Nonlinear Oscillations in Power Circuits

-Short History and Recent Research-

Kohshi Okumura

Department of Electrical Engineering
Kyoto University
Yoshida, Sakyo-ku, Kyoto, Japan

Abstract This paper presents the overview of studies on the abnormal oscillations which have been occurred in the power transmission systems in Japan. It also focuses on the recent studies of the dynamics of the subharmonic oscillation of order 1/3 in a nonlinear three-phase circuit which is the fundamental model of the series-compensated transmission line.

1 Introduction

As the research on the nonlinear oscillations in power circuits has a long history, we have to know it to develop the new area in future. We overview the brief history of the studies on the so-called abnormal oscillations in Japanese transmission system before we describe the recent studies on the nonlinear oscillations in three-phase circuits.

1.1 Short history

In April 1927 the Inawashiro transmission system which connected between the fourth water power station in Fukushima prefecture and Hatogaya substation in Tokyo city caused the abnormal electric oscillation. The length was 220km. The transmission voltage was 154kV. When the tests of transmission to examine the cause were performed in October 1927, the huge abnormal oscillation was observed when the voltage of the generator went up to 5800V. This oscillation accompanied with beats(20 times per 10 seconds) was named the undamped electric oscillation by Dr. Goto, who clarified that it was caused by the nonlinear magnetizing characteristic of the transformer cores and the capacitance between the lines and the ground[1, 2, 3]. After this, focusing on the higher harmonic abnormal oscillations, the research had been continued[4].

After the second world war, the series capacitor-compensated transmission lines were going to be constructed in order to compensate the inductance of the lines. In 1956 the test of Suriko-Singu transmission line in Wakayama prefecture observed that there occurred the subharmonic oscillations of order 1/3(abbreviated as 1/3-subharmonic oscillation) when lightly or no-loaded transformers were switched on. The transmission systems were approximated by single-phase circuits and several subharmonic oscillations were studied. What had been clear was that the 1/3-subharmonic oscillation was generated by the nonlinearity of the magnetic characteristic of the transformer and series capacitors in the lines. Most of the researches on the 1/3-subharmonic oscillation had been done by means of an analog computer in those days[6]. Afterward the 1/3-subharmonic oscillations were studied analytically and experimentally[5, 7, 8, 9, 10]. In a distribution system the abnormal oscillation due to ferroresonance has recently been

reported. This oscillation was caused by the nonlinear characteristics of the transformer cores and the capacitors in the circuit which protects semiconductor-switches[11].

In recent years there is a report that non-periodic oscillations are observed in 400kV power transmission system in France . The paper says that the precise analysis is required to understand the cause of the non-periodic oscillations and that the recent theory of nonlinear dynamics is needed to understand the mechanism of the oscillations to be avoided[12, 13, 14].

In this article the particular attention is paid to the 1/3-subharmonic oscillations occurred in the three-phase series resonance circuit which approximates the three-phase series compensated transmission systems. The method of analysis is overviewed. The experimental results are also described in comparison with theoretical results[15, 16, 17].

2 Nonlinear three-phase circuit

We are needed to abstract from the large scale power circuits the essential part of the circuit which causes the nonlinear oscillations. In this sense the series capacitor-compensated transmission system with no-loaded or lightly loaded transformers is approximated by the three phase-circuit as shown in Fig.1 if the inductances of the transmission lines are neglected. The nonlinear three inductors are the approximations of the wye-delta connected transformers. The three-phase circuit is coupled with nonlinear inductors, or nonlinearly coupled circuits. Studying the three-phase circuits, we have also the possibility to find bifurcation phenomena occurred in higher dimensional dynamical system. The circuit equation can be represented by

$$\frac{d\mathbf{y}}{dt} = \mathbf{f}(\mathbf{y}, t) = \mathbf{e}(t) + \mathbf{A}\mathbf{y} + \mathbf{F}(\mathbf{y}) \tag{1}$$

where $\mathbf{y} \in \mathbf{R}^5$ is the state vector of which elements are the flux-interlinkages of the transformer cores and the capacitor voltages. The three-phase source voltage vector is denoted by a periodic function $\mathbf{e}(t) \in \mathbf{R}^5$. The elements of the constant matrix $\mathbf{A} \in \mathbf{R}^{5\times 5}$ mean capacitances and resistances of the circuit. The vector-valued function $\mathbf{F}$ represents the nonlinear magnetizing characteristics of the transformer cores. The problem is to obtain the periodic solutions corresponding to the 1/3-subharmonic oscillations.

3 Analysis and experiments

In this section the methods for obtaining the periodic solutions are described. The first method is called asymptotic method developed by Krylov, Bogoliubov and Mitropolsky(abbreviated as KBM method), which is rather classical. The second method is the homotopy method based on the shooting method to solve the two-points boundary value problem in ordinary differential equations. The former method obtains the determining equation which gives the amplitudes and phases of the oscillations. The determining equation is based on the solution of the generating systems of the circuit equation. On the other hand, in the latter method the numerical integration of the circuit equation is carried out and Poincare mapping is used to follow the solution curves. Hence we

are able to investigate the bifurcation phenomena from the eigen values and vectors of Jacobian matrix. In the following the outline of the above two methods are going to simply described.

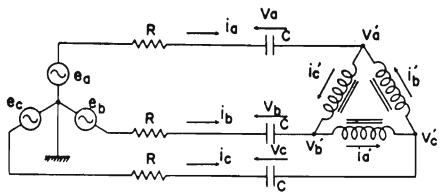


Fig. 1. Nonlinear three-phase circuit.

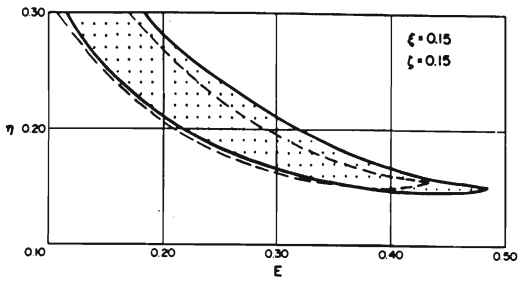


Fig. 2. Region where 1/3-subharmonic oscillation occurs in synchronism. Dashed line by analog computer and real line by KBM method.

3.1 KBM method and its extension

The original asymptotic method can not be applied to the analysis of the 1/3-subharmonic oscillations which have found to be the multi-mode oscillations since the periodic solution corresponding to the 1/3-subharmonic oscillations is not found out. Hence we need not only to extend the asymptotic method to analyze the multi-mode oscillations, but also to transform the original circuit equation into the nonlinear differential equation which can be applied to KBM method. The first thing we must do is to transform the circuit equation into the 0-,d-,q-coordinate expression, in which the stationary points of the resultant differential equations are corresponding to the sinusoidal steady state of the original system. The abnormal oscillations are governed by the variations $\mathbf{x} \in bfR^5$ from the stationary points. Hence we have another nonlinear differential equation in which the variables are the variation of the flux-interlinkages and that of the capacitor voltages. Clearly the abnormal oscillations arerepresented by the nonlinear differential equation of the variation expressed by

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{C}\mathbf{x} + \epsilon \mathbf{g}(\mathbf{x}, \tau) \tag{2}$$

where the eigenvalues of the matrix $\mathbf{C} \in \mathbf{R}^{5\times 5}$ are determined to be pure imaginary numbers $\pm j2/3, \pm j4/3$. Hence the unperturbed system of Eq.(2) is in inner resonance state. This assumption is approximately hold. The extension of KBM method is made in a following way. The periodic solution $\mathbf{x}$ of Eq.(2) is given by the power series of small parameter ϵ

$$\mathbf{x} = \mathbf{x}^{(0)}(\mathbf{a}, \boldsymbol{\theta}; \tau) + \epsilon \mathbf{x}^{(1)}(\mathbf{a}, \boldsymbol{\theta}; \tau) + \cdots \tag{3}$$

where $\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \cdots$ are the periodic functions with period 2π of τ. The valuables $\mathbf{a} \in \mathbf{R}^2$ and $\boldsymbol{\theta} \in \mathbf{R}^2$ are the amplitude and phase, respectively and are assumed to be determined by the simultaneous differential equation

$$\frac{d\mathbf{a}}{d\tau} = \epsilon \mathbf{A}_1(\mathbf{a}, \boldsymbol{\theta}) + \epsilon^2 \mathbf{A}_2(\mathbf{a}, \boldsymbol{\theta}) + \cdots \tag{4}$$

$$\frac{d\boldsymbol{\theta}}{d\tau} = \epsilon\boldsymbol{\Theta}_1(\mathbf{a}, \boldsymbol{\theta}) + \epsilon^2\boldsymbol{\Theta}_2(\mathbf{a}, \boldsymbol{\theta}) + \cdots \tag{5}$$

Substitution of Eqs.(3)to (5) into Eq.(2) and equating the same power of ϵ gives us a series of nonlinear differential equations of $\mathbf{x}^{(k)}(k = 0, 1, \cdots)$. The conditions that these series of the differential equations have no secular terms determines $\mathbf{A}_1, \boldsymbol{\Theta}_1, \cdots$. In the steady state the right hand sides of Eqs.(4) and (5) become zero. Hence we have the determining equation by which the amplitude $\mathbf{a}$ and phase $\boldsymbol{\theta}$ are able to be determined.

When we analyze the 1/3-subharmonic oscillation, there is possibility not to find out any solution which accounts for the real experimental results. In this case we must proceed to the higher approximation. When we approximate the magnetizing characteristics of the nonlinear inductors to the third power of the fluxinterlinkages, we can have the periodic solution of the 1/3-subharmonic oscillations. However, if we want to have the periodic solution corresponding to the 1/2-subharmonic oscillation we must proceed to the second approximation which requires the more complicate processes of computations. About thirty years ago, the software which allowed us to have an algebraic computation was not useful. In the last ten years, however, the software for computer algebra has made a remarkable progress and easier processing to the higher approximation has become possible.

In Fig.2 the parameter region is shown on E-η plain in which the 1/3-subharmonic oscillations occurs. The parameters E corresponds to the amplitude of the three-phase voltage source, η to the elastance of the capacitor, ξ to the resistance, ζ to the resistance in the delta connection of the nonlinear inductors. It becomes clear that there are three kinds of 1/3-subharmonic oscillations, which are the 1/3-subharmonic oscillation with beats, synchronized 1/3-subharmonic oscillation and the single-phase 1/3-subharmonic oscillation in which only one of three inductors is active. We can see that the 1/3-subharmonic oscillation occurs in symmetry as well as in unsymmetry.

3.2 Experiments

We also present the former experimental results which interpreted analytical results. In order to confirm the three kinds of the 1/3-subharmonic oscillations, the experimental three-phase circuit is set up as shown in Fig.1. The results are shown in Fig.3, in which the synchronized 1/3-subharmonic oscillation is confirmed the dotted small region. In the horizontally hatched region there occurred the single-phase 1/3-subharmonic oscillation and in the obliquely hatched region 1/3-subharmonic oscillation with beats. In these regions the modes of oscillations are changed with variation of the circuit parameters. That is to say, the bifurcation phenomena of the 1/3-subharmonic oscillations occur. As far as KBM method is concerned, it can not be applicable to find the exact bifurcation points in the parameter space. From the standpoint of the determination of the bifurcation points KBM method has its own limitation. We are now in a position to use another effective method for more precise analysis.

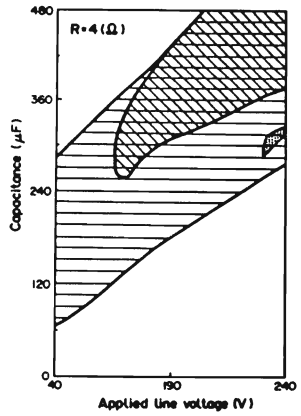


Fig. 3. Region where 1/3-subharmonic oscillations occur. By experiment.

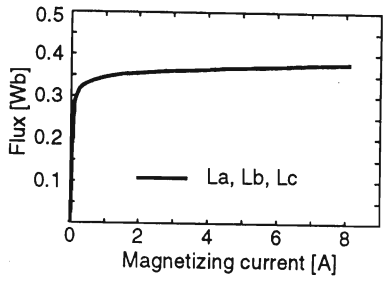


Fig. 4. Magnetizing characteristic of nonlinear inductors.

4 More precise analysis and experiment

4.1 Shooting and Homotopy method

Shooting method is one of the methods to solve the two-points boundary problem of an ordinary differential equation. The homotopy method is effective to find the solution of nonlinear equation because it has global convergent region. Combination of these two methods provides the periodic solutions as well as the bifurcation points. The combined method is powerful to find them, although it is very time-consuming to reach solutions. However, workstations with high quality and high speed have recently been very common. It has become possible to compute the bifurcation diagram in the parameter region by running it for two or three days or more.

Let the periodic solution be $\mathbf{y}(t) \in \mathbf{R}^5$ and have the period T. Then the solution of Eq.(1). The periodic solution satisfies the boundary condition

$$\mathbf{y}(0) = \mathbf{y}(T). \tag{6}$$

The integration of Eq.(1) gives

$$\mathbf{y}(T) = \int_0^T \mathbf{f}(\mathbf{y}, s)ds + \mathbf{y}(0). \tag{7}$$

Letting the right-hand side be $\mathbf{T}(\mathbf{y}_0)$ where $\mathbf{y_0} = \mathbf{y}(0)$, we have

$$\mathbf{y}_0 = \mathbf{T}(\mathbf{y}_0). \tag{8}$$

Eq.(8) is the nonlinear equation of the variable $\mathbf{y}_0$. Using homotopy method we try to find the solution. The homotopy function is defined by

$$\mathbf{H}(\mathbf{y}_0, p) = p\mathbf{F}(\mathbf{y}_0) + (1-p)[\mathbf{F}(\mathbf{y}_0) - \mathbf{F}(\mathbf{a})] \tag{9}$$

where p is the homotopy parameter and

$$\mathbf{F}(\mathbf{y}_0) = \mathbf{y}_0 - \mathbf{T}(\mathbf{y}_0). \tag{10}$$

By varying the value of p we can reach the solution at $p = 1$. When we try to obtain the bifurcation point in the parameter region, the homotopy function is given by

$$\mathbf{H}(\mathbf{y}_0, \mu) = \mathbf{F}(\mathbf{y}_0, \mu) \tag{11}$$

where μ is the parameter of the circuit.

4.2 Analytical and experimental results

Fig.4 illustrates the nonlinear magnetizing characteristics of the nonlinear inductors in the experimental circuit. Three inductors have almost identical characteristic. Precise experiment shows the three modes of the 1/3-subharmonic oscillations in Fig.5. Mode M_1 is the 1/3-subharmonic oscillation in which one nonlinear inductor is active in its generation. Mode M_2 has two active inductors and mode M_3 has three active inductors. In mode M_3 the 1/3-subharmonic oscillation accompanies with beats.

Fig. 6 shows the analytical results. The diagram of the bifurcation of the 1/3-subharmonic oscillations is illustrated where ψ_a is the fluxinterlinkage of the phase a and E_m is the parameter corresponding to the voltage of the source. We can see from this diagram that the 1/3-subharmonic oscillation occurs by the saddle-node bifurcation S. With increasing E_m there occurs the pitch-folk bifurcation P and appears period-doubling bifurcation D. Through the period doubling bifurcation the stable solution disappears. The waveform of mode M_1 is varying with the increase of the parameter E_m as shown in Fig. 7 together with the frequency spectrum in Fig. 8. From this spectrum we can see that the 1/3-subharmonic oscillations changes into the chaos through pitch-folk and period-doubling bifurcations.

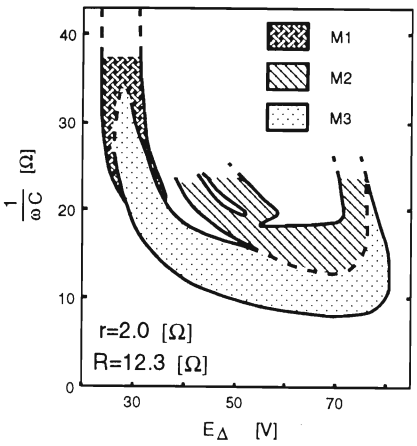


Fig.5. Region where 1/3-subharmonic oscillations occur. By experiment.

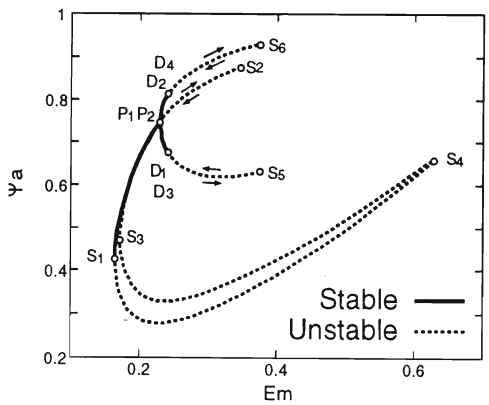
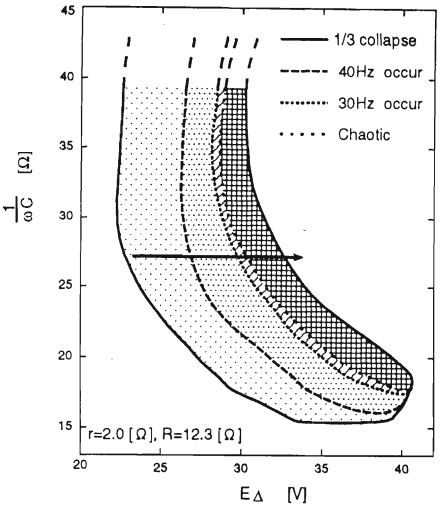


Fig.6. Bifurcation diagram of 1/3-subharmonic oscillation of mode M_1.



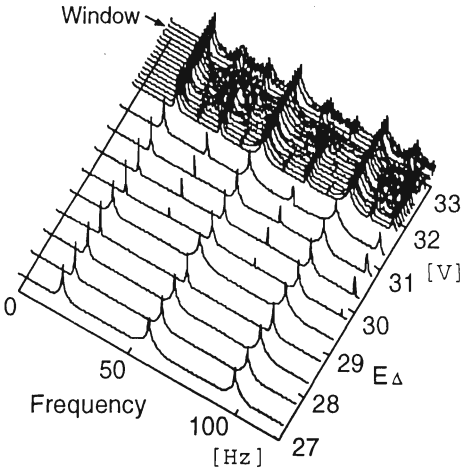


Fig.7. Region of 1/3-subharmonic oscillations of mode M_1 (experiment).

Fig.8. Spectrum of 1/3-subharmonic oscillation of mode M_1 (experiment).

In the next step the two-phase-like 1/3-subharmonic oscillations(mode M_2) are examined. Fig.9 is the solution curves of mode M_2 . The Neimark-Sacker bifurcations appears between the saddle-node bifurcations. The experimental results are shown in Fig.10. The synchronized, almost periodic and chaotic 1/3-subharmonic oscillations occur in two phases. In comparison with the experimental results Fig.11 illustrates the computational results by means of the homotopy method. We can see the U-like region in each figure. The large difference is that there are experimental regions in which the stable periodic regions exist in the middle of the U-like region. This difference may be based on the small mismatch of the parameters between three phases of the experimental circuit. The further observations in the experiments have brought us 11 kinds of 1/3-subharmonic oscillations as follows:

1. *Mode M_3(Symmetrical mode): Three inductors are active.*
   - (a) *synchronized 1/3-subharminc oscillation*
   - (b) *almost periodic(beat) 1/3-subharmonic oscillation*
   - (c) *chaotic 1/3-subharmonic oscillation*
2. *Mode M_2(unsymmetrical mode): Two inductors are active.*
   - (a) *synchronized 1/3-subharmnic oscillation with DC component*
   - (b) *almost periodic 1/3-subharmonic oscillation with DC component*
   - (c) *chaotic 1/3-subharmonic oscillation*
     - i. *without DC component*
     - ii. *with DC component*
3. *Mode M_1(unsymmetrical mode): One inductor is active.*
   - (a) *synchronized 1/3-subharmonic oscillation*
     - i. *without DC component*
     - ii. *with DC component*

(b) chaotic 1/3-subharmonic oscillation

i. without DC component

ii. with DC component

It is one of the important problems to discriminate between the almost periodic and chaotic 1/3-subharmonic oscillations by any analytical method or to find analytically the bifurcation points from almost periodic 1/3-harmonic to chaotic 1/3-harmonic oscillation.

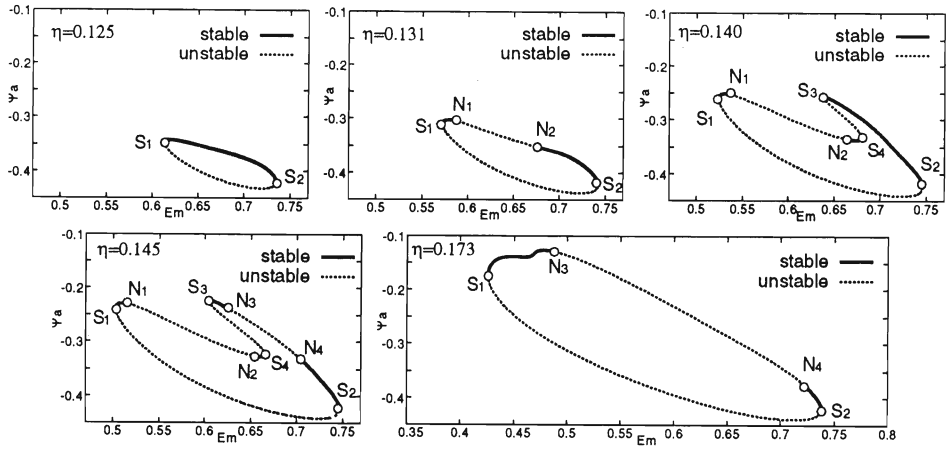


Fig.9. Bifurcation-diagram of 1/3-subharmonic oscillation of mode M_2(experiment).

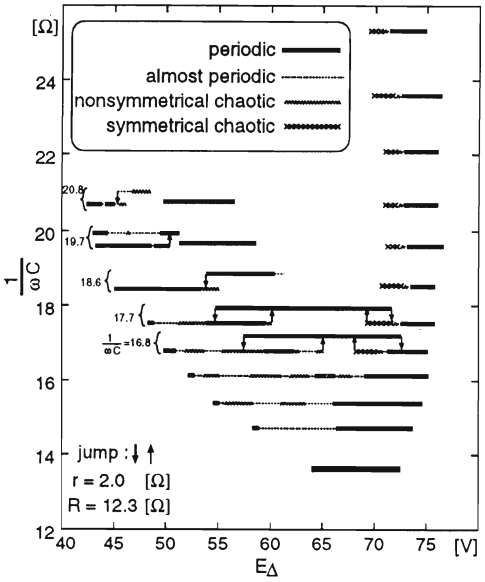


Fig.10. Region of 1/3-subharmonic oscillations of mode M_2 (experiment).

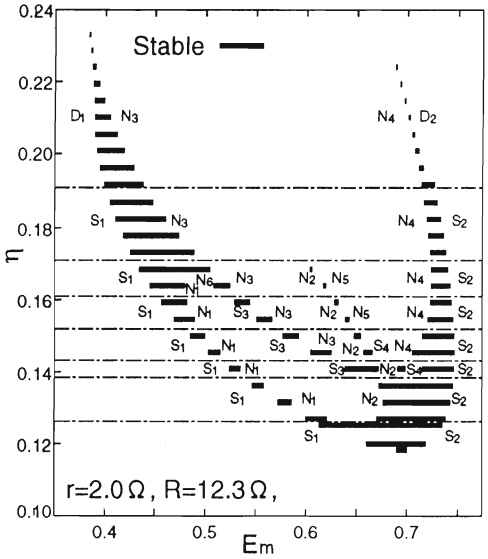


Fig.11. Region of 1/3-subharmonic oscillation of mode M_2(homotopy method).

5 Conclusion

The history of researches on nonlinear oscillations in power circuits have been briefly described in a particular attention to Japanese transmission system. To be general, the power system is a higher dimensional system. There are hence many and various nonlinear problems in power systems which have been left unsolved so far. However, the theory of nonlinear dynamics, the development of numerical methods such as homotopy method, interval method and electronic measuring system will presumably provide clear solution.

References

[1] M. Kameda: "The Special Electrical Oscillation of Transmission System", Journal of IEEJ, pp.880-887, August 1930.

[2] M. Goto: "Undamped Electric Oscillation of An Alternator Connected to A Transmission Line", Researches of the Electrotechnical Laboratory No.281, 1930.

[3] M. Goto: "'Undamped electric oscillation and electric instability of a transmission system," Journal of IEEJ Vol.51, 1931.

[4] T. Norimatsu:"Researches on Electrical Instability of Power Transmission System caused by Saturation of Transformer", Researches of Electrotechnical Laboratory, No.613, September 1961.

[5] A. Kishima: "Subharmonic Oscillations in Three-Phase Circuit", the Memoirs of the Faculty of Engineering, Kyoto University, Vol. XXX, Pt. 1,pp. 26-44, January 1968.

[6] I. A. Wright: "Three-Phase Subharmonic Oscillations in Symmetrical Power Systems", IEEE PAS, pp.1295-1304, 1971.

[7] K. Okumura and A. Kishima: "Nonlinear Oscillation in Three-Phase Circuit", Trans. IEEJ, Vol.B-96, No. 12, pp.599-606, December 1976.

[8] K. Okumura and A. Kishima: "Subharmonic Oscillation in Three-Phase Circuit", Trans. IEEJ, Vol.B-97, No. 4, pp.199-206, April 1977.

[9] K. Okumura, M. Nakamura and A. Kishima: " Experimental and Analytical Study on 1/3-Subharmonic Oscillation in Three-Phase Circuits", Trans. IEEJ, Vol.107B, No. 1, pp.1-8, January 1987.

[10] K. Okumura, S. Fukuda and A. Kishima: "Analysis of Frequency Entrainment in Three-Phase Circuits", Trans. IEEJ, Vol.107B, No. 3, pp.115-122, March 1987.

[11] Nisshin Electric Co.Ltd. Test report 92C-60, November 1992.

[12] C. Kieny: "Application of the bifurcation theory in studying and understanding the global behaviour of a ferroresonant electric power circuit", Trans, IEEE on Power Delivery, Vol.6, No. 2, April 1991.

[13] J. R. Marti and A. C. Soudack; "Ferroresonance in power systems: Fundamental solution", Proc. IEE-C, Vol. 138, No. 4, July 1991.

[14] A. E. A Araujio, A. C. Soudac and J. R. Marti: "Ferroresonance in power systems: Chaotic behaviour", Proc. IEE-C, Vol. 140, No.3, May 1993.

[15] T. Hisakado,T. Yamada and K. Okumura: "Single-Phase 1/3-Subharmonic Oscillations in Symmetrical Three-Phase Circuit", Trans. IEICE, Vol.J79-A, No.9, pp.1553-1561, September 1996.

[16] T. Hisakado and K. Okumura:"Two-Phase 1/3-Subharmonic Oscillations in Symmetrical Three-Phase Circuit", Trans. IEICE, Vol.J80-A, No.2, pp.355-362,February 1997.

[17] T. Hisakado and K. Okumura:"Bifurcation Phenomena of Harmonic Oscillatios in Three-Phase Circuit", Trans. IEICE on Fundamentals, Vol.E80-A, No.6, pp1127-1134, June 1997.