AN ABSTRACT FOR "THE SEMI-ABSOLUTE ANABELIAN GEOMETRY OF GEOMETRICALLY PRO-P ARITHMETIC FUNDAMENTAL GROUPS OF ASSOCIATED LOW-DIMENSIONAL CONFIGURATION SPACES"

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Let $n \in \mathbb{Z}_{>1}$; (g, r) a pair of nonnegative integers such that $2g-2+r > 0$; p a prime number; k a number field or a p-adic local field; $X^{\log}$ a smooth log curve over k of type (g, r). In the present paper, we study the n-th log configuration space $X_n^{\log}$ associated to $X^{\log} \to \mathrm{Spec}(k)$. Write U_S for the interior of a log scheme $S^{\log}$. The log scheme $X_n^{\log}$ may be thought of as a compactification of the usual n-th configuration space U_{X_n} associated to the smooth curve U_X. It is known that the function field of U_X may be reconstructed group-theoretically

- from its profinite arithmetic fundamental group whenever U_X is of strictly Belyi type or,
- from its geometrically pro-Σ arithmetic fundamental group, where Σ is a set of prime numbers of cardinality ≥ 2 that contains p, equipped with the auxiliary data constituted by the collection of decomposition groups associated to the closed points of U_X, regardless of whether or not U_X is of strictly Belyi type.

By contrast, in the present paper, we reconstruct the function field of U_X group-theoretically from various geometrically pro-p arithmetic fundamental groups associated to U_{X_n}, equipped with the auxiliary data constituted by the collection of decomposition groups associated to the closed points of the underlying scheme X_n of $X_n^{\log}$.

Our main result is as follows:

Theorem 1. (Semi-absolute bi-anabelian formulation) *Let $n \in \mathbb{Z}_{>1}$; (g, r) a pair of nonnegative integers such that $2g-2+r > 0$; $\Sigma_\Delta, \Sigma_{\mathrm{Gal}}$ sets of prime numbers such that $\Sigma_\Delta \subseteq \Sigma_{\mathrm{Gal}}$, and $\Sigma_\Delta, \Sigma_{\mathrm{Gal}}$ are of cardinality 1 or equal to the set of prime numbers. Let $\mathscr{B} = (\Pi_n, G, \mathcal{D}_n)$ be a PGCS-collection of type $(g, r, n, \Sigma_\Delta, \Sigma_{\mathrm{Gal}})$. That is to say, Π_n is a profinite group; G is a quotient of Π_n; $\mathcal{D}_n$ is a set of subgroups of Π_n; there exist a prime number $p \in \Sigma_\Delta$, a generalized sub-p-adic local field k, an algebraic closure $\bar{k}$ of k, a smooth log curve $X^{\log}$ over k of type (g, r), and an isomorphism*

$$\alpha\colon \Pi_n \xrightarrow{\sim} \Pi_{U_{X_n}} \stackrel{\mathrm{def}}{=} \begin{cases} \pi_1(U_{X_n})^{\Sigma_\Delta} & (\text{if } \Sigma_\Delta = \Sigma_{\mathrm{Gal}}) \\ \pi_1(U_{X_n})^{[p]} & (\text{if } \Sigma_\Delta \subsetneq \Sigma_{\mathrm{Gal}}) \end{cases}$$

— where $\pi_1(U_{X_n})^{\Sigma_\Delta}$ denotes the maximal pro-Σ_Δ quotient of $\pi_1(U_{X_n})$, and $\pi_1(U_{X_n})^{[p]}$ denotes the maximal geometrically pro-p quotient of $\pi_1(U_{X_n})$ — such that, if we write $G_k \stackrel{\mathrm{def}}{=} \mathrm{Gal}(\bar{k}/k)$ and $K \subseteq \bar{k}$ for the maximal pro-Σ_{Gal} subextension of $\bar{k}/k$ (so $G_k^{\Sigma_{\mathrm{Gal}}} = \mathrm{Gal}(K/k)$), then the natural outer action $G_k \stackrel{\mathrm{out}}{\curvearrowright} \pi_1(U_{X_n} \times_k \bar{k})^{\Sigma_\Delta}$ factors

through the natural surjection $G_k \twoheadrightarrow G_k^{\Sigma_{\mathrm{Gal}}}$*, and* α *induces a commutative diagram*

$$\begin{array}{ccc} \Pi_n & \xrightarrow[\alpha]{\sim} & \Pi_{U_{X_n}} \\ \downarrow & \circlearrowright & \downarrow \\ G & \overset{\sim}{\dashrightarrow} & G_k^{\Sigma_{\mathrm{Gal}}}, \end{array}$$

where the lower horizontal arrow is an isomorphism, as well as a bijection

$$\mathcal{D}_n \xrightarrow{\sim} \{D \subseteq \Pi_{U_{X_n}} \mid D \text{ is a decomposition group associated to some } x \in X_n(K)\}.$$

Suppose that $X(k)$ *is a nonempty set, and that* (g, r, n) *is tripodally ample, i.e., one of the following conditions (i), (ii), (iii) holds:*

$$\text{(i) } n \in \mathbb{Z}_{>3}; \quad \text{(ii) } n \in \mathbb{Z}_{>2}, r \neq 0; \quad \text{(iii) } (g, r, n) = (0, 3, 2).$$

Write $\mathrm{Aut}(U_{X_n})$ *for the set of automorphisms of the scheme* U_{X_n} *and* $\mathrm{Aut}(\mathscr{B})$ *for the set of automorphisms of the PGCS-collection* $\mathscr{B}$*, considered up to composition with an inner automorphism arising from* $\mathrm{Ker}(\Pi_n \twoheadrightarrow G)$*. Then any* α *as above induces a bijection*

$$\mathrm{Aut}(U_{X_n}) \xrightarrow{\sim} \mathrm{Aut}(\mathscr{B}).$$