

Figure 3: Profiles of horizontal (a) and vertical (b) surface displacement rates at different times. The velocity profiles at t = 1 year (red), 5 years (blue), 10 years (green), 25 years (purple), and 50 years (light blue) after the megathrust earthquake are shown. In each diagram, the broken line represents the profile of coseismic displacement (the scale is on the right-hand side), but the sign of it is reversed for comparison. The inset of (b) is the enlargement of the displacement-rate profiles near the trench.

Figure 4: Time histories of horizontal (a) and vertical (b) surface displacement rates at x =

60 km (red), 120 km (blue), and 180 km (green). The signs of the horizontal displacement rates at x = 120 km and 180 km and the vertical displacement rates at x = 60 km and 180 km are reversed for comparison. The black curve shows a manual fitting to each time-history curve by an exponential function with an offset term, $a\exp(-t/b\tau)+c$. The number in a colored rectangle shows the value of parameter b for each fitting.

Figure 5: Dependence of the effective relaxation time constants $b_m(\xi)\tau$ of decaying components on wavenumber. In the present case, the system has three time constants (m = 1, 2, 3) for viscoelastic relaxation, each of which depends on wavenumber. The values of relaxation time constants are determined by the structural parameters given in Table 1, and does not depend on the locations of the source and observation points as well as on the deformation mode i and k (see Appendix). It should be noted that the relaxation time constant curves do not intersect with each other. The dotted line represents the nominal (intrinsic) Maxwell relaxation time (i.e., b_m = 1) of the asthenosphere, defined by the ratio of viscosity to rigidity in the asthenosphere. The relaxation time constant of the *SH* problem (for a strike slip component) for the same structure model is also shown by a broken curve; the solid curves correspond to the relaxation time constants of the *P-SV* problem (for a dip slip component).

Figure 6: Weight of each mode $a_{ikm}(z,\xi;d)$ (see Eq. A2 in Appendix). The weight is indicated by the color scale at the lower right. The values of a_{ikm} for the surface displacement (z = 0) with the source depth d = 10 km are shown as a function of wavenumber ξ for each pair of i and k. In each diagram, the profile of the weight that corresponds to each relaxation time constant (m = 1, 2, 3) is shown. The functional forms of the relaxation time constants in each diagram are exactly the same as those in Fig. 5. The cases of i = 1 and 2 correspond to the components of horizontal and vertical displacements, respectively (see Eq. 1); k originally corresponds to the order of the Bessel function, but it reduces to trigonometric functions in the two-dimensional problem (Fukahata and Matsu'ura 2005) (Eqs. A1 and A3). Note that the weight of each mode changes several orders of magnitude, which results in the characteristic behavior of viscoelastic relaxation.