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Algebraic points on Shimura curves of $\Gamma_0(p)$-type (IV)

By

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Abstract

In previous articles, we proved that there are no points rational over a fixed number field on the Shimura curve of $\Gamma_0(p)$-type for every sufficiently large prime number p under a mild assumption. In this article, (1) we generalize the previous result to an infinite family of number fields, and (2) give examples not satisfying the mild assumption as mentioned above.

§ 1. Introduction

Let B be an indefinite quaternion division algebra over $\mathbb{Q}$, let $d(B)$ be its discriminant, and let M^B be the Shimura curve over $\mathbb{Q}$ associated to B. For a prime number p not dividing $d(B)$, let $M_0^B(p)$ be the Shimura curve of $\Gamma_0(p)$-type over $\mathbb{Q}$ associated to B. Then there is a natural finite morphism

$$\pi^B(p) : M_0^B(p) \longrightarrow M^B$$

over $\mathbb{Q}$ (see [4, §1]). We see $M^B(\mathbb{R}) = \emptyset$ and $M_0^B(p)(\mathbb{R}) = \emptyset$ by [6, Theorem 0]. Let k be a number field. In previous articles [2], [3], [4], we proved that the set $M_0^B(p)(k)$ of k-rational points is empty for every sufficiently large prime number p under a mild assumption (see §2). In this article, we generalize the previous result to an infinite family of number fields in Theorem 1.1 below. In §3, we prove Theorem 1.1 by a method of classical algebraic number theory. In §4, we give examples not satisfying the mild assumption as mentioned above.

Before stating the main result, we give some notation and convention. A prime $\mathfrak{q}$ of k is *of odd degree* if the residue field of $\mathfrak{q}$ is an extension of $\mathbb{F}_q$ of odd degree, where q is the residue characteristic of $\mathfrak{q}$. Throughout this article, we assume that all number fields are contained in $\mathbb{C}$. For simplicity, we say that

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- k satisfies NCH if k does not contain the Hilbert class field of any imaginary quadratic field,
- a prime $\mathfrak{q}$ of k satisfies OD(B) if $\mathfrak{q}$ is of odd degree and its residue characteristic q satisfies $B \otimes_{\mathbb{Q}} \mathbb{Q}(\sqrt{-q}) \ncong \mathrm{M}_2(\mathbb{Q}(\sqrt{-q}))$, and
- k satisfies EOD(B) if there is a prime of k satisfying OD(B).

Here, note that NCH (resp. OD, resp. E) is an abbreviation of "not contain Hilbert" (resp. "odd degree," resp. "exist"). Let $\mathcal{P}_{\mathrm{OD}(B)}(k)$ be the set of primes of k satisfying OD(B). We have $\mathcal{P}_{\mathrm{OD}(B)}(k) \neq \emptyset$ if and only if k satisfies EOD(B). The main result of this article is:

Theorem 1.1. *Suppose that k is Galois over $\mathbb{Q}$ and satisfies* NCH, EOD(B). *Let $n \geq 1$ be an integer. Then there are infinitely many extensions k' of k of $[k' : k] = 2^n$ and constants $C(B, k')$ depending on B, k' such that $M_0^B(p)(k') = \emptyset$ for any prime number $p > C(B, k')$.*

In Theorem 1.1, the assumption is important, and the essential part of the proof is to find infinitely many quadratic extensions of k satisfying NCH, EOD(B).

§ 2. Result for a fixed number field

We review a result in [3]. Let $\mathcal{M}^{new}(k)$ be the set of prime numbers which split completely in k. Let $\mathcal{N}^{new}(k)$ be the set of primes of k which divide some prime number in $\mathcal{M}^{new}(k)$. Fix a finite subset $\emptyset \neq \mathcal{S}^{new}(k) \subseteq \mathcal{N}^{new}(k)$ which generates the ideal class group of k. Let h_k (resp. $\mathcal{O}_k$) be the class number of k (resp. the ring of integers of k). For each prime $\mathfrak{q} \in \mathcal{S}^{new}(k)$, fix an element $\alpha_{\mathfrak{q}} \in \mathcal{O}_k \setminus \{0\}$ satisfying $\mathfrak{q}^{h_k} = \alpha_{\mathfrak{q}}\mathcal{O}_k$. Let $\mathbf{Ram}(k)$ be the set of prime numbers which are ramified in k. For a prime $\mathfrak{q}$ of k, let $\mathrm{N}(\mathfrak{q}) := \sharp(\mathcal{O}_k/\mathfrak{q})$. For an integer $n \geq 1$, let

$$\mathcal{FR}(n) := \left\{ \beta \in \mathbb{C} \;\middle|\; \beta^2 + a\beta + n = 0 \text{ for some integer } a \in \mathbb{Z} \text{ with } |a| \leq 2\sqrt{n} \right\}.$$

From now to the end of this article, suppose that k is Galois over $\mathbb{Q}$. Let

$$\mathcal{E}(k) := \left\{ \varepsilon_0 = \sum_{\sigma \in \mathrm{Gal}(k/\mathbb{Q})} a_\sigma \sigma \in \mathbb{Z}[\mathrm{Gal}(k/\mathbb{Q})] \;\middle|\; a_\sigma \in \{0, 8, 12, 16, 24\} \right\},$$
$$\mathcal{M}_1^{new}(k) := \{ (\mathfrak{q}, \varepsilon_0, \beta_{\mathfrak{q}}) \mid \mathfrak{q} \in \mathcal{S}^{new}(k),\ \varepsilon_0 \in \mathcal{E}(k),\ \beta_{\mathfrak{q}} \in \mathcal{FR}(\mathrm{N}(\mathfrak{q})) \},$$
$$\mathcal{M}_2^{new}(k) := \left\{ \mathrm{Norm}_{k(\beta_{\mathfrak{q}})/\mathbb{Q}}(\alpha_{\mathfrak{q}}^{\varepsilon_0} - \beta_{\mathfrak{q}}^{24h_k}) \in \mathbb{Z} \;\middle|\; (\mathfrak{q}, \varepsilon_0, \beta_{\mathfrak{q}}) \in \mathcal{M}_1^{new}(k) \right\} \setminus \{0\},$$
$$\mathcal{N}_0^{new}(k) := \{ \text{ prime divisors of some of the integers in } \mathcal{M}_2^{new}(k) \},$$
$$\mathcal{T}^{new}(k) := \{ \text{ prime numbers divisible by some prime in } \mathcal{S}^{new}(k) \} \cup \{2, 3\},$$
$$\mathcal{N}_1^{new}(k) := \mathcal{N}_0^{new}(k) \cup \mathcal{T}^{new}(k) \cup \mathbf{Ram}(k).$$

Note that the set $\mathcal{N}_1^{new}(k)$ is finite. In this setting, we obtained:

Theorem 2.1 ([3]). *Suppose that k satisfies* NCH, EOD(B). *Let p be a prime number satisfying $p > \min_{\mathfrak{q}\in\mathcal{P}_{\mathrm{OD}(B)}(k)} 4\mathrm{N}(\mathfrak{q})$, $p \nmid 13d(B)$, $p \notin \mathcal{N}_1^{new}(k)$.*

(1) *If $B \otimes_{\mathbb{Q}} k \cong \mathrm{M}_2(k)$, then $M_0^B(p)(k) = \emptyset$.*

(2) *If $B \otimes_{\mathbb{Q}} k \ncong \mathrm{M}_2(k)$, then $M_0^B(p)(k) \subseteq \{$elliptic points of order 2 or 3$\}$.*

Note that an upper bound of $\mathcal{N}_1^{new}(k)$ is estimated in [1]. We can easily eliminate elliptic points of $M_0^B(p)(k)$ with the aid of the classification of their fields of moduli as follows:

Proposition 2.2. *Let K be a subfield of $\mathbb{C}$. If $M_0^B(p)(K)$ has an elliptic point of order 2 (resp. 3), then K contains $\mathbb{Q}(\sqrt{-1})$ (resp. $\mathbb{Q}(\sqrt{-3})$).*

Proof. Let $x \in M_0^B(p)(K)$ be an elliptic point of order 2 (resp. 3). Then x is a CM point by $\mathbb{Z}[\sqrt{-1}]$ (resp. $\mathbb{Z}[\frac{-1+\sqrt{-3}}{2}]$) in the sense of [5, Definition 5.5]. Let $\mathbb{Q}(x)$ be the number field generated over $\mathbb{Q}$ by the coordinates of x on $M_0^B(p)$. Since $M_0^B(p)(\mathbb{Q}) = \emptyset$, we have $\mathbb{Q}(x) \neq \mathbb{Q}$. Then $\mathbb{Q}(x) = \mathbb{Q}(\sqrt{-1})$ (resp. $\mathbb{Q}(\sqrt{-3})$) by [5, Theorem 5.12]. □

A number field satisfying NCH contains neither $\mathbb{Q}(\sqrt{-1})$ nor $\mathbb{Q}(\sqrt{-3})$. Therefore:

Theorem 2.3. *Suppose that k satisfies* NCH, EOD(B). *Then $M_0^B(p)(k) = \emptyset$ for any prime number p satisfying $p > \min_{\mathfrak{q}\in\mathcal{P}_{\mathrm{OD}(B)}(k)} 4\mathrm{N}(\mathfrak{q})$, $p \nmid 13d(B)$, $p \notin \mathcal{N}_1^{new}(k)$.*

§ 3. Proof of Theorem 1.1

Recall that k is a finite Galois extension of $\mathbb{Q}$. The key to proving Theorem 1.1 is:

Proposition 3.1. *Suppose that k satisfies* NCH. *Let $\mathfrak{q}$ be a prime of k of odd degree.*

(1) *There are infinitely many quadratic extensions k' of k such that*

  (i) *k' is Galois over $\mathbb{Q}$,*

  (ii) *k' satisfies* NCH, *and*

  (iii) *there is a prime $\mathfrak{q}'$ of k' of odd degree which divides $\mathfrak{q}$.*

(2) *Let $n \geq 1$ be an integer. Then there are infinitely many extensions k' of k of $[k' : k] = 2^n$ such that*

  (i) *k' is Galois over $\mathbb{Q}$,*

(ii) k' *satisfies* NCH, *and*

(iii) *there is a prime* $\mathfrak{q}'$ *of* k' *of odd degree which divides* $\mathfrak{q}$.

Corollary 3.2. *Suppose that* k *satisfies* NCH, EOD(B). *Let* $n \geq 1$ *be an integer. Then there are infinitely many extensions* k' *of* k *of* $[k' : k] = 2^n$ *such that*

(i) k' *is Galois over* $\mathbb{Q}$, *and*

(ii) k' *satisfies* NCH, EOD(B).

Proof. Let $\mathfrak{q} \in \mathcal{P}_{\mathrm{OD}(B)}(k)$. Applying Proposition 3.1 (2) to $k, \mathfrak{q}$, we obtain the result. □

Theorem 1.1 is a consequence of Theorem 2.3 and Corollary 3.2. More precisely:

Theorem 3.3. *Suppose that* k *satisfies* NCH, EOD(B). *Let* $n \geq 1$ *be an integer. Then there are infinitely many extensions* k' *of* k *of* $[k' : k] = 2^n$ *such that*

(i) k' *is Galois over* $\mathbb{Q}$,

(ii) k' *satisfies* NCH, EOD(B), *and*

(iii) $M_0^B(p)(k') = \emptyset$ *for any prime number* p *satisfying* $p > \min_{\mathfrak{q}' \in \mathcal{P}_{\mathrm{OD}(B)}(k')} 4\mathrm{N}(\mathfrak{q}')$, $p \nmid 13d(B)$, $p \notin \mathcal{N}_1^{new}(k')$.

The rest of this section is devoted to proving Proposition 3.1. For a prime $\mathfrak{q}$ of k and a finite Galois extension k' of k, let $e_{\mathfrak{q}}(k'/k)$ be the ramification index of $\mathfrak{q}$ in k'/k. From now to the end of this section, suppose that the assumption in Proposition 3.1 holds. Let q be the residue characteristic of $\mathfrak{q}$. For a non-zero integer $N \in \mathbb{Z}$, consider the following conditions:

(C1) N is square free.

(C2) $\left(\frac{N}{q}\right) = 1$ if $q \neq 2$; $\quad N \equiv 1 \bmod 8$ if $q = 2$.

(C3) $q \mid N$ if $q \neq 2$; $\quad N \equiv 2, 3 \bmod 4$ if $q = 2$.

Here, $\left(\frac{N}{q}\right) \in \{0, 1, -1\}$ is the Legendre symbol. For an integer $N \in \mathbb{Z} \setminus \{0\}$ satisfying (C1), let

$$W_N := \mathbb{Q}(\sqrt{N}).$$

Note that if two distinct integers $N_1, N_2 \in \mathbb{Z} \setminus \{0\}$ satisfy (C1), then $W_{N_1} \neq W_{N_2}$.

Lemma 3.4. *Let* $N \in \mathbb{Z} \setminus \{0\}$ *be an integer satisfying* (C1).

(1) (i) *We have* (C2) *if and only if* q *splits in* W_N.

(ii) *We have* (C3) *if and only if* q *is ramified in* W_N.

(2) *Let* $\mathfrak{q}'$ *be a prime of the composite field* kW_N *above* $\mathfrak{q}$.

(i) *If* (C2) *holds, then* $\mathfrak{q}'$ *is of odd degree.*

(ii) *If* (C3) *holds and if* $2 \nmid e_q(k/\mathbb{Q})$, *then* $\mathfrak{q}'$ *is of odd degree.*

Proof. (1) Easy.
(2) If $kW_N = k$, then the assertion is trivial. Assume otherwise i.e. $kW_N \neq k$.
(i) The prime $\mathfrak{q}$ splits in kW_N since q splits in W_N. Therefore $\mathfrak{q}'$ is of odd degree.
(ii) By (1) (ii), we have $e_q(W_N/\mathbb{Q}) = 2$. Since $2 \nmid e_q(k/\mathbb{Q})$, we have $e_{\mathfrak{q}}(kW_N/k) = 2$. Therefore $\mathfrak{q}'$ is of odd degree. □

Let $\mathcal{V}_{1;2}$ (resp. $\mathcal{V}_{1;2,3}$) be the set of integers $N \in \mathbb{Z} \setminus \{0\}$ satisfying (C1) and (C2) (resp. (C1) and ((C2) or (C3))). Then $\mathcal{V}_{1;2} \subseteq \mathcal{V}_{1;2,3}$.

Lemma 3.5. *We have* $\sharp\mathcal{V}_{1;2} = \infty$ *and* $\sharp\mathcal{V}_{1;2,3} = \infty$.

Proof. It suffices to prove $\sharp\mathcal{V}_{1;2} = \infty$. This follows from the Dirichlet prime number theorem. □

Let

$$\mathcal{V} := \begin{cases} \mathcal{V}_{1;2} & \text{if } 2 \mid e_q(k/\mathbb{Q}), \\ \mathcal{V}_{1;2,3} & \text{if } 2 \nmid e_q(k/\mathbb{Q}). \end{cases}$$

Then $\sharp\mathcal{V} = \infty$. Let

$$\mathcal{V}_0 := \{\, N \in \mathcal{V} \mid kW_N \text{ satisfies NCH} \,\}.$$

Lemma 3.6.

(1) *We have* $\sharp\mathcal{V}_0 = \infty$.

(2) *We have* $\sharp\{\, kW_N \mid N \in \mathcal{V}_0 \text{ and } kW_N \neq k \,\} = \infty$.

Proof. (1) For any $N \in \mathcal{V} \setminus \mathcal{V}_0$, there is an imaginary quadratic field J_N such that kW_N contains the Hilbert class field H_N of J_N. Since $H_N \not\subseteq k$, we have $k \subsetneq kH_N \subseteq kW_N$. Then $kH_N = kW_N$ and $[kW_N : k] = 2$ because $[W_N : \mathbb{Q}] = 2$. Therefore $h_{J_N} = [H_N : J_N] = \frac{1}{2}[H_N : \mathbb{Q}] \leq \frac{1}{2}[kH_N : \mathbb{Q}] = \frac{1}{2}[kW_N : \mathbb{Q}] = [k : \mathbb{Q}]$. There are only finitely many such imaginary quadratic fields J_N, because their class numbers are bounded above. We also have $kH_N = kW_N \supseteq W_N$. Now, assume $\sharp\mathcal{V}_0 < \infty$. Then $\sharp(\mathcal{V} \setminus \mathcal{V}_0) = \infty$. This implies that finitely many number fields contain infinitely many quadratic fields, which is a contradiction.

(2) The assertion follows from (1). □

Proof of Proposition 3.1. (1) The assertion follows from Lemmas 3.4 (2) and 3.6 (2).

(2) By applying (1) successively, we obtain the result. □

§ 4. Examples not satisfying the assumption

In this section, we give examples of number fields not satisfying the assumption of Theorem 3.3.

Lemma 4.1. *Let p, q be prime numbers. Assume $p \equiv -1 \bmod 8$.*

(1) *The following conditions are equivalent:*

(i) *q splits in $\mathbb{Q}(\sqrt{-p})$.*

(ii) *($q \neq 2$ and $\left(\frac{-p}{q}\right) = 1$) or $q = 2$.*

(iii) $\left(\frac{q}{p}\right) = 1$.

(2) *The following conditions are equivalent:*

(i) *q is ramified in $\mathbb{Q}(\sqrt{-p})$.*

(ii) $q = p$.

(iii) $\left(\frac{q}{p}\right) = 0$.

(3) *The following conditions are equivalent:*

(i) *q is inert in $\mathbb{Q}(\sqrt{-p})$.*

(ii) $\left(\frac{q}{p}\right) = -1$.

Proof. (1) Since $p \equiv -1 \bmod 8$, the prime number 2 splits in $\mathbb{Q}(\sqrt{-p})$ and we have $\left(\frac{2}{p}\right) = 1$. Suppose $q \neq 2$. Then $\left(\frac{-p}{q}\right) = \left(\frac{-1}{q}\right)\left(\frac{p}{q}\right) = (-1)^{\frac{q-1}{2}}\left(\frac{q}{p}\right)(-1)^{\frac{(p-1)(q-1)}{4}} = \left(\frac{q}{p}\right)$. Here, the last equality holds because $p \equiv -1 \bmod 4$. Then the assertion holds.

(2) The discriminant of $\mathbb{Q}(\sqrt{-p})$ is $-p$, as required.

(3) The assertion follows from (1) and (2). □

Let p_1, p_2 be distinct prime numbers satisfying

$$p_1 \equiv -1 \bmod 8, \quad p_2 \equiv -1 \bmod 8,$$

and let

$$k_{p_1,p_2} := \mathbb{Q}(\sqrt{-p_1}, \sqrt{-p_2}).$$

Fix a prime number q. Let e_q (resp. f_q, resp. g_q) be the ramification index of q in $k_{p_1,p_2}/\mathbb{Q}$ (resp. the degree of the residue field extension above q in $k_{p_1,p_2}/\mathbb{Q}$, resp. the number of primes of k_{p_1,p_2} above q).

Lemma 4.2.

(1) *The following conditions are equivalent:*

(i) f_q *is odd.*

(ii) $f_q = 1$.

(iii) $(e_q, f_q, g_q) = (1, 1, 4)$ *or* $(2, 1, 2)$.

(2) *The following conditions are equivalent:*

(i) $(e_q, f_q, g_q) = (1, 1, 4)$.

(ii) q *splits in both* $\mathbb{Q}(\sqrt{-p_1})$ *and* $\mathbb{Q}(\sqrt{-p_2})$.

(iii) $\left(\dfrac{q}{p_1}\right) = \left(\dfrac{q}{p_2}\right) = 1$.

(3) *The following conditions are equivalent:*

(i) $(e_q, f_q, g_q) = (2, 1, 2)$.

(ii) *(q is ramified in $\mathbb{Q}(\sqrt{-p_1})$ and splits in $\mathbb{Q}(\sqrt{-p_2})$) or (q splits in $\mathbb{Q}(\sqrt{-p_1})$ and is ramified in $\mathbb{Q}(\sqrt{-p_2})$).*

(iii) *($\left(\dfrac{q}{p_1}\right) = 0$ and $\left(\dfrac{q}{p_2}\right) = 1$) or ($\left(\dfrac{q}{p_1}\right) = 1$ and $\left(\dfrac{q}{p_2}\right) = 0$).*

Proof. (1) If $(e_q, f_q, g_q) = (4, 1, 1)$, then q is ramified in both $\mathbb{Q}(\sqrt{-p_1})$ and $\mathbb{Q}(\sqrt{-p_2})$. By Lemma 4.1 (2), we have $q = p_1 = p_2$. This is a contradiction. Therefore $(e_q, f_q, g_q) \neq (4, 1, 1)$, and the assertion holds.

(2), (3) The assertion follows from Lemma 4.1 (1), (2). □

Lemma 4.3. *If $d(B) = p_1p_2$, then the following conditions are equivalent:*

(i) $B \otimes_{\mathbb{Q}} \mathbb{Q}(\sqrt{-q}) \ncong \mathrm{M}_2(\mathbb{Q}(\sqrt{-q}))$.

(ii) *At least one of p_1, p_2 splits in $\mathbb{Q}(\sqrt{-q})$.*

(iii) $\left(\dfrac{-q}{p_1}\right) = 1$ *or* $\left(\dfrac{-q}{p_2}\right) = 1$.

(iv) $\left(\dfrac{q}{p_1}\right) = -1$ *or* $\left(\dfrac{q}{p_2}\right) = -1$.

Proof. For $i = 1, 2$, we have $\left(\dfrac{-1}{p_i}\right) = -1$ since $p_i \equiv -1 \bmod 4$. Then (iii) and (iv) are equivalent. The rest is obvious (cf. Lemma 3.4 (1) (i)). □

Concerning EOD(B), we have:

Proposition 4.4. *If $d(B) = p_1p_2$, then k_{p_1,p_2} does not satisfy* EOD(B).

Proof. The assertion follows from Lemmas 4.2 and 4.3. □

As for NCH, we have:

Proposition 4.5. *The following conditions are equivalent:*

(i) k_{p_1,p_2} *does not satisfy* NCH.

(ii) $h_{\mathbb{Q}(\sqrt{-p_1})} = 1$ *or* $h_{\mathbb{Q}(\sqrt{-p_2})} = 1$.

(iii) $p_1 = 7$ *or* $p_2 = 7$.

Proof. [(i)$\Longrightarrow$(ii)] Let $H(i)$ be the Hilbert class field of $\mathbb{Q}(\sqrt{-p_i})$ for $i = 1, 2$. By the assumption, we have $k_{p_1,p_2} \supseteq H(1)$ or $H(2)$. We may assume $k_{p_1,p_2} \supseteq H(1)$. Suppose $h_{\mathbb{Q}(\sqrt{-p_1})} \neq 1$. Then $H(1) \supsetneq \mathbb{Q}(\sqrt{-p_1})$, and so $k_{p_1,p_2} = H(1)$. Let $\mathfrak{p}_2$ be a prime of $\mathbb{Q}(\sqrt{-p_1})$ above p_2. Since p_2 is unramified in $\mathbb{Q}(\sqrt{-p_1})/\mathbb{Q}$, the prime $\mathfrak{p}_2$ is ramified in $k_{p_1,p_2}/\mathbb{Q}(\sqrt{-p_1})$. This contradicts $k_{p_1,p_2} = H(1)$. Therefore $h_{\mathbb{Q}(\sqrt{-p_1})} = 1$.

[(ii)$\Longrightarrow$(i)] Clear.

[(ii)$\Longleftrightarrow$(iii)] All the imaginary quadratic fields of class number one are $\mathbb{Q}(\sqrt{-N})$, where $N \in \{1, 2, 3, 7, 11, 19, 43, 67, 163\}$. Then the assertion holds. □

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References

[1] Arai, K., An effective bound of p for algebraic points on Shimura curves of $\Gamma_0(p)$-type, *Acta Arith.* **164** (2014), 343–353.

[2] Arai, K., Algebraic points on Shimura curves of $\Gamma_0(p)$-type (II), *Manuscripta Math.*, published online.

[3] Arai, K., Algebraic points on Shimura curves of $\Gamma_0(p)$-type (III), *preprint*, available at the web page (http://arxiv.org/pdf/1303.5270v1.pdf).
[4] Arai, K. and Momose, F., Algebraic points on Shimura curves of $\Gamma_0(p)$-type, *J. Reine Angew. Math.* **690** (2014), 179–202.
[5] González, J. and Rotger, V., Non-elliptic Shimura curves of genus one, *J. Math. Soc. Japan* **58** (2006), no. 4, 927–948.
[6] Shimura, G., On the real points of an arithmetic quotient of a bounded symmetric domain, *Math. Ann.* **215** (1975), 135–164.