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ASYMPTOTIC LINEAR STABILITY OF BENNEY-LUKE LINE SOLITARY WAVES IN 2D

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The Benney-Luke equation is an approximation model of small amplitude long water waves with finite depth originally derived by Benney and Luke [1] as a model for 3D water waves. Let L be the horizontal length scale of motion and h be the depth of water. Suppose that the amplitude parameter ϵ and the long-wave parameter $\mu=(h/L)^2$ are small and $\epsilon=\mu$. Then the water wave equation

$$\tag{1} \begin{cases} \eta_t+\nabla\phi\cdot\nabla\eta=\phi_z \quad \text{for } z=h+\eta\,, \\ \phi_t+\dfrac{1}{2}(|\nabla\phi|^2+(\phi_z)^2)+g\eta=0 \quad \text{for } z=h+\eta\,, \\ \Delta\phi+\phi_{zz}=0 \quad \text{for } 0<z<h+\eta\,, \\ \phi_z(x,y,0)=0\,, \end{cases}$$

can be reduced to the Benney-Luke equation

$$\tag{2} \partial_t^2\Phi-\Delta\Phi+\mu(a\Delta^2\Phi-b\Delta\partial_t^2\Phi)+\epsilon\{(\partial_t\Phi)(\Delta\Phi)+\partial_t(|\nabla\Phi|^2)\}=0 \quad \text{on } \mathbb{R}\times\mathbb{R}^2$$

with $a=1/6$ and $b=1/2$. Here ϕ is the velocity potential of the water and the variable Φ is the nondimensional velocity potential on the bottom satisfying

$$\phi(t,x,y,0)=\epsilon L\sqrt{gh}\Phi\left(\frac{\sqrt{gh}}{L}t,\frac{x}{L},\frac{y}{L}\right).$$

The parameters a and b should be positive and satisfy $a-b=\sigma-1/3$, where σ is the Bond number. See [1] and also [14, 20] for the derivation of (2) from the water wave equation with surface tension.

We remark that (2) is an isotropic model for propagation of water waves whereas KdV, BBM and KP equations are unidirectional models. See e.g. [2, 3, 4] for the other bidirectional models of 2D and 3D water waves. Since the Benney-Luke equation is isotropic, it could be more useful to describe nonlinear interactions of waves at high angle than the KP equations.

The solution $\Phi(t)$ of the Benney-Luke equation (2) formally satisfies the energy conservation law

$$\tag{3} E(\Phi(t),\partial_t\Phi(t))=E(\Phi_0,\Psi_0) \quad \text{for } t\in\mathbb{R},$$

where

$$E(\Phi,\Psi):=\int_{\mathbb{R}^2}\left\{|\nabla\Phi|^2+\mu a(\Delta\Phi)^2+\Psi^2+\mu b|\nabla\Psi|^2\right\}dxdy\,,$$

and (2) is globally well-posed in the energy class $(\dot{H}^2(\mathbb{R}^2)\cap\dot{H}^1(\mathbb{R}^2))\times H^1(\mathbb{R}^2)$ (see [26]). If $a>b>0$, then (2) has a stable ground state for c satisfying $0<c^2<1$ ([20, 25]).

For the sake of simplicity, we let $\epsilon=\mu=1$ in (2) by using the change of variable $\mu^{-1/2}(t,x,y)\mapsto(t,x,y)$ and $\mu^{-1/2}\epsilon\Phi\mapsto\Phi$.

The Benney Luke equation (2) has a 3-parameter family of line solitary wave solutions

$$\Phi(t,x,y)=\varphi_c(x\cos\theta+y\sin\theta-ct+\gamma)\,,\quad \pm c>1\,,\quad \gamma\in\mathbb{R}\,,\quad \theta\in[0,2\pi)\,, \tag{4}$$

where

$$\varphi_c(x)=\frac{2(c^2-1)}{c\alpha_c}\tanh(\frac{\alpha_c}{2}x)\,,\quad \alpha_c=\sqrt{\frac{c^2-1}{bc^2-a}}\,,$$

and

$$q_c(x):=\varphi_c'(x)=\frac{c^2-1}{c}\operatorname{sech}^2(\frac{\alpha_c x}{2})$$

is a solution of

$$(bc^2-a)q_c''-(c^2-1)q_c+\frac{3c}{2}q_c^2=0\,. \tag{5}$$

In this article, we report transverse linear stability of the line solitary waves in the weak surface tension case ($0<a<b$). In view of [27, 28], line solitary waves are expected to be unstable if $0<b<a$ and $0<c^2<1$. Stability of solitary waves to the 1-dimensional Benney-Luke equation is studied by [24] for the strong surface tension case $a>b>0$ and by [18] for the weak surface tension case $b>a>0$.

Since (2) is isotropic and translation invariant, we may assume $\theta=\gamma=0$ in (4) without loss of generality. Let $\Psi=\partial_t\Phi$, $A=I-a\Delta$ and $B=I-b\Delta$. Then in the moving coordinate $z=x-ct$, the Benney-Luke equation (2) can be rewritten as

$$\begin{cases}\partial_t\Phi=c\partial_z\Phi+\Psi\,,\\ \partial_t\Psi=c\partial_z\Psi+B^{-1}A\Delta\Phi-B^{-1}(\Psi\Delta\Phi+2\nabla\Phi\cdot\nabla\Psi)\,,\end{cases} \tag{6}$$

Let $r_c(z)=-cq_c(z)$. Linearizing (6) around $(\Phi,\Psi)=(\varphi_c(z),r_c(z))$, we have

$$\partial_t\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}=\mathcal{L}\begin{pmatrix}\Phi\\ \Psi\end{pmatrix}\,, \tag{7}$$

$$\mathcal{L}=\mathcal{L}_0+V\,,\quad \mathcal{L}_0=\begin{pmatrix}c\partial_z & 1\\ B^{-1}A\Delta & c\partial_z\end{pmatrix}\,,$$

$$V=-B^{-1}\begin{pmatrix}0 & 0\\ v_{1,c} & v_{2,c}\end{pmatrix}\,,\quad v_{1,c}=2r_c'(z)\partial_z+r_c(z)\Delta\,,\quad v_{2,c}=2q_c(z)\partial_z+q_c'(z)\,. \tag{8}$$

Before we state our results, we introduce several notations. For an operator A, we denote by $\sigma(A)$ the spectrum of the operator A. For a Banach space X, let $B(X)$ be the space of all linear continuous operators from X to itself and $\|T\|_{B(X)}=\sup_{\|x\|_X=1}\|Tu\|_X$.

Let $L^2_\alpha(\mathbb{R}^2)=L^2(\mathbb{R}^2;e^{2\alpha x}dxdy)$, $L^2_\alpha(\mathbb{R})=L^2(\mathbb{R};e^{2\alpha x}\,dx)$ and let $H^k_\alpha(\mathbb{R}^2)$ and $H^k_\alpha(\mathbb{R})$ be Hilbert spaces with the norms

$$\|u\|_{H^k_\alpha(\mathbb{R}^2)}=\left(\|\partial_x^k u\|^2_{L^2_\alpha(\mathbb{R}^2)}+\|\partial_y^k u\|^2_{L^2_\alpha(\mathbb{R}^2)}+\|u\|^2_{L^2_\alpha(\mathbb{R}^2)}\right)^{1/2}\,,$$

$$\|u\|_{H^k_\alpha(\mathbb{R})}=\left(\|\partial_x^k u\|^2_{L^2_\alpha(\mathbb{R})}+\|u\|^2_{L^2_\alpha(\mathbb{R})}\right)^{1/2}\,.$$

We consider linear stability of (7) in a weighted space $X:=H^1_\alpha(\mathbb{R}^2)\times L^2_\alpha(\mathbb{R}^2)$. Let $\mathcal{L}(\eta)u(z)=e^{-iy\eta}\mathcal{L}(e^{iy\eta}u(z))$ for $\eta\in\mathbb{R}$. Note that V is independent of y. For each small $\eta\neq 0$, the operator $\mathcal{L}(\eta)$ has two stable eigenvalues.

Theorem 1. ([19, Theorem 2.1]) *Let $0 < a < b$ and $k \in \mathbb{N}$. Fix $c > 1$ and $\alpha \in (0, \alpha_c)$. Then there exist a positive constant η_0 and functions $\lambda(\eta) \in C^\infty([-\eta_0, \eta_0])$,*

$$\zeta(\cdot,\eta) \in C^\infty([-\eta_0,\eta_0]; H^k_\alpha(\mathbb{R}) \times H^{k-1}_\alpha(\mathbb{R}))\,, \quad \zeta^*(\cdot,\eta) \in C^\infty([-\eta_0,\eta_0]; H^k_{-\alpha}(\mathbb{R}) \times H^{k-1}_{-\alpha}(\mathbb{R}))$$

such that

$$\mathcal{L}(\eta)\zeta(z,\eta) = \lambda(\eta)\zeta(z,\eta)\,, \quad \mathcal{L}(\eta)^*\zeta^*(z,\eta) = \lambda(-\eta)\zeta^*(z,\eta)\,,$$

$$\lambda(\eta) = i\lambda_1\eta - \lambda_2\eta^2 + O(\eta^3)\,, \tag{9}$$

$$\zeta(\cdot,\eta) = \zeta_1 + i\lambda_1\eta\zeta_2 + O(\eta^2) \quad \text{in } H^k_\alpha(\mathbb{R}) \times H^{k-1}_\alpha(\mathbb{R}), \tag{10}$$

$$\zeta^*(\cdot,\eta) = \zeta_2^* - i\lambda_1\eta\zeta_1^* + O(\eta^2) \quad \text{in } H^k_{-\alpha}(\mathbb{R}) \times H^{k-1}_{-\alpha}(\mathbb{R}), \tag{11}$$

$$\overline{\lambda(\eta)} = \lambda(-\eta)\,, \ \overline{\zeta(z,\eta)} = \zeta(z,-\eta)\,, \ \overline{\zeta^*(z,\eta)} = \zeta^*(z,-\eta) \quad \text{for } \eta \in [-\eta_0,\eta_0] \text{ and } z \in \mathbb{R}, \tag{12}$$

where λ_1 and λ_2 are positive constants, $A_0 = 1 - a\partial_z^2$, $B_0 = 1 - b\partial_z^2$ and

$$\zeta_1 = \begin{pmatrix} q_c \\ r_c' \end{pmatrix}, \quad \zeta_2 = \begin{pmatrix} \int_z^\infty \partial_c q_c \\ -\partial_c r_c \end{pmatrix},$$

$$\zeta_1^* = c\begin{pmatrix} -B_0\partial_c r_c - 2q_c\partial_c q_c - q_c' \int_{-\infty}^z \partial_c q_c \\ B_0 \int_{-\infty}^z \partial_c q_c \end{pmatrix}, \quad \zeta_2^* = \begin{pmatrix} A_0 q_c' \\ -B_0 r_c \end{pmatrix}.$$

Remark 1. We remark that $\mathcal{L}(0)$ is a linearized operator of the 1-dimensional Benney-Luke equation around $\varphi_c(z)$ and that ζ_1 and ζ_2 belong to the generalized kernel of $\mathcal{L}(0)$. More precisely,

$$\mathcal{L}(0)\zeta_1 = 0\,, \quad \mathcal{L}(0)\zeta_2 = \zeta_1\,, \quad \mathcal{L}(0)^*\zeta_1^* = \zeta_2^*\,, \quad \mathcal{L}(0)^*\zeta_2^* = 0\,,$$

$$\ker_g(\mathcal{L}(0)) := \cup_{j=1}^\infty \ker\left(\mathcal{L}(0)^j\right) = \operatorname{span}\{\zeta_1, \zeta_2\}\,,$$

$$\ker_g(\mathcal{L}(0)^*) := \cup_{j=1}^\infty \ker\left((\mathcal{L}(0)^*)^j\right) = \operatorname{span}\{\zeta_1^*, \zeta_2^*\}\,,$$

in a weighted space $L^2_\alpha(\mathbb{R})$ with $\alpha \in (0, \alpha_c)$. The eigenvalue $\lambda = 0$ for $\mathcal{L}(0)$ splits into two stable eigenvalues $\lambda(\pm\eta)$ for $\mathcal{L}(\eta)$ with $\eta \neq 0$.

In the exponentially weighted space $H^1_\alpha(\mathbb{R}) \times L^2_\alpha(\mathbb{R})$, the value $\lambda = 0$ is an isolated eigenvalue of $\mathcal{L}(0)$ and there exists a $\beta > 0$ such that

$$\sigma(\mathcal{L}(0)) \setminus \{0\} \subset \{\lambda \in \mathbb{C} \mid \Re\lambda \le -\beta\}$$

provided $c > 1$ and c is sufficiently close to 1. See Lemma 2.1, Theorem 2.3 and Appendix B in [18].

Remark 2. For the KP-II equation, the spectrum of the linearized operator around a 1-line soliton near $\lambda = 0$ can be obtained explicitly thanks to the integrability of the equation (see [5, 17]). In [19], we use the Lyapunov-Schmidt method to find resonant eigenmodes of the linearized operator.

Remark 3. The eigenfunctions $\zeta_k(\cdot,\eta)$ and $\zeta_k^*(\cdot,\eta)$ ($k = 1, 2$) do not belong to $H^1(\mathbb{R}) \times L^2(\mathbb{R})$ because they are exponentially growing as $z \to -\infty$. This is a reason why we study spectral stability of $\mathcal{L}$ in the exponentially weighted space X.

Let α be a small positive number. Then there exist an $\eta_0 > 0$ and $\mathcal{P}(\eta_0) \in B(H^1_\alpha(\mathbb{R}^2) \times L^2_\alpha(\mathbb{R}^2))$ such that $P(\eta_0)$ is a spectral projection onto the subspace corresponding to the continuous eigenvalues $\{\lambda(\eta)\}_{-\eta_0 \le \eta \le \eta_0}$. Let $\mathcal{Q}(\eta_0) = I - \mathcal{P}(\eta_0)$ and $Z = \mathcal{Q}(\eta_0)(H^1_\alpha(\mathbb{R}^2) \times L^2_\alpha(\mathbb{R}^2))$. If $\mathcal{L}$ is spectrally stable, then the restriction of $e^{t\mathcal{L}}$ to Z is exponentially stable.

Theorem 2. ([19, Theorem 2.2]) *Let $0 < a < b$, $c > 1$ and $\alpha \in (0, \alpha_c)$. Consider the operator $\mathcal{L}$ in the space $X = H^1_\alpha(\mathbb{R}^2) \times L^2_\alpha(\mathbb{R}^2)$. Assume that there exist positive constants β and η_0 such that*

$$\text{(H)} \qquad \sigma(\mathcal{L}|_Z) \subset \{\lambda \mid \Re\lambda \le -\beta\},$$

where $\mathcal{L}|_Z$ is the restriction of the operator $\mathcal{L}$ to Z. Then for any $\beta' < \beta$, there exists a positive constant C such that

$$\|e^{t\mathcal{L}}\mathcal{Q}(\eta_0)\|_{B(X)} \le Ce^{-\beta' t} \quad \text{for any } t \ge 0. \tag{13}$$

The semigroup estimate (13) follows from the assumption (H) and the Geahart-Prüss theorem [9, 23] which tells us that the boundedness of C^0-semigroup in a Hilbert space is equivalent to the uniform boundedness of the resolvent operator on the right half plane. See also [10, 11, 12].

Time evolution of the continuous eigenmodes $\{e^{t\lambda(\eta)+iy\eta}g(z,\eta)\}_{-\eta_0\le\eta\le\eta_0}$ can be considered as a linear approximation of non-uniform phase shifts of modulating line solitary waves. For the KP-II equation, modulations of the local amplitude and the angle of the local phase shift of a line soliton are described by a system of Burgers' equations (see [17, Theorems 1.4 and 1.5]). In [19], we find the first order asymptotics of solutions for the linearized equation (7) is described by a wave equation with a diffraction term and it tends to a constant multiple of the x-derivative of the line solitary wave as $t \to \infty$.

Theorem 3. ([19, Theorem 2.3]) *Let $0 < a < b$, $c > 1$, α be as in Theorem 2 and $(\Phi_0, \Psi_0) \in H^2_\alpha(\mathbb{R}^2) \times H^1_\alpha(\mathbb{R}^2)$. Assume* (H). *Then a solution of* (7) *with $(\Phi(0), \partial_t\Phi(0)) = (\Phi_0, \Psi_0)$ satisfies*

$$\left\| \begin{pmatrix} \partial_z\Phi(t,z,y) \\ \partial_t\Phi(t,z,y) \end{pmatrix} - (H_t * W_t * f)(y) \begin{pmatrix} q_c'(z) \\ r_c'(z) \end{pmatrix} \right\|_{L^2_\alpha(\mathbb{R}_z)L^\infty(\mathbb{R}_y)} = O(t^{-1/4}) \quad \text{as } t \to \infty,$$

where $f(y) = \langle cB_0\Psi_0(\cdot,y) - A_0\partial_z\Phi_0(\cdot,y), q_c(\cdot)\rangle_{L^2(\mathbb{R})}$, $H_t(y) = (4\pi\lambda_2 t)^{-1/2}e^{-y^2/4\lambda_2 t}$, $\kappa_1 = \frac{\lambda_1}{2}\frac{d}{dc}E(q_c, r_c)$ and $W_t(y) = (2\kappa_1)^{-1}$ for $y \in [-\lambda_1 t, \lambda_1 t]$ and $W_t(y) = 0$ otherwise.

We remark that if $f(y)$ is well localized and $\int_{\mathbb{R}} f(y)\,dy \ne 0$, then $H_t * W_t * f(y) \simeq (2\kappa_1)^{-1}\int_{\mathbb{R}} f(y)\,dy$ on any compact intervals in y as $t \to \infty$. The first order asymptotics of solutions to (7) suggests that the local phase shift of line solitary waves propagates mostly at constant speed toward $y = \pm\infty$.

If $c > 1$ and close to 1, then the assumption (H) is valid and the spectrum of $\mathcal{L}$ near 0 is similar to that of the linearized KP-II operator. To be more precise, let us introduce the scaled parameters and variables

$$\lambda = \epsilon^3\Lambda, \quad c^2 = 1+\epsilon^2, \quad \hat{z} = \epsilon z, \quad \hat{y} = \epsilon^2 y, \quad \xi = \epsilon\hat{\xi}, \quad \eta = \epsilon^2\hat{\eta}, \tag{14}$$

and translate the solitary wave profile $q_c(x)$ as

$$q_c(z) = \epsilon^2\theta_\epsilon(\hat{z}), \quad \theta_\epsilon(\hat{z}) = \frac{1}{c}\operatorname{sech}^2\left(\frac{\hat{\alpha}_\epsilon\hat{z}}{2}\right), \quad \hat{\alpha}_\epsilon = \frac{1}{\sqrt{bc^2 - a}}. \tag{15}$$

Let

$$\hat{\alpha}_0 = (b-a)^{-1/2}, \quad \theta_0(\hat{z}) = \operatorname{sech}^2(\frac{\hat{\alpha}_0}{2}\hat{z}),$$

$$\mathcal{L}_{KP} = -\frac{1}{2}\{(b-a)\partial_{\hat{z}}^3 - \partial_{\hat{z}} + \partial_{\hat{z}}^{-1}\partial_{\hat{y}}^2 + 3\partial_{\hat{z}}(\theta_0\cdot)\}.$$

We remark that the operator $\mathcal{L}_{KP}$ is the linearization of the KP-II equation

$$2\partial_t u + (b-a)\partial_x^3 u + \partial_x^{-1}\partial_y^2 u + \frac{3}{2}\partial_x(u^2) = 0 \tag{16}$$

around its line soliton solution $\theta_0(x - \frac{t}{2})$. The linearized operator $\mathcal{L}_{KP}$ has continuous eigenvalues $\lambda_{KP}(\eta) = \frac{i\eta}{\sqrt{3}}\sqrt{1 + i\gamma_1\eta}$ in $L^2(\mathbb{R}^2; e^{2\hat{\alpha}_0 x}\,dxdy)$.

Theorem 4. ([19, Theorem 2.4]) *Let $c = \sqrt{1+\epsilon^2}$, $\alpha = \hat{\alpha}\epsilon$ and $\hat{\alpha} \in (0, \hat{\alpha}_0/2)$. Then there exist positive constants ϵ_0, η_0, $\hat{\beta}$ and a smooth function $\lambda_\epsilon(\eta)$ such that if $\epsilon \in (0, \epsilon_0)$, then*

$$\sigma(\mathcal{L}) \setminus \{\lambda_\epsilon(\eta) \mid \eta \in [-\epsilon^2\eta_0, \epsilon^2\eta_0]\} \subset \{\lambda \in \mathbb{C} \mid \Re\lambda \le -\hat{\beta}\epsilon^3\}\,, \tag{17}$$

$$\lim_{\epsilon \downarrow 0} |\epsilon^{-3}\lambda_\epsilon(\epsilon^2\eta) - \lambda_{KP}(\eta)| = O(\eta^3) \quad \text{for } \eta \in [-\eta_0, \eta_0], \tag{18}$$

$$\|e^{t\mathcal{L}}\mathcal{Q}(\epsilon^2\eta_0)\|_{B(X)} \le K e^{-\hat{\beta}\epsilon^3 t} \quad \text{for any } t \ge 0, \tag{19}$$

where K is a constant that does not depend on t.

Finally, we will explain the strategy to prove Theorem 4. Since the dispersion relation for the linearization of (2) around 0 is

$$\omega^2 = (\xi^2 + \eta^2)\frac{1 + a(\xi^2+\eta^2)}{1 + b(\xi^2+\eta^2)}\,, \quad |\nabla\omega| \le 1\,,$$

and X is an exponentially weighted space whose weight function is biased in the direction of motion of a line solitary wave, we have $\|e^{t\mathcal{L}_0}\|_{B(X)} \lesssim e^{-\alpha(c-1)t}$ for $t \ge 0$ and $\sigma(\mathcal{L}_0) \subset \{\lambda \in \mathbb{C} \mid \Re\lambda \le -\beta\epsilon^3\}$ for a $\beta > 0$. To prove Theorem 4, we need to take the influence of the potential V into account. Since $\lim_{(\xi,\eta)\to(0,0)} |\nabla\omega(\xi,\eta)| = 1$ and $\nabla\omega(\xi,\eta) \parallel (\xi,\eta)$, we see that $c - \omega_\xi(\xi,\eta)$ is smaller in the frequency regime

$$A_{low} = \{(\xi,\eta) \mid |\xi| \lesssim \epsilon^{1-0}\,,\ |\eta| \lesssim \epsilon^{2-0}\}$$

than in any other region and the effect of the potential is negligible in the frequency regime A_{low}^c. In A_{low}, we can deduce the eigenvalue problem

$$\mathcal{L}\begin{pmatrix} u \\ v \end{pmatrix} = \lambda \begin{pmatrix} u \\ v \end{pmatrix} \tag{20}$$

to $\mathcal{L}_{KP}\partial_{\tilde{z}} u = \Lambda \partial_{\tilde{z}} u$ and make use of the spectral stability results for the KP-II equation ([17]).

We remark that for 1-dimensional long wave models, non-existence of unstable modes for the linearized operator around solitary waves has been proved by utilizing spectral stability of KdV solitons (e.g. [7, 15, 16, 18, 21, 22]) and [19] is the first result which proves linear stability of line solitary waves making use of the spectral stability of KP-II line solitons.

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