# A Classification of the 3rd Order Oscillators with Respect to Chaos


Masami KURAMITSU

*Department of Electrical Engineering, Graduate School of Engineering.*
*Kyoto University, Yoshida-honmachi, Sakyo-ku. Kyoto, 606-8501, JAPAN*
*Phone:+81-75-753-5276, FAX:+81-75-752-0624,*
*E-mail:kuramitu@kuee.kyoto-u.ac.jp*



**Abstract.** The author proposes two necessary conditions under which the 3rd order oscillators with cubic nonlinear active elements generate chaos. Using these conditions, these oscillators are classificated into two groups according to whether they generate chaos or not, and this result is confirmed by computer simulations.


## 1. Introduction

Electrical circuit is one of the fields where the chaos was found earliest[1]. Since then many types of the 3rd order oscillators generating chaos have been reported[2-11]. However, most of them are found by trial or chance, and the conditions for generating chaos have not yet been made clear. In this article the author considers a group of the 3rd order oscillators which consist of five two-terminal elements; four of them are linear capacitors, inductors, and one resistor, and only one element is a cubic nonlinear active resistor. There exist 11 types of oscillators as shown in Fig.1. Two conditions for generating chaos are proposed. Using these conditions, these oscillators are classified into two groups according to whether they generate chaos or not, and this result is confirmed by computer simulations.

## 2. A Group of the 3rd Order Oscillators

In order to consider the conditions for generating chaos, we restrict ourselves to the simple and natural oscillators which consist of the following five elements.

1. Three capacitors and inductors are positive. i.e., natural ones.

2. One voltage-controlled nonlinear resistor has the cubric active characteristics, i.e.,

$$i = f(v) = \mu(-v + v^3/3) \tag{1}$$

3. One linear resistor may be positive or negative.

By using these elements, we obtain 11 types of complete circuits as shown in Fig.1. A1 is the Chua's circuit. It is our object to find oscillators generating chaos theoretically.

## 3. Conditions for Generating Chaos

Chaos is roughly considered to be the situation where all the equiliblium points, limit cycles and tori in a cirtain region of the phace space are unstable and the trajectory is obliged to wander in that region forever. On the other hand, as is well known, the behavior of an oscillator is rather simple for weak nonlinearity. As the nonlinearity becomes strong, complicated phenomena may occur. Such is the chaos. These changes of phenomena are considerd to accompany the change of the property(number and stability) of the equilibrium points inevitably. To see this situation, we change one parameter $\mu$ of (1).

According to these physical considerations, we propose two following conditions as the necessary ones to generate chaos. The expression is slightly different from those in [12] based on our simulation results and the suggestion of [10].

- Condition I: The number of equilibrium points and/or their stability changes in a complicated manner as the parameter of activity $\mu$ is increased.
- Condition II: There exists a parameter value $\mu$ for which three equilibrium points exist, and all of them are unstable.

## 4. Classification of the 3rd Order Oscillators

The equilibrium points of oscillators shown in Fig.1 are determined by the resistive circuits obtained with the capacitors open-circuited and the inductors short-circuited. These resistive circuits are divided into three types A, B, and C as shown in Fig.2. Their equilibrium points are given by the nonlinear and the linear resistive characteristics as shown in Fig.3. Therefore, we can classify these oscillators into four groups as shown in Fig.1. Groups A and B in Fig.1 have three equilibrium points given by A and B of Fig.2 & 3, respectively. Both C containing two capacitors and D containing one capacitor in Fig.1 have only one equilibrium points as shown in C of Fig.2 & 3.

## 5. Oscillators Generating Chaos

As the oscillators of C and D types in Fig.1 have only one equilibrium point, they are considered not to generate chaos according to Condition II. When a linear resistor is positive, two equilibrium points other than the origin are stable in B type. So, they can not generate chaos according to Condition II, either. Therefore, only three oscillators of A type have the possibility to generate chaos. Now, we apply Condition I to them. By deriving variational equations from circuit equations, we obtain the 3rd order characteristic equation generally as follows.

$$\lambda^3 + p\lambda^2 + q\lambda + r = 0 \tag{2}$$

where $\lambda$ is the characteristic root. The stability conditions are given by

$$p = p(\mu) > 0, r = r(\mu) > 0, s = s(\mu) = pq - r > 0 \tag{3}$$

By the calculation, $p$ and $r$ are found to be linear functions of $\mu$, and $s$ is a quardatic one. Situations of graphs of $p(\mu), r(\mu)$,and $s(\mu)$ depends on the system parameters.

When they situates as shown in Fig.4, we can see that the stability of three equilibrium points changes in the most complicated manner as $\mu$ increses. Therefore, when we denote the the roots of $p=0, r=0$, and $s=0$, in Fig.4b as $p_1', r_1'$,and $s_1', s_2'$, respectively, the conditions for generating chaos as follows.

1. $s(0)$ has two real roots. 2. $s_1' > s_2' > r_1' \& p_1'$ 3. $s_1' > \mu > s_2'$ (4)

For A1 and A2, we can choose parameter values so as to satisfy (4) and realize Fig.4. On the other hand, for A3 we can never realize Fig.4 because $p_1' > s_2'$ identically. As a result, we can predict that A1 and A2 can generate chaos when a linear resistance is positive. In fact, chaos can be easily found in A1 and A2 for the parameters satisfying (4), while chaos has never been found in A3.

In the same manner, when a linear resistor is negative, group A and B have the possibility to generate chaos according to Condition II. Using Condition I, A2, A3 and all of group B, i.e., six types of oscillators are expected to generate chaos. An example of the complicated change of stability for group B is shown in Fig.5. This prediction was confirmed by computer simulation. Some examples of chaotic attractors are shown in Fig.6. No exceptions of this classification have been found by the author and reported so far in the literatures.

## 6. Conclusion

In this article, the author has proposed two necessary conditions under which the 3rd order oscillators with a cubic nonlinear active element generate chaos. Using these conditions, 11 types of oscillators which consists of five two-terminal elements were classified into two groups according to whether they generate chaos or not. This result was confirmed by computer simulation.

This article is the revised one which presented in Proc.of NOLTA1995 (International Symposium on Nonlinear Theory and its Applications) [12]. The subsequent results are repoted in [13,14].

[Acknowledgement] The author would like to express his appreciation to his students, Messers. K.Mori, A.Aratani, T.Yamada, T.Oku, T.Ikeda, S.Shizuka, Y.Hatta, T.Doi, K.Isaji, M.Hiraoka and S.Nakase for their valuable assistance.

## References

[1] Y.Ueda: The Road to Chaos, Aerial Press, 1992.

[2] T.Matsumoto, L.O.Chua & M.Komuro: The Double Scroll, IEEE Trans. CAS, CAS-32, 8, pp.797-818 (1985).

[3] M.Shinriki, M.Yamamoto & S.Mori: Multimode oscillations in a modified van der Pol oscillator containing a positive nonlinear conductance, Proc. IEEE, 69, pp.394-395 (1981).

[4] M.Itoh: Chaotic attractor in nonlinear circuit,Thechnical report of IEICE, NLP85-45 (1985) (in Japanese).

[5] N.Inaba, S.Mori & T.Saito: Period doubling bifurcation and chaos in a constrained circuit, Thechnical report of IEICE, NLP86-33 (1986) (in Japanese).

[6] N.Inaba, T.Saito & S.Mori: Chaotic phenomena in a circuit with a negative resistance and an ideal switch of diode, Trans. IEICE, E-70, pp.744-755, Aug (1987).

[7] L.O.Chua: The genesis of Chua's circuit, Archiv Elektronic Ubertransgungstechnik, 46, 4, pp.250-257 (1992).

[8] L.O.Chua: Fundamental Theory and Application, IEEE Trans. CAS, CAS-40, 10&11 (1993).

[9] Chua's Circuit: A Paradigm for Chaos, edited by R.N.Madan, World Scientific, 1993.

[10] L.Pivka & V.Spany: Boundary Surfaces and Basin Bifurcations in Chua's Circuit, J.of Circuits, Systems, and Computers, 3, 2, pp.441-470 (1993).

[11] M.Kuramitsu & K.Mori: A Simple Circuit Generating Chaos, Proc. of NOLTA1994, pp.93-96.

[12] M.Kuramitsu: A Classification of the 3rd Order Oscillators with Respect to Chaos, Proc. of NOLTA1995, pp.559-602.

[13] M.Kuramitsu & S.Sizuka: Parameter Region Where the 3rd Order Oscillator Generates Chaos, Proc. of NOLTA1996, pp.137-140.

[14] M.Kuramitsu & K.Isaji: Admittance Representation of the Necessary Conditions for Chaotic Oscillators, Proc. of NOLTA1997, pp.1101-1104.

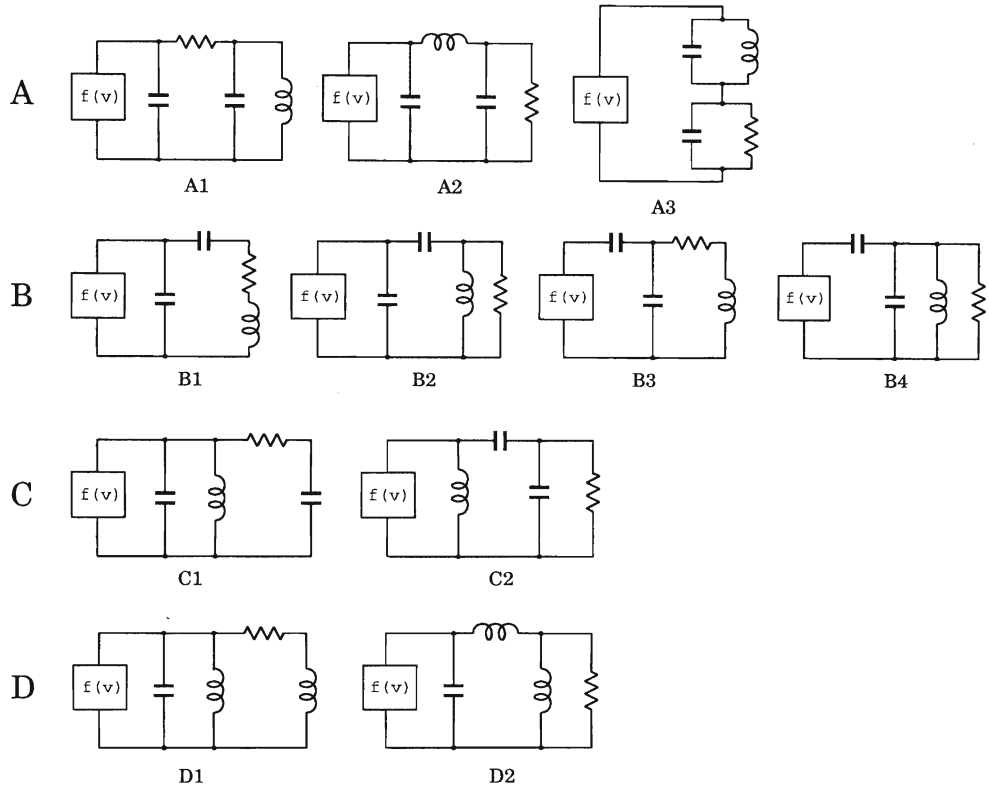


Fig. 1: Classification of 11 types of 3rd order oscillators containing voltage-controlled nonlinear resistor.

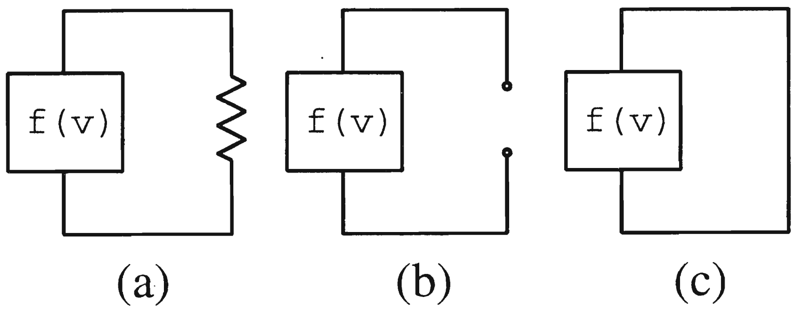


Fig. 2: Resistive circuits for determining equilibrium points.

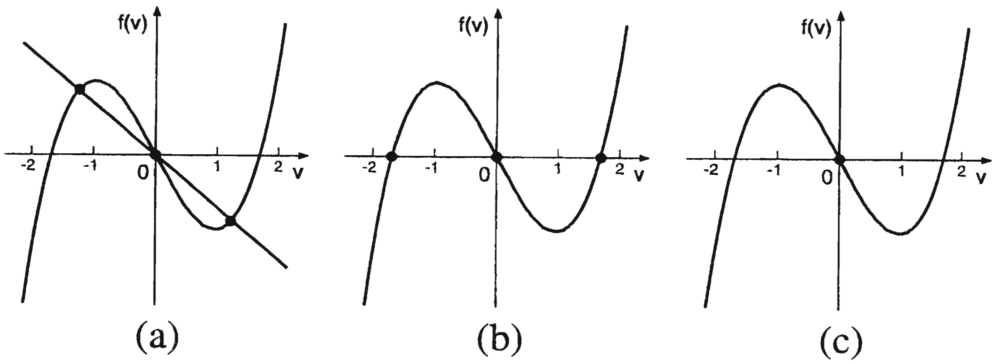


Fig. 3: Characteristics of resistiors for determining equilibrium points.

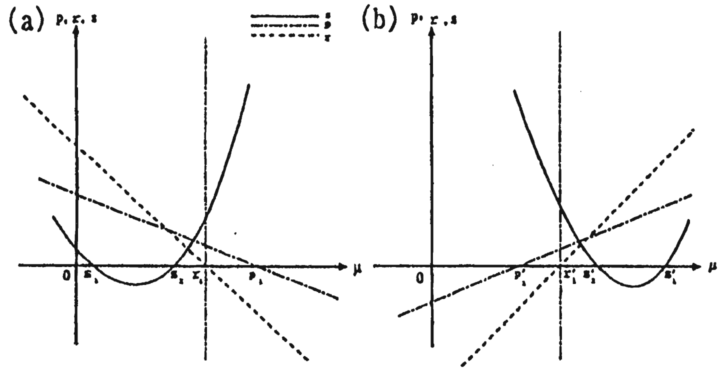


Fig. 4: Stability conditions of A type oscillators.
(a)Equilibrium point of origin
(b)Equilibrium point other than the origin

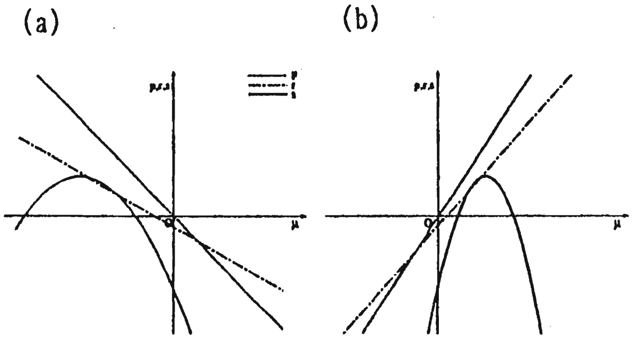


Fig. 5: Stability conditions of B type oscillators.
(a)Equilibrium point of origin
(b)Equilibrium point other than the origin

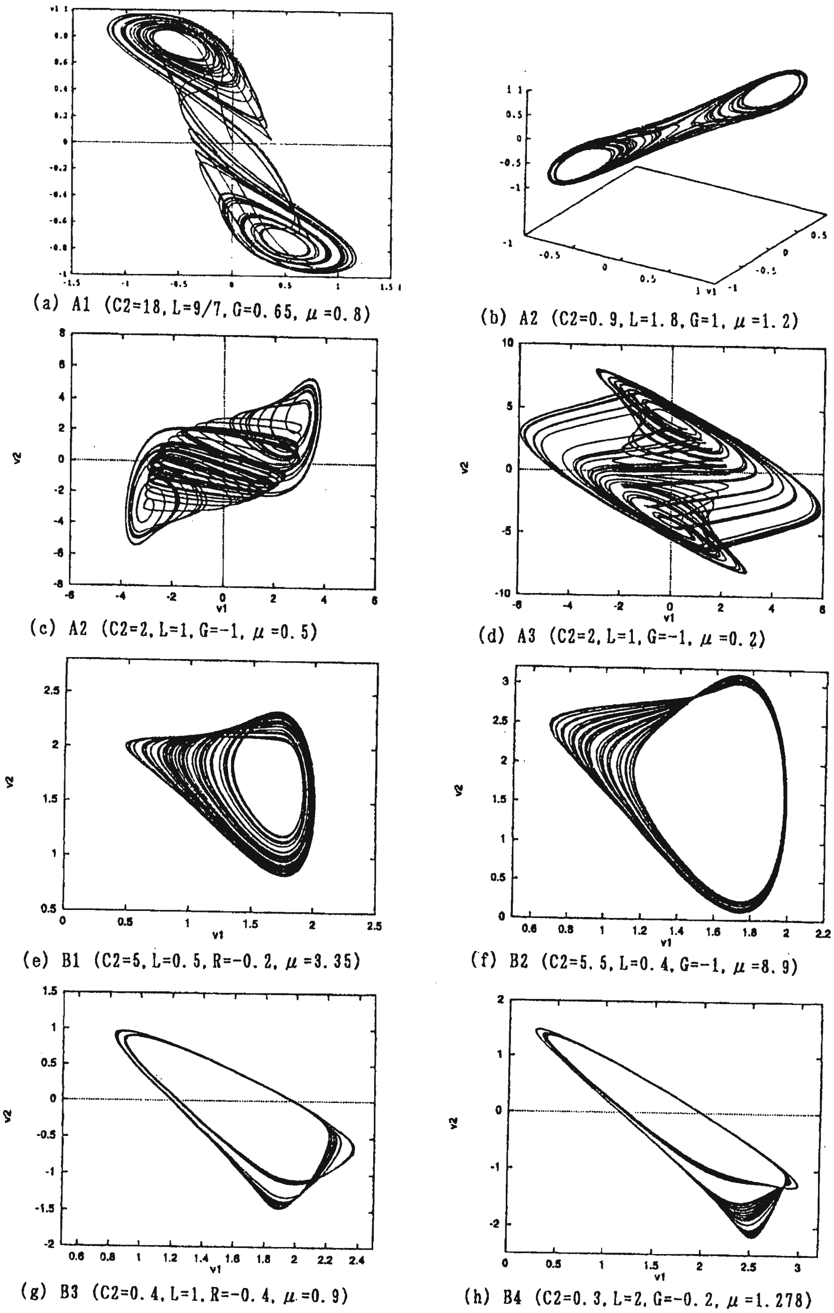


Fig. 6: Some examples of the chaotic attractors ($C_1 = 1.0$).