Forced Chaos in Nerve Membranes and its Modeling and Application to Chaotic Parallel Distributed Processing

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ABSTRACT

Nerve membranes respond to periodically forcing stimulation not only periodically but also chaotically, depending upon parameter values of the force. We review the forced chaos in nerve membranes and its modeling with a simple one-dimensional map. We also discuss possible information processing, or chaotic PDP (Parallel Distributed Processing) with spatio-temporal chaos in networks composed of such one-dimensional maps.

1 Introduction

This world is full of many rhythms and oscillations. Mutual interactions of rhythms and oscillations seem to make the world rich and attractive. The typical and simplest example of such an interaction is a forced oscillation, or periodic forcing of an oscillator. Generally speaking, a forced oscillation generates a variety of phenomena including beautiful deterministic chaos in spite of simplicity of the interaction.

In Japan, great researches on forced oscillations were achieved in the field of electrical engineering. In particular, it is well known that Y. Ueda discovered a strange attractor in the forced Duffing - Van del Pol equations on November 27, 1961 [28, 50]. Since then, deterministic chaos has been studied actively in various fields of engineering in Japan [9] (see also Proceedings of International Symposiums on Nonlinear Theory and its Applications (NOLTA)).

Such studies on chaos in Japan have been producing many practical applications including even consumer products for household appliances like kerosene fan heaters, dish washers, air conditioners and microwave ovens [9]. Although these applications of chaos are interesting, the importance of chaos for engineering should be more profound because concepts of deterministic chaos are greatly influencing basic theories in engineering on prediction, control, computation, information and so on [9, 21, 56]. "Chaos engineering", which is defined to be generic studies on theoretical and technological foundations for possible applications of deterministic chaos, fractal and complex systems, was advocated by the present author first in 1990 for basic studies of *applied chaos* and *applicable chaos* [6, 7, 9, 10]. Here, *applied chaos* and *applicable chaos* mean , respectively, applications of known theories on deterministic chaos to concrete examples of complex phenomena , and generalization and systematization of mathematical structures common to such concrete complex phenomena toward possible applications in the future [9].

This article reviews a chaotic forced oscillation, which is generated by periodic forcing of a nerve oscillator, and its modeling with a simple one-dimensional and bimodel map. We also discuss possible information processing with spatio–temporal chaos in neural networks composed of such chaotic maps, which is an important research subject on chaos and computation in chaos engineering.

2 Forced Chaos in Nerve Membranes

Squid giant axons have been widely used in electrophysiological experiments because they are giant with diameters between $500\mu m$ and $750\mu m$ and structurally simple without myelin sheaths [12, 23]. It has been shown by experiments with squid giant axons that a chaotic forced oscillation, or chaotic response to periodic force can be easily and reproducibly observed not only in oscillatory axons but also in resting axons when axons are stimulated by periodic current stimulations [11, 12, 13, 35, 36]. Further, different routes to chaotic forced oscillations have also been found experimentally with squid giant axons, namely, (1) "successive period-doubling bifurcations" where the period of a periodic forced oscillation increases in the form of 2^n times at each bifurcation point until an infinity period is realized in a chaotic forced oscillation, (2) "intermittency" where chaotic bursts occur intermittently among apparently periodic phases, (3) "collapse of quasi-periodicity" where a chaotic forced oscillation is produced through collapse of a 2-dimensional torus representing a quasi-periodic forced oscillation.

Chaotic forced oscillations and the routes to forced chaos in the squid giant axons can be described by solutions of nerve ordinary differential equations of the Hodgkin-Huxley equations [23] and the FitzHugh-Nagumo equations [18, 38]. The fact that the nerve equations can reproduce the forced chaos experimentally observed in squid giant axons implies that such chaotic phenomena in nerve membranes can be understood in the framework of the deterministic nonlinear dynamics.

These studies have clarified that biological neurons are entirely different from simple linear threshold neurons like the McCulloch–Pitts model [37] widely used in artificial neural networks at least in the meaning that the former has excitable nonlinear "dynamics" with threshold, which can produce forced chaos [12, 13, 35, 36] but the latter can't.

3 Simple Model of Forced Chaos in Nerve Membranes

Although forced chaos in the squid giant axons can be well described quantitatively with the Hodgkin-Huxley equations and qualitatively with the FitzHugh-Nagumo equations, these equations are too complicated as a model to represent constituent elements in large-scaled artificial neural networks. Therefore, a simple mapping model which can qualitatively reproduce the forced chaos in the nerve membranes was proposed on the basis of the properties of graded responses, and relative refractoriness and its accumulation in the nerve membranes [14]. The equation of the model is given as follows:

$$x(t+1) = f[s(t) - \alpha \sum_{d=0}^{t} k^d g\{x(t-d)\} - \Theta] \tag{1}$$

where $x(t+1)$ is the continuous output of the neuron at the discrete time $t+1$; t shows the discrete time steps ($t = 0, 1, 2, \cdots$); f is the continuous output function usually assumed to be the logistic function$f(y) = 1/\{1 + \exp(-y/\varepsilon)\}$ with the steepness parameter ε; $s(t)$ is the strength of the stimulation at t; α is a positive parameter; k is the damping factor of the refractoriness between 0 and 1; g is a refractory function describing the relationship between the analog output and the magnitude of the refractory effect to the following stimulation; Θ is the resting threshold.

The continuous output function f represents the property of a continuous stimulus-response curve in nerve membranes [18]. The term $\alpha \sum_{d=0}^{t} k^d g\{x(t-d)\}$ in Eq.(1) corresponds to accumulated refractory effects of the nerve membranes. It is assumed here that refractory effects due to past series of output firings are superimposed with exponential temporal decay [15, 39].

Eq.(1) can be transformed to the following simpler one-dimensional map by defining the internal state $y(t+1) \equiv s(t) - \alpha \sum_{d=0}^{t} k^d g\{x(t-d)\} - \Theta$ [14] :

$$y(t+1) = ky(t) - \alpha g[f\{y(t)\}] + a \tag{2}$$

where $y(t+1)$ is the internal state ; $a = (A - \Theta)(1 - k)$ and A is the strength of the stimulation which is temporally constant in electrophysiological experiments stimulating squid giant axons by periodic pulses with a constant amplitude. The output $x(t+1)$ of the neuron is calculated with the internal state $y(t+1)$ as follows :

$$x(t+1) = f\{y(t+1)\} \tag{3}$$

The one–dimensional map of Eq.(2) can qualitatively reproduce not only periodic response but also chaotic one experimentally observed with squid giant axons.

4 Chaotic Neural Network Model and Computation with Spatio–temporal Chaos

A model of the forced chaos in nerve membranes in Eq. (1) can be extended to one of synchronous chaotic neural networks[3, 4, 6, 14] by considering an additional physiological property on spatio–temporal summation of both external inputs and feedback inputs from other constituent neurons.

The dynamics of the i-th chaotic neuron integrated in a synchronous chaotic neural network with M external inputs and N constituent neurons is generally described as follows [4, 6, 14]:

$$x_i(t+1) = f_i\Big[\sum_{j=1}^{M} v_{ij} \sum_{d=0}^{t} k_e^d A_j(t-d) + \sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_f^d x_j(t-d) - \alpha \sum_{d=0}^{t} k_r^d g\{x_i(t-d)\} - \Theta_i\Big], \tag{4}$$

where $x_i(t+1)$ is the output of the i-th neuron with a continuous value between 0 and 1 at discrete time $t+1$; t shows the discrete time steps ; f_i is the continuous output function ; $A_j(t-d)$ is the strength of the j-th external input at discrete time $t-d$; v_{ij} and w_{ij} are synaptic weights to the i-th neuron from the j-th external input and from the j-th constituent neuron, respectively;k_e, k_f and k_r are the decay factors taking values between 0 and 1 for the external inputs, the feedback inputs and the refractoriness, respectively; g is the refractory function usually assumed to be $g(x) = x$ for the sake of simplicity.

Equation(4) can be transformed into the following simplified and simultaneous form under the assumption of the exponential temporal decay of input and refractory effects in the form of k_e^d, k_f^d and k_r^d in eq.(4) [4, 6, 14]:

$$\xi_i(t+1) = \sum_{j=1}^{M} v_{ij} A_j(t) + k_e \xi_i(t), \tag{5}$$

$$\eta_i(t+1) = \sum_{j=1}^{N} w_{ij} x_j(t) + k_f \eta_i(t), \tag{6}$$

$$\zeta_i(t+1) = -\alpha g\{x_i(t)\} + k_r \zeta_i(t) - \theta_i, \tag{7}$$

where $\xi_i(t+1)$ $(\equiv \sum_{j=1}^{M} v_{ij} \sum_{d=0}^{t} k_e^d A_j(t-d))$, $\eta_i(t+1)$ $(\equiv \sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_f^d x_j(t-d))$ and $\zeta_i(t+1)$ $(\equiv -\alpha \sum_{d=0}^{t} k_r^d g\{x_i(t-d)\} - \Theta_i)$ are internal state terms for the external inputs, the feedback inputs from the constituent neurons in the network and the refractoriness, respectively and $\theta_i \equiv \Theta_i(1-k_r)$. The output $x_i(t+1)$ of the i-th neuron is calculated by transformation from the internal state to the output value through the output function f_i, namely $x_i(t+1) = f_i\{\xi_i(t+1) + \eta_i(t+1) + \zeta_i(t+1)\}$. The model of eqs. (5) – (7) includes some of conventional discrete–time neuron models such as linear threshold neurons with the Heaviside output function, or the McCulloch–Pitts neurons [37] and analog neurons with the logistic output function used in backpropagation neural networks with many applications [41] as special cases of the model by changing parameter values of the model; namely, the model of the synchronous chaotic neural networks is a natural extension of the conventional discrete–time neural networks for introducing chaotic dynamics into these usual discrete–time neural network models in order to study possible functions and computational roles of spatio–temporal chaos in artificial and biological information processing by comparing the behavior of the synchronous chaotic neural networks with that of the conventional neural networks [3, 4, 6, 14].

Generally speaking, the model of synchronous chaotic neural networks generates various complex and computational neurodynamics with spatio–temporal chaos in the level of neural networks, through nonlinear interactions with synaptic connections among elemental neurons with their own chaotic dynamics in the level of single neurons. Here, hierarchical interactions between low-dimensional chaos in single neurons and possibly high-dimensional spatio-temporal chaos in a global neural network may also play an important role to create such rich neurodynamics.

In particular, the spatio–temporal dynamics with refractoriness or equivalently a self–recurrent inhibitory connection in each neuron produces a kind of chaotic itinerancy [32, 49] with possibly fractal structure of a global strange attractor in the phase space without getting stuck and stopping at equilibrium points corresponding to local minima of a computational energy function [24]. This is because any neurons can't keep firing with a high output value due to accumulation of refractory effects if the decay factor k_r for the refractoriness is sufficiently close to 1 [2]. This property has been applied to dynamical association of spatio–temporal patterns in associative memory neural networks [2, 4] and to dynamical searching for good approximate solutions to such NP-hard problems as traveling salesman problems and quadratic assignment problems in combinatorial optimization neural networks [16, 17, 22, 48, 55]. The associative dynamics of synchronous chaotic neural networks with the self–organization rule [52, 53] is similar to nonlinear behavior experimentally observed in an olfactory system [19, 44].

The chaotic neurodynamics can be harnessed or tamed to converge to a fixed point as follows. If certain information on distances from the network state to a target state in an appropriate space is available, the information can be used for adapting parameters by a kind of feedback control [47]. If such information is not available as in the case of NP–hard problems, on the other hand, a kind of deterministic simulated annealing with transient chaos, which we call *chaotic simulated annealing*, was shown to work to some extent [16, 17, 48], although a more sophisticated adaptive annealing scheme is desirable for practical applications.

As this simple model of the chaotic neural networks is a synchronous version derived on the basis of the experiment with periodic forcing [5, 11, 12, 13, 35, 36], it has little direct relation to biological neural networks. However, from the viewpoint of theoretical and engineering studies, the model has abundant and curious spatio–temporal mapping dynamics with such engineering applicability as dynamical associative memory, combinatorial optimization and self–organization. In fact, computation with synchronous chaotic neural networks, or chaotic PDP (Parallel Distributed Processing) [9] is one of important research subjects in chaos engineering [7, 9, 10].

For more biological modeling of the brain dynamics, we can extend the model of synchronous chaotic neural networks further to an asynchronous version with continuous, or analog time intervals between spikes of action potentials [29, 30]. The model of asynchronous chaotic neural networks can be utilized not only for studying possibilities of spatio–temporal coding and information processing with timing of the spikes and continuous interspike intervals [29, 30, 31, 34] but also for modeling various biological neurons distributed from integrators to coincidence detectors [1, 45] by changing values of the parameters such as time constants of exponential temporal decay. Recently, roles and operation modes of cortical neurons are becoming an important research subject [1, 8, 42, 45, 46]. When the time constants of exponential temporal decay in the model of asynchronous chaotic neural networks are much smaller than the average interspike interval of input pulses, each neuron effectively operates as a coincidence detector. For example, emergence of *dynamical cell assemblies* in neural networks composed of coincidence detector neurons was first demonstrated by the model of the asynchronous chaotic neural networks [20]. Here, *the dynamical cell assembly* means a group of neurons linked temporally by events of coincidence detection of incident spikes at each neuron and correlated firings resulting from successive events of such coincidence detection among neurons ; the emergence mechanism of such a dynamical assembly is a kind of forced oscillation in the network level, or response of a neural network composed of coincidence detector neurons to an external spatio-temporal input [20].

Moreover, asynchronous chaotic neural networks with additional properties like the latency for generating action potentials and global negative feedback produce a new kind of spatio–temporal chaos as a deterministic spatio–temporal point process in neural networks composed of coincidence detector neurons [51].

5 Possibility of a New Kind of Brain–like Computing Systems Based upon Chaotic Neural Networks

Almost all computers existing today are digital computers. Probable reasons why digital computers have prevailed in the world just within a half century since the invention of the first machine ENIAC in 1946 are existence of the firm principle of the *Turing Machine* and rapid progress of hardware technology on digital integrated circuits.

The possible limits of performances on digital computers, however, are coming within sight recently. One factor results just from the hardware architecture of present digital computers, namely the synchronous system design. The extremely rapid advance of speed and integration in the synchronous digital hardware is beginning to make manifest the problems of delays of signal propagation and electrical power consumption peculiar to the synchronous VLSI system design [40].

Another fundamental problem of digital computation, namely digital uncomputability of real numbers and deterministic chaos has been clarified by understanding chaotic dynamics. A seed generating complexity of deterministic chaos exists in complexity of real numbers specifying the initial condition, as typically shown with the Bernoulli shift map and the Logistic map. On the other hand, almost all real numbers can't be computed by Turing machines because the set of algorithm and that of real numbers are countably infinite and uncountable, respectively. Furthermore, since the pseudo–orbit tracing property can not be guaranteed in many chaotic systems, numerically approximate calculation on long–term behavior of deterministic chaos by digital computers suffers from difficulty in this sense too.

There exist possibilities to invent a new kind of computing systems which might break the barriers limiting the performances of the present digital computers explained above, by learning from the dynamical brain. In fact, we can get two clues for the purpose from the brain, namely analog hardware implementation of deterministic chaos with nerve membranes and asynchronous computing with spatio–temporal pulses of action potentials in biological neural networks.

As summarized in section 2, real nerve membranes are chaotic analog devices which dynamically realize deterministic chaos directly on the basis of their own nonlinear excitable characteristics. Although chaotic behaviour produced by such analog devices, or nerve membranes is with inevitable fluctuation like thermal noise and channel noise, it implies implementation technique of deterministic chaos entirely different from digital implementation with numerically approximate calculation. In other words, analog hardware devices can implement deterministic chaos directly by its own dynamical property.

It is one of important characteristics of deterministic chaos that simple nonlinear systems can frequently generate very complicated behavior and possibly complicated functions. The simplicity of the nonlinear property leads to an advantage that such chaotic systems can be easily and directly implemented by simple hardware. In fact, since the equations of the chaotic neural networks are simple enough, they are easily implemented by various analog electronic circuits [25, 26, 27, 33, 43]. For example, an electronic IC chip with nine chaotic neurons based upon switched–capacitor circuit technique has been designed and implemented in a standard $2\mu m$ n–well CMOS IC fabrication process [25, 26] and electronic realization of the computational spatio–temporal dynamics of the synchronous chaotic neural networks has been confirmed [25, 26, 27].

The second point to be noted is asynchronous characteristics of spatio–temporal spikes in real neural networks. On the aspect, a model of asynchronous chaotic neural networks and the dynamical cell assembly hypothesis on the neural information processing [20] may provide mechanisms of flexible asynchronous computation making a breakthrough to conventional synchronous computing systems. In particular, asynchronous chaotic neural networks composed of coincidence detector neurons generate an interesting chaotic point process with spatio-temporal spikes [51]. Since such networks can operate multiple functions with temporal spike coding and functional connectivity [54], it is an important future problem to examine possibility of altering the network functional mode through such a new kind of spatio–temporal point–process chaos.

6 Conclusion

We have reviewed chaotic forced oscillations in nerve membranes and models of synchronous and asynchronous chaotic neural networks. The models may have potential to create a new kind of brain-like computing systems beyond present digital and synchronous computers.

References

[1] M. Abeles, Role of the Cortical Neuron : Integrator or Coincidence Detector ?, Israel J. Med. Sci. **1** (1982) 83 – 92.

[2] M. Adachi and K. Aihara, *Associative Dynamics in a Chaotic Neural Network*, Neural Networks, **10/1** (1997) 83 – 98.

[3] K. Aihara, Neural Computers (Tokyo Denki University Press, 1988, Tokyo (in Japanese)).

[4] K. Aihara, Chaotic Neural Networks, in Bifurcation Phenomena in Nonlinear Systems and Theory of Dynamical Systems (Ed. H.Kawakami), 143, (World Scientific, 1990, Singapore).

[5] K. Aihara (Ed.), Chaos in Neural Systems (Tokyo Denki University Press, 1993, Tokyo (in Japanese); the Korean–language edition, 1994, Ohm Ltd., Seoul).

[6] K. Aihara (Ed.), Neural, Fuzzy and Chaotic Computing – Introduction to a New Generation of Analog Computing (Ohm-sha, 1993, Tokyo (in Japanese); the Korean–language edition, 1994, Dae Kwang, Seoul).

[7] K. Aihara (Ed.), Applied Chaos and Applicable Chaos, (Science-sha, 1994, Tokyo (in Japanese); the Korean–language edition, 1996, Ohm Ltd., Seoul).

[8] K.Aihara and K.Judd, Author's Response, Neural networks **9/4** (1996) 724-725.

[9] K. Aihara and R. Katayama, Chaos Engineering in Japan, Communications of the ACM **38/11** (1995) 103 – 107.

[10] K. Aihara and R. Katayama, A special issue on Chaos Engineering, J. of Intelligent & Fuzzy Systems **5/1** (1997) John Wiley & Sons, Inc.

[11] K. Aihara, G. Matsumoto and Y. Ikegaya, Periodic and Non–Periodic Responses of a Periodically Forced Hodgkin–Huxley Oscillator, J. Theor. Biol. **109** (1984) 249 – 269.

[12] K. Aihara and G. Matsumoto, Chaotic Oscillations and Bifurcations in Squid Giant Axons, in Chaos (Ed. A.V. Holden), (Manchester University Press, Manchester and Princeton University Press, 1986, Princeton) 257 – 269.

[13] K. Aihara, T. Numajiri, G. Matsumoto and M. Kotani, Structures of Attractors in Periodically Forced Neural Oscillators, Physics Letters A, **116** (1986) 313 – 317.

[14] K. Aihara, T. Takabe and M. Toyoda, Chaotic Neural Networks, Physics Letters A, **144/6/7** (1990) 333 – 340.

[15] E.R.Caianiello, Outline of a Theory of Thought-Processes and Thinking Machines, J. Theor. Biol. **2** (1961) 204 – 235.

[16] L. Chen and K. Aihara, Chaotic Simulated Annealing by a Neural Network Model with Transient Chaos, Neural Networks **8/6** (1995) 915 – 930.

[17] L. Chen and K. Aihara, Chaos and Asymptotical Stability in Discrete – time Neural Networks, Physica D **104** (1997) 286 – 325.

[18] R. FitzHugh, Mathematical Models of Excitation and Propagation in Nerve, in Biological Engineering (Ed. H.P.Schwan), 1, (McGraw–Hill, 1969, N.Y.).

[19] W. J. Freeman, Tutorial on Neurobiology – From Single Neurons to Brain Chaos, Int. J. of Bifurcation and Chaos, **2/3** (1992) 3, 451 – 481.

[20] H. Fujii, H. Ito, K. Aihara, N. Ichinose and M. Tsukada, Dynamical Cell Assembly Hypothesis – Theoretical Possibility of Spatio–Temporal Coding in the Cortex, Neural Networks, **9/8** (1996) 1303 – 1350.

[21] C. Grebogi and J. A. Yorke (eds.), The Impact of Chaos on Science and Society, (United Nations University Press, 1997, Tokyo).

[22] M.Hasegawa, T.Ikeguchi and K.Aihara, Combination of Chaotic neurodynamics with the 2-opt Algorithm to Solve Traveling Salesman Problems, Physical Review Letters, **79/12** (1997) 2344 – 2347.

[23] A. L. Hodgkin and A. F. Huxley, A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve, J.Physiol. (London) **117** (1952) p. 500.

[24] J.Hopfield and D.Tank, Neural Computation of Decisions in Optimization Problems, Biol. Cybern. **52** (1985) 141 – 152.

[25] Y. Horio, K. Suyama, A. Dec and K. Aihara, Switched-Capacitor Chaotic Neural Networks for Traveling Salesman Problem, Proc. WCNN94 **4** (1994) 690.

[26] Y.Horio and K.Suyama, Experimental Observations of 2–and 3–Neuron Chaotic Neural Networks Using Switched–Chapacitor Chaotic Neuron IC Chip, Trans. IEICE **E78-A/4** (1995) 529 – 535.

[27] Y.Horio and K.Aihara, Chaotic Neural Network Integrated Circuit, Seitai No Kagaku **48/1** (1997) 67 (in Japanese).

[28] K. Horiuchi and T. Koga (Eds.), Special Issue on Engineering Chaos, Trans. IEICE **E73/6** (1990).

[29] N. Ichinose and K. Aihara, Asynchronous Chaotic Neural Networks, in Towards the Harnessing of Chaos (ed. by M.Yamaguti) (Elsevier Science Publishers B.V., 1994, Amsterdam) 353–356.

[30] N. Ichinose and K. Aihara, An Analysis on Pulse Propagation Dynamics in Asynchronous Chaotis Neural Networks, Trans. IEICE **J78–A/3** (1995) 373 – 380 (in Japanese); the English–language translation, Electronics and Communications in Japan, Part 3, **78/11** (1995) 52 – 59, Scripta Technica, Inc., John Wiley, New York.

[31] K. Judd and K. Aihara, Pulse Propagation Networks : A Neural Network Model That Uses Temporal Coding by Action Potentials, Neural Networks, **6/2** (1993) 203 – 215; the Errata, Neural Networks **7/9** (1994) 1491.

[32] K. Kaneko, Clustering, Coding, Switching, Hierarchical Ordering, and Control in Network of Chaotic Elements, Physica D **41** (1990) 137.

[33] N. Kanou, Y. Horio, K. Aihara and S. Nakamura, A Current–Mode Circuit of a Chaotic Neuron Model, Trans. IEICE **E76–A/4** (1993) 642 – 644.

[34] W. Maass, Advances in Neural Information Processing Systems, (Ed. G. Tesauro, D. Touretzky and T. Leen), **7** (1995) 185 – 190.

[35] G. Matsumoto, K. Aihara, Y. Hanyu, N. Takahashi, S. Yoshizawa and J. Nagumo, Chaos and Phase Locking in Normal Squid Axons, Physics Letters A **123** (1987) 162 – 166.

[36] G. Matsumoto, K. Aihara, M. Ichikawa and A. Tasaki, Periodic and Nonperiodic Responses of Membrane Potentials in Squid Giant Axons During Sinusoidal Current Stimulation, J. Theor. Neurobiol. **3** (1984) 1 – 14.

[37] W. S. McCulloch and W. H. Pitts, A Logical Calculus of the Ideas Immanent in Neural Nets, Bull. Math. Biophys. **5** (1943) 115 – 133.

[38] J. Nagumo, S. Arimoto and S. Yoshizawa, An Active Pulse Transmission Line Simulating Nerve Axon, Proc. IRE **50** (1962) 2061.

[39] J. Nagumo, and S.Sato, On a Response Characteristic of a Mathematical Neuron Model, Kybernetics **10** (1972) 155 – 164.

[40] T.Nanya, Survey on Asynchronous VLSI System Design Techniques, Future Electron Devices Journal, **6/3** (1995) 11 – 16 (in Japanese).

[41] D. E. Rumelhart, J. L. McClelland and the PDP Research Group, Parallel Distributed Processing, (MIT Press, 1986, Cambridge).

[42] M. N. Shadlen and W. T. Newsome, Noise, neural Codes and Cortical Organization, Current Opinion in Neurobiology, **4** (1994) 569 – 579.

[43] K. Shimizu, K. Aihara and M. Kotani, An Electronic Circuit Model of Chaotic Neural Networks, Electronics and Communications in Japan, Part 3, **74/1** (1991) 51–64, Scripta Technica, Inc., John Wiley, New York.

[44] C. A. Skarda and W. J. Freeman, How Brains Make Chaos in order to Make Sense of the World, Behavioral and Brain Science, **10** (1987) 61.

[45] W. R. Softky and C. Koch, The Hightly Irregular Firing of Cortical Cells is Inconsistent with Temporal Integration of Random ESPSs, J.Neuroscience **13** (1993) 334 – 350.

[46] H.Stowell, Naming the Neuron in Vain, Neural Networks **9/4** (1996) 723 – 725.

[47] H.Takahashi, Chaos and Memory, in Chaos Seminar (ed. K.Aihara) (Kaibundo, Tokyo, 1994) 127 (in Japanese).

[48] I. Tokuda, K. Aihara and T. Nagashima, Global Bifurcation Structure of Chaotic Neural Networks and its Application to Traveling Salesman Problems, Neural Networks **10/9** (1997) 1673–1690.

[49] I. Tsuda, Chaotic Itinerancy as a Dynamical Basis of Hermeneutics in Brain and Mind, World Futures **32** (1991) 167.

[50] Y. Ueda, The Road to Chaos (Aerial Press, 1992).

[51] M. Watanabe and K. Aihara, Chaotic in Neural Networks Composed of Coincidence Detector Neurons, Neural Networks, **10/8** (1997) 1353 – 1359.

[52] M. Watanabe, K. Aihara and S. Kondo, Automatic Learning in Chaotic Neural Networks, Trans. of IEICE **J78–A/6** (1995) 686 – 691.

[53] M. Watanabe, K. Aihara and S. Kondo, Self–organization Dynamics in Chaotic Neural Networks, in Control and Chaos (Ed. by K. Judd et al.) 320–333, Birkhäuser, 1997.

[54] M.Watanabe, K.Aihara and S.Kondo, A Dynamical Neural Network with Temporal Coding and Functional Connectivity, Biol. Cybern. (in press).

[55] T. Yamada and K. Aihara, Nonlinear Neurodynamics and Combinatorial Optimization in Chaotic Neural Networks, J. Intelligent and Fuzzy Systems, **5/1** (1997) 53 – 68.

[56] M. Yamaguti (ed.), Towards the Harnessing of Chaos (Elsevier, 1994, Amsterdam).