Integrable deformations of string sigma models and generalized supergravity

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This thesis is mainly devoted to studying integrable deformations of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring and generalized supergravity which might be a new low-energy effective theory of string theory recently discovered.

Integrable deformations of the $\mathbf{AdS}_5 \times \mathbf{S}^5$ superstring

In chapter 2, we start to review the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring and then introduce the homogeneous Yang-Baxter (YB) deformation of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring. The deformation is known as a systematic way which describes integrable deformations of the two-dimensional non-linear sigma models. A feature of the method is that a YB deformation is specified by taking a classical r-matrix solving the classical YB equation (CYBE).

The derivation of full expressions of the YB deformed backgrounds is an important task. In this thesis, we give a general formula for YB deformed backgrounds by performing a supercoset construction. Remarkably, the formula has a very simple form. In order that the YB deformed backgrounds solve the usual supergravity equations, r-matrices need to satisfy the unimodularity condition which is discovered by Borsato and Wulff. By using our formula, we present various YB deformed $\mathrm{AdS}_5 \times \mathrm{S}^5$ backgrounds associated with unimodular r-matrices and show that these are indeed solutions of type IIB supergravity.

If a given r-matrix doesn't satisfy the unimodularity condition, the deformed background solves the generalized supergravity equations (GSE) which include an extra vector field I as well as the usual supergravity fields. We consider non-unimodular YB deformations and show the deformed backgrounds are solutions of the GSE. We further observe that some of them are reduced to the original background after performing the generalized TsT transformations.

YB deformations as duality transformations

In chapter 3, we explain that the homogeneous YB deformation can be interpreted as a string duality transformation. We first review Double Field Theory (DFT) which is a manifestly T-duality covariant formulation for the massless sector of string theory. After that, we show that the YB deformation with a classical r-matrix $r = \frac{1}{2}\, r^{ij}\, T_i \wedge T_j$ satisfying the CYBE, is equivalent to the β-deformation with the deformation parameter

$$\boldsymbol{r}^{mn} = 2\,\eta\, r^{ij}\, \hat{T}_i^m\, \hat{T}_j^n\,.$$

Therefore, the YB deformations can be regarded as a string duality transformation. This implies that the deformation might also be expressed as the generalized diffeomorphism which is the gauge symmetry of the DFT. Indeed, in section 3.3, we present the generalized diffeomorphism parameters which produce various β-twisted backgrounds.

Once the homogeneous YB deformations are realized as duality transformations, we can apply β-transformations to almost all backgrounds. In section 3.4, we consider β-transformations of $\text{AdS}_3 \times \text{S}^3 \times \text{T}^4$ backgrounds supported by H-flux and obtain various solutions as (generalized) supergravity. In this way, the β-transformation is a new useful tool to generate solutions of (generalized) supergravity.

T-folds from YB deformations

The DFT enables us to discuss the global structure of non-geometric spacetimes which are stringy geometries whose the structure group contains T-duality transformations. Such spacetime is called T-fold in some literature.

In chapter 4, by computing monodromy matrices for various YB-deformed backgrounds and a non-Abelian T-dual background, we show that these backgrounds can be regarded as non-geometric backgrounds involving non-geometric Q-fluxes. In particular, an extra vector field I appearing in GSE implies the existence of the trace of the Q-flux. Therefore, the generalized supergravity intrinsically has a T-fold structure. Importantly, as long as the r-matrix solves the CYBE, the deformed background is a solution of the DFT. Therefore, the YB deformation is a systematic way to obtain solutions with Q-fluxes in the DFT.

Weyl invariance of a bosonic string on the generalized superbravity backgrounds

Tseytlin and Wulff have shown that the GSE follows from the requirement of the κ-symmetry in the Green-Swartz formalism. Thanks to the result, at the classical level, string theory is consistently defined on the generalized supergravity backgrounds. However, the quantum consistency of string theories defined on such backgrounds is not apparent.

As explained in chapter 3, the homogeneous YB deformations could be reformulated in the DFT. This implies that the DFT can reproduce both the usual and generalized supergravity from a single action. It is well known that the DFT leads to the standard supergravity equations by taking a solution of the section condition which all spacetime fields don't depend on the dual coordinates. In the case of the GSE, the situation is a bit different. In section 5.1, we show that the associated dilaton has the linear dual coordinate dependence

$$\Phi_* = \Phi + I^i \tilde{Y}_i \,,$$

where Φ is the usual dilaton and $\tilde{Y}_i$ is the dual coordinate associated with the Killing direction I. Remarkably, the linear dual coordinate dependence is consistent with the section condition. Therefore, the generalized supergravity is realized as a non-standard solution of the section condition in the DFT.

The above modification of the dilaton teaches us how we generalize the well-known counterterm to cancel out the Weyl anomaly of a string in the generalized supergravity backgrounds. When we consider a string in the usual supergravity backgrounds, the counterterm to cancel the Weyl anomaly was proposed by Fradkin and Tseytlin and is given by

$$S_{\text{FT}} = \int \mathrm{d}^2\sigma \sqrt{-\gamma} R^{(2)} \Phi \,,$$

where $R^{(2)}$ is the Ricci scalar for the world-sheet metric $\gamma_{\alpha\beta}$. In the case of the generalized supergravity backgrounds, we propose a possible counterterm

$$S_{\text{FT}}^{(*)} = \int \mathrm{d}^2\sigma \sqrt{-\gamma} R^{(2)} \Phi_* \,.$$

Indeed, the counterterm cancels out the Weyl anomaly of a bosonic string on generalized supergravity backgrounds. In particular, we can show that the resulting counterterm is definitely local. In this sense, the string theories can be consistently defined on the generalized supergravity backgrounds.