## AN ABSTRACT FOR "THE SEMI-ABSOLUTE ANABELIAN GEOMETRY OF GEOMETRICALLY PRO-P ARITHMETIC FUNDAMENTAL GROUPS OF ASSOCIATED LOW-DIMENSIONAL CONFIGURATION SPACES"

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Let  $n \in \mathbb{Z}_{>1}$ ; (g, r) a pair of nonnegative integers such that 2g - 2 + r > 0; p a prime number; k a number field or a p-adic local field;  $X^{\log}$  a smooth log curve over k of type (g, r). In the present paper, we study the n-th log configuration space  $X_n^{\log}$  associated to  $X^{\log} \to \operatorname{Spec}(k)$ . Write  $U_S$  for the interior of a log scheme  $S^{\log}$ . The log scheme  $X_n^{\log}$  may be thought of as a compactification of the usual n-th configuration space  $U_{X_n}$  associated to the smooth curve  $U_X$ . It is known that the function field of  $U_X$  may be reconstructed group-theoretically

- from its profinite arithmetic fundamental group whenever  $U_X$  is of strictly Belyi type or,
- from its geometrically pro- $\Sigma$  arithmetic fundamental group, where  $\Sigma$  is a set of prime numbers of cardinality  $\geq 2$  that contains p, equipped with the auxiliary data constituted by the collection of decomposition groups associated to the closed points of  $U_X$ , regardless of whether or not  $U_X$  is of strictly Belyi type.

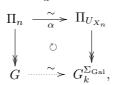
By contrast, in the present paper, we reconstruct the function field of  $U_X$  grouptheoretically from various geometrically pro-*p* arithmetic fundamental groups associated to  $U_{X_n}$ , equipped with the auxiliary data constituted by the collection of decomposition groups associated to the closed points of the underlying scheme  $X_n$ of  $X_n^{\log}$ .

Our main result is as follows:

**Theorem 1. (Semi-absolute bi-anabelian formulation)** Let  $n \in \mathbb{Z}_{>1}$ ; (g, r) a pair of nonnegative integers such that 2g-2+r > 0;  $\Sigma_{\Delta}$ ,  $\Sigma_{\text{Gal}}$  sets of prime numbers such that  $\Sigma_{\Delta} \subseteq \Sigma_{\text{Gal}}$ , and  $\Sigma_{\Delta}$ ,  $\Sigma_{\text{Gal}}$  are of cardinality 1 or equal to the set of prime numbers. Let  $\mathscr{B} = (\Pi_n, G, \mathcal{D}_n)$  be a PGCS-collection of type  $(g, r, n, \Sigma_{\Delta}, \Sigma_{\text{Gal}})$ . That is to say,  $\Pi_n$  is a profinite group; G is a quotient of  $\Pi_n$ ;  $\mathcal{D}_n$  is a set of subgroups of  $\Pi_n$ ; there exist a prime number  $p \in \Sigma_{\Delta}$ , a generalized sub-p-adic local field k, an algebraic closure  $\overline{k}$  of k, a smooth log curve  $X^{\log}$  over k of type (g, r), and an isomorphism

$$\alpha \colon \Pi_n \xrightarrow{\sim} \Pi_{U_{X_n}} \stackrel{\text{def}}{=} \begin{cases} \pi_1 (U_{X_n})^{\Sigma_\Delta} & (if \ \Sigma_\Delta = \Sigma_{\text{Gal}}) \\ \pi_1 (U_{X_n})^{[p]} & (if \ \Sigma_\Delta \subsetneq \Sigma_{\text{Gal}}) \end{cases}$$

- where  $\pi_1(U_{X_n})^{\Sigma_{\Delta}}$  denotes the maximal pro- $\Sigma_{\Delta}$  quotient of  $\pi_1(U_{X_n})$ , and  $\pi_1(U_{X_n})^{[p]}$ denotes the maximal geometrically pro-p quotient of  $\pi_1(U_{X_n})$  — such that, if we write  $G_k \stackrel{\text{def}}{=} \operatorname{Gal}(\bar{k}/k)$  and  $K \subseteq \bar{k}$  for the maximal pro- $\Sigma_{\operatorname{Gal}}$  subextension of  $\bar{k}/k$  (so  $G_k^{\Sigma_{\operatorname{Gal}}} = \operatorname{Gal}(K/k)$ ), then the natural outer action  $G_k \stackrel{\text{out}}{\frown} \pi_1(U_{X_n} \times_k \bar{k})^{\Sigma_{\Delta}}$  factors through the natural surjection  $G_k \twoheadrightarrow G_k^{\Sigma_{\text{Gal}}}$ , and  $\alpha$  induces a commutative diagram



where the lower horizontal arrow is an isomorphism, as well as a bijection

 $\mathcal{D}_n \xrightarrow{\sim} \{D \subseteq \Pi_{U_{X_n}} \mid D \text{ is a decomposition group associated to some } x \in X_n(K)\}.$ Suppose that X(k) is a nonempty set, and that (g, r, n) is tripodally ample, i.e., one of the following conditions (i), (ii), (iii) holds:

(i)  $n \in \mathbb{Z}_{>3}$ ; (ii)  $n \in \mathbb{Z}_{>2}, r \neq 0$ ; (iii) (g, r, n) = (0, 3, 2).

Write  $\operatorname{Aut}(U_{X_n})$  for the set of automorphisms of the scheme  $U_{X_n}$  and  $\operatorname{Aut}(\mathscr{B})$  for the set of automorphisms of the PGCS-collection  $\mathscr{B}$ , considered up to composition with an inner automorphism arising from  $\operatorname{Ker}(\Pi_n \twoheadrightarrow G)$ . Then any  $\alpha$  as above induces a bijection

$$\operatorname{Aut}(U_{X_n}) \xrightarrow{\sim} \operatorname{Aut}(\mathscr{B}).$$