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Dirichlet form approach to interacting particle systems with long range interactions on $\mathbb{Z}^d$

By

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Abstract

In this paper we present a general theorem of constructing interacting particle systems with long range interactions on discrete spaces. It can be applied to the system that interaction between particles is given by the logarithmic potential. If its equilibrium measure μ is translation invariant we can construct the system whose particles have a summable jump rate. In addition the decay order of jump rate is restricted by the growth order of the 1-correlation function of the measure μ in general cases. The results are the discrete counter part of the results in [2]. In addition we discuss Glauber dynamics whose equilibrium measures are associated with these long range interactions.

§ 1. Introduction

Spitzer [12] and Liggett [5, 6] started the studies of infinite particle systems of jump type with interaction from around 1970's. Let $\mathbb{Z}$ be the set of integers. In most of these systems, particles are moving on the lattice such as $\mathbb{Z}^d$, and their configuration space, for instance $\{0,1\}^{\mathbb{Z}^d}$, are compact with the product topology. Then the systems are described by Feller processes on the configuration space. In this paper the configuration space is taken as $X = \{\xi = \sum_i \delta_{s^i}; s^i \in S, \xi(K) < \infty$ for all compact sets $K \subset \mathbb{R}^d\}$, where δ_a stands for the delta measure at a. X is endowed with the vague topology. Then X is a Polish space. In the following, a point in $\mathbb{Z}^d$ will be denoted by x or y,

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while a configuration over $\mathbb{Z}^d$ will be denoted by ξ or η. Let $n_\xi(x)$ be a function denoting the number of particles at x for ξ. We define

$$\xi^{xy} := \xi + (-\delta_x + \delta_y)\mathbf{1}_{\{n_\xi(x)\geq 1\}} \quad \text{and} \quad \xi \setminus x := \xi - \delta_x \mathbf{1}_{\{n_\xi(x)\geq 1\}},$$

where $\mathbf{1}_\omega$ denotes the indicator of ω; $\mathbf{1}_\omega = 1$ if ω is satisfied and $\mathbf{1}(\omega) = 0$ otherwise. In the following we introduce a non-negative measurable function $c(\xi, x; y)$, $(\xi, x, y) \in X \times \mathbb{Z}^d \times \mathbb{Z}^d$. The function c controls the jump rate of the particle at x to y under the configuration ξ and is called a rate function. Suppose that $c(\xi, x; y) = 0$ if $n_\xi(x) = 0$, and

$$c(\xi, x; y) = p(x, y) + p(x, y)\frac{d\mu_y}{d\mu_x}(\xi \setminus x)\frac{\rho^1(y)}{\rho^1(x)}, \quad \text{if } n_\xi(x) \geq 1,$$

with some positive symmetric measurable function p on $\mathbb{Z}^d \times \mathbb{Z}^d$ and a probability measure μ on X. Here μ_x is the conditional probability measure defined by

$$\mu_x = \mu\,(\,\cdot \setminus x |\, n_\xi(x) \geq 1) \quad \text{for } x \in \mathbb{Z}^d, \tag{1.1}$$

$\rho^1(x)$ is the 1-correlation function of μ defined by $\rho^1(x) = \int_X \xi(x)d\mu$ for $x \in \mathbb{Z}^d$ and $d\mu_y/d\mu_x$ is the Radon-Nikodym derivative of μ_y with respect to μ_x. Then we introduce the linear operator Ω on the space of local functions $\mathscr{D}_\circ$ in (2.1) by the following.

$$\Omega f(\xi) = \sum_{x\in\mathbb{Z}^d} n_\xi(x) \sum_{y\in\mathbb{R}^d} c(\xi, x; y)[f(\xi^{xy}) - f(\xi)].$$

In addition we define the associated bilinear form $\mathfrak{E}$ on $\mathfrak{D}_\infty$ in (2.2) by the following.

$$\mathfrak{E}(f, g) = \frac{1}{2}\int_X d\mu \sum_{x\in\mathbb{Z}^d} n_\xi(x) \sum_{y\in\mathbb{Z}^d} p(x, y)\{f(\xi^{xy}) - f(\xi)\}\{g(\xi^{xy}) - g(\xi)\}. \tag{1.2}$$

Then by using Liggett's theorem [6], we can construct the Feller process generated by the closure of Ω under suitable assumptions on the rate function c. This process describe an interacting particle system of jump type on $\mathbb{Z}^d$. The rate function c satisfies the following detailed balance condition in this situation.

$$c(\xi^{xy}, y; x) = c(\xi, x; y)\frac{\rho^1(x)}{\rho^1(y)}\frac{d\mu_x}{d\mu_y}(\xi \setminus y), \quad x, y \in \mathbb{Z}^d.$$

Hence μ is a reversible measure of these dynamics. In addition the bilinear form $\mathfrak{E}$ in (1.2) is the Dirichlet form associated with these dynamics. However we can not apply the above argument by Liggett's theorem to construct infinite particle systems with long range interactions such as the ones associated with the logarithmic potential. This is because a value of the rate function of some configuration diverges for the strong effects from particles faraway. To conquer the difficulties we need to use another methods.

The diffusion processes on general Polish spaces that may be non locally compact are constructed by the Dirichlet form theory in Kusuoka [4], Ma-Röckner [8], Osada [9] and others. The infinite particle system of jump type with interaction on continuum space was also constructed by Kondratiev-Lytvynov-Röckner [3], Lytvynov-Ohlerich [7], Esaki [2] and others. In [2] we give the general theory to construct interacting particle systems. The systems include some Kawasaki dynamics for determinantal random point fields associated with the operator K whose eigenvalue set $\mathrm{Spec}(K)$ contain 1. In this paper we apply the method to construct interacting particle systems with long range interactions on the discrete space $\mathbb{Z}^d$. The main theorem in this paper is the discrete counter part of the results in [2].

This paper is organized as follows. In Section 2 we introduce some notations and our main theorem in this paper. We give the proof of our main result in Section 3. In Section 4 we discuss the results for exclusion case. We introduce the definition of the discrete version quasi-Gibbs measures and give a sufficient condition of closability of our bilinear form in the exclusion case in Section 5. In Section 6 we introduce a known result related to the process constructed in this paper. We give comments of the L^2-generator associated with our Dirichlet form in Section 7. In Section 8 we consider the Glauber dynamics whose equilibrium measures associated with the above long range interaction potentials.

§ 2. Set up and main results

Let S be a subset in $\mathbb{Z}^d$ such that $0 \in S$. The distance on S is denoted by d. Let $X = \{\xi = \sum_i \delta_{s^i}; \xi(K) < \infty$ for all compact sets $K \subset S\}$, where δ_a stands for the delta measure at a. We endow X with the vague topology. Then X is a Polish space. We call X the configuration space over S. For any $\xi \in X$ there exist a function $n_\xi : S \to \mathbb{N} \cup \{0\}$ such that $\xi = \sum_{x \in S} n_\xi(x)\delta_x$. We note that $n_\xi(x)$ denotes the number of particles at x for ξ. We introduce a bilinear form to describe our infinite particle system. For $n \in \mathbb{N} \cup \{\infty\}$ let $X^n = \{\xi \in X; \xi(S) = n\}$. Let $S^\infty = \{(x^n)_{1 \le n < \infty}; (x^n)_{1 \le n < \infty}$ have no cluster points in $S\}$. We introduce a map $\boldsymbol{x}_n = (x^1, x^2, \dots, x^n); X^n \to S^n$ such that $\xi = \sum_{k=1}^n \delta_{x^k(\xi)}$. We call the map a S^n-coordinate of ξ. Let for $1 \le n \le \infty$,

$$D^n[f,g](\boldsymbol{x}) = \frac{1}{2} \sum_{j=1}^{n} \sum_{y \in S} \nabla_j^y f(\boldsymbol{x}) \nabla_j^y g(\boldsymbol{x}) p(x^j, y),$$

where for $\boldsymbol{x} = (x^1, \dots, x^n) \in S^n$ and $f : S^n \to \mathbb{R}$, we set $\nabla_j^y f(\boldsymbol{x}) = f(\boldsymbol{x}_j^y) - f(\boldsymbol{x})$ and $\boldsymbol{x}_j^y = (x^1, \dots, x^{j-1}, y, x^{j+1}, \dots, x^n)$. Here $p : S^2 \to [0, \infty)$ satisfies that $p(x,y)$ depends only on the distance between x and y and $\sum_{y \in \mathbb{Z}^d} p(0,y) < \infty$. Let $\pi_A : X \to X$ be $\pi_A(\xi) = \xi(\cdot \cap A)$ for a subset $A \subset S$. To simplify notations we write π_r and π_r^c

instead of π_{S_r} and $\pi_{S_r^c}$ respectively, where $S_r = \{x \in S; d(0,x) \le r\}$. We set $X_r^n = \{\xi \in X; \xi(S_r) = n\}$. We note that $X = \sum_{n=0}^{\infty} X_r^n$. A function $\boldsymbol{x}_{r,n} = (x^1, \dots, x^n) : X_r^n \to S_r^n$ is called a S_r^n-coordinate (or a coordinate on X_r^n) of ξ if $\pi_r(\xi) = \sum_{k=1}^n \delta_{x^k(\xi)}$ holds. For $\mathfrak{f} : X \to \mathbb{R}$ a function $f_{r,\xi}^n(x) : X \times S_r^n \to \mathbb{R}$ is called the S_r^n-representation of $\mathfrak{f}$ if $f_{r,\xi}^n$ satisfies the following :

(1) $f_{r,\xi}^n(x)$ is a permutation invariant function on S_r^n for each $\xi \in X$.

(2) $f_{r,\xi_{(1)}}^n(x) = f_{r,\xi_{(2)}}^n(x)$ if $\pi_r^c(\xi_{(1)}) = \pi_r^c(\xi_{(2)})$, $\xi_{(1)}, \xi_{(2)} \in X_r^n$.

(3) $f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi)) = \mathfrak{f}(\xi)$ for $\xi \in X_r^n$, where $\boldsymbol{x}_{r,n}(\xi)$ is a S_r^n-coordinate of ξ.

(4) $f_{r,\xi}^n(x) = 0$ for $\xi \notin X_r^n$.

Note that $f_{r,\xi}^n$ is unique and $\mathfrak{f}(\xi) = \sum_{n=0}^{\infty} f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi))$. When $\mathfrak{f}$ is $\sigma[\pi_r]$-measurable, S_r^n-representations are independent of ξ. In this case we often write f_r^n instead of $f_{r,\xi}^n$. Let $\mathscr{B}_r^* = \{\mathfrak{f} : X \to \mathbb{R};\ \mathfrak{f} \text{ is } \sigma[\pi_r]\text{-measurable}\}$ and $\mathscr{B}_r = \{\mathfrak{f} \in \mathscr{B}_r^*; \mathfrak{f} \text{ is bounded}\}$. We set

$$\mathscr{B}_\infty^* = \bigcup_{r=1}^{\infty} \mathscr{B}_r^*, \quad \mathscr{B}_\infty = \bigcup_{r=1}^{\infty} \mathscr{B}_r.$$

Moreover we set

$$\mathscr{D}_\circ = \{\mathfrak{f} \in \mathscr{B}_\infty^*; f_{r,\xi}^n(x) \text{ are continuous on } S_r^n \text{ for all } n, r, \xi\}, \tag{2.1}$$

where $f_{r,\xi}^n$ are S_r^n-representations of $\mathfrak{f}$. For $\mathfrak{f}, \mathfrak{g} \in \mathscr{D}_\circ$ we set $\mathbb{D}[\mathfrak{f}, \mathfrak{g}] : X \to \mathbb{R}$ by

$$\mathbb{D}[\mathfrak{f}, \mathfrak{g}](\xi) = \begin{cases} D^n[f^n, g^n](\boldsymbol{x}_n(\xi)) & \text{for } \xi \in X^n, 1 \le n \le \infty, \\ 0 & \text{for } \xi \in X^0. \end{cases}$$

Here $\boldsymbol{x}_n$ is a S^n-coordinate and f^n is the permutation invariant function on $\mathbb{R}^{(n)}$ such that $f(\xi) = f^n(\boldsymbol{x}_n(\xi))$ for all $\xi \in X^n$. We set g^n similarly. Note that such f^n and g^n are unique for each n ($1 \le n \le \infty$) and $\mathbb{D}$ is well defined. We set

$$\begin{aligned} \mathscr{E}(\mathfrak{f}, \mathfrak{g}) &= \int_X \mathbb{D}[\mathfrak{f}, \mathfrak{g}](\xi) d\mu, \\ \mathscr{D}_\infty &= \{\mathfrak{f} \in \mathscr{D}_\circ \cap L^2(X, \mu); \mathscr{E}(\mathfrak{f}, \mathfrak{f}) < \infty\}. \end{aligned} \tag{2.2}$$

We say a nonnegative permutation invariant function ρ^n on S^n is the n-correlation function of μ if

$$\sum_{(x^1,\dots,x^{k_1}) \in A_1^{k_1}} \cdots \sum_{(x^{n-k_m+1},\dots,x^n) \in A_m^{k_m}} \rho^n(x^1, \dots, x^n) = \int_X \prod_{i=1}^m \frac{\xi(A_i)!}{(\xi(A_i) - k_i)!} d\mu$$

for any sequence of disjoint bounded measurable subsets $A_1, \ldots, A_m \subset S$ and a sequence of natural numbers $k_1, \ldots, k_m$ satisfying $k_1 + \cdots + k_m = n$.

Permutation invariant functions $\sigma_r^n : S_r^n \to \mathbb{R}^+$ are called density functions of μ if

$$\frac{1}{n!} \sum_{x \in S_r^n} f_r^n(x) \sigma_r^n(x) = \int_{X_r^n} \mathfrak{f}(\xi) d\mu(\xi)$$

for all bounded $\sigma[\pi_r]$-measurable functions $\mathfrak{f}$. Here $f_r^n : S_r^n \to \mathbb{R}$ is the permutation invariant function such that $f_r^n(\boldsymbol{x}_{r,n}(\xi)) = \mathfrak{f}(\xi)$ for $\xi \in X_r^n$, where $\boldsymbol{x}_{r,n}$ is a S_r^n-coordinate.

We assume:

$$(\mathscr{E}, \mathscr{D}_\infty) \text{ is closable on } L^2(X, \mu), \tag{A.1}$$

$$\sigma_r^k \text{ is bounded for all } k, r \in \mathbb{N}. \tag{A.2}$$

$$\sum_{n=1}^{\infty} n\mu(X_r^n) < \infty \text{ for all } r \in \mathbb{N}. \tag{A.3}$$

By (A.1) we denote by $(\mathscr{E}, \mathscr{D})$ the closure of $((\mathscr{E}, \mathscr{D}_\infty), L^2(X, \mu))$. We assume following additional conditions;
(B.0) There exists a function $p_0(r)$ on $(0, \infty)$ such that $p(x, y) \le C_1 p_0(d(x, y))$ for μ-a.s. $\xi \in X$ and all $x, y \in S$.

$$\rho^1(x) = O\left(|x|^\kappa\right) \text{ as } |x| \to \infty \text{ for some } \kappa \ge 0. \tag{B.1}$$

$$p_0(r) = O(r^{-(d+\alpha)}) \text{ as } r \to \infty \text{ for some } \alpha > \kappa. \tag{B.2}$$

$$\frac{\mathrm{Var}\left[\xi(S_r)\right]}{\left(\mathbb{E}\left[\xi(S_r)\right]\right)^2} = O\left(r^{-\delta}\right) \quad \text{as } r \to \infty \text{ for some } \delta > 0. \tag{B.3}$$

Now we state our main theorem:

Theorem 2.1. *Suppose that (A.1)–(A.3), (B.0)–(B.3) hold. Then* $(\mathscr{E}, \mathscr{D})$ *is a quasi-regular Dirichlet form on* $L^2(X, \mu)$.

By virtue of [8, Theorem IV.3.5 and Theorem IV.5.1] we can show the following proposition.

Corollary 2.2. *Suppose that (A.1)–(A.3), (B.0)–(B.3) hold. Then there exists a special standard process* $\{\mathbb{P}_\xi\}_{\xi \in X}$ *associated with* $((\mathscr{E}, \mathscr{D}), L^2(X, \mu))$. *Moreover* $\{\mathbb{P}_\xi\}_{\xi \in X}$ *is reversible with invariant measure* μ.

Remark. (i) Condition (B.1) and (B.2) imply that there exists a constant C_2 such that

$$\sum_{x \in S} \rho^1(x)p(x,y) < C_2\rho^1(y), \tag{2.3}$$

for all compact subset A. The property (2.3) is necessary to construct the infinite particle systems of independent jump type processes. Hence Conditions (B.1) and (B.2) are reasonable.
(ii) The LHS of (B.3) is represented by the 1 and 2-correlation functions of μ by the following:

$$\frac{\mathrm{Var}\left[\xi(S_r)\right]}{\left(\mathbb{E}\left[\xi(S_r)\right]\right)^2} = \frac{\sum_{x\in S_r}\rho^1(x) - \sum_{(x^1,x^2)\in S_r^2}\left(\rho^1(x^1)\rho^1(x^2) - \rho^2(x^1,x^2)\right)}{\left(\sum_{x\in S_r}\rho^1(x)\right)^2}.$$

By the expression we can check that (B.4) holds if μ is the Poisson random point field with respect to Lebesgue measure or μ is a determinantal point field.

§ 3. Proof of Theorem 2.1

We need to show the following lemma.

Lemma 3.1. *Let $\mathfrak{f} \in \mathscr{B}_r$. Then we have $\mathfrak{f} \in \mathscr{D}_\infty$.*

Proof. First we check $\mathscr{E}(\mathfrak{f},\mathfrak{f}) < \infty$. It is enough to show this condition in the case when p_0 is decrease.

$$\mathscr{E}(\mathfrak{f},\mathfrak{f}) = \frac{1}{2}\int_X d\mu \sum_{k=1}^{\infty}\sum_{y\in S}\left\{\nabla_k^y \widetilde{f}(\boldsymbol{x}_\infty(\xi))\right\}^2 p(x^k(\xi),y), \tag{3.1}$$

Here the symmetric function $\widetilde{f}$ is the associated function with a local function f on X such that $f(\xi) = \widetilde{f}((x^j(\xi))_{j\in\mathbb{N}})$, where $\boldsymbol{x}_\infty = (x^j(\xi))_{j\in\mathbb{N}}$ is the S^∞-coordinate of ξ. We divide the right hand side of (3.1) into three terms as $I_1 + I_2 + I_3$. Here we set

$$I_1 = \frac{1}{2}\sum_{n=0}^{\infty}\int_{X_r^n} d\mu \left\{\sum_{x^m(\xi)\in S_r}\sum_{y\in S_r}\{\nabla_m^y f_r^n(\boldsymbol{x}_{r,n}(\xi))\}^2 p(x^m(\xi),y)\right\},$$

$$I_2 = \frac{1}{2}\sum_{n=0}^{\infty}\int_{X_r^n} d\mu \left\{\sum_{x^m(\xi)\in S_r}\sum_{y\in S_r^c}\{\nabla_m^* f_r^n(\boldsymbol{x}_{r,n}(\xi))\}^2 p(x^m(\xi),y)\right\},$$

$$I_3 = \frac{1}{2}\sum_{n=0}^{\infty}\int_{X_r^n} d\mu \left\{\sum_{x^m(\xi)\in S_r^c}\sum_{y\in S_r}\left\{\nabla_{n+1}^y f_r^n(\boldsymbol{x}_{r,n}(\xi))\right\}^2 p(x^m(\xi),y)\right\},$$

where for $\boldsymbol{x}_{r,n}(\xi) = (x^1(\xi), \ldots, x^n(\xi))$ we set

$$\nabla_j^* f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi)) = f_{r,\xi}^{n-1}(\boldsymbol{x}_{r,n}^{\langle j \rangle}(\xi)) - f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi)),$$
$$\nabla_{n+1}^y f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi)) = f_{r,\xi}^{n+1}(\boldsymbol{x}_{r,n}(\xi) \cdot y) - f_{r,\xi}^n(\boldsymbol{x}_{r,n}(\xi)),$$

and for $\boldsymbol{x} = (x^j)_{j=1}^n \in S^n$ we set

$$\boldsymbol{x}^{\langle j \rangle} = (x^1, \ldots, x^{j-1}, x^{j+1}, \ldots, x^n) \in S^{n-1},$$
$$\boldsymbol{x} \cdot y = (x^1, \ldots, x^n, y) \in S^{n+1}.$$

Firstly we calculate I_1.

$$(3.2)\quad I_1 \le \sum_{n=0}^\infty \int_{X_r^n} d\mu \left\{ \sum_{x^m(\xi) \in S_r} \sum_{y \in S_r} \left\{ f_r^n(\{\boldsymbol{x}_{r,n}(\xi)\}_m^y)^2 + f_r^n(\boldsymbol{x}_{r,n}(\xi))^2 \right\} p(x^m(\xi), y) \right\}$$
$$\le \sum_{n=0}^\infty \int_{X_r^n} d\mu \left\{ \sum_{x^m(\xi) \in S_r} \sum_{y \in S_r} 2C_{\mathfrak{f}}^2 p(x^m(\xi), y) \right\}$$
$$\le C_4 \sum_{n=0}^\infty \int_{X_r^n} \sum_{x \in S_r} n_\xi(x) d\mu = C_4 \sum_{n=0}^\infty n\mu\left(X_r^n\right) < \infty,$$

where $C_3 = C_3(\mathfrak{f}) = \sup_{\xi \in X} \mathfrak{f}(\xi) < \infty$ and $C_4 = C_4(\mathfrak{f}, p) = 2C_3^2 \cdot \sum_{y \in \mathbb{Z}^d} p(0, y) < \infty$. I_2 is calculated by the same way of I_1. In addition we calculate I_3.

(3.3)

$$I_3 \le \sum_{n=0}^\infty \int_{X_r^n} d\mu \left\{ \sum_{x^m(\xi) \in S_r^c} \sum_{y \in S_r} \left\{ f_r^{n+1}(\boldsymbol{x}_{r,n}(\xi) \cdot y)^2 + f_r^n(\boldsymbol{x}_{r,n}(\xi))^2 \right\} p(x^m(\xi), y) \right\}$$
$$\le 2C_{\mathfrak{f}} \int_X d\mu \left\{ \sum_{x^m(\xi) \in S_r^c} \sum_{y \in S_r} p(x^m(\xi), y) \right\}.$$

For a integer $v \ge r+1$, $x \in S_v \setminus S_{v-1}$ and $y \in S_r$ we can check

$$p(x, y) \le C_1 p_0(d(x, y)) \le C_1 p_0(v - r).$$

Hence we have

$$(3.4)\qquad \text{The RHS of (3.3)} \le 2C_{\mathfrak{f}} \int_X d\mu \left\{ \sum_{v=r+1}^\infty \sum_{x^m(\xi) \in S_v \setminus S_{v-1}} \sum_{y \in S_r} C_1 p_0(v - r) \right\}$$
$$\le 2C_{\mathfrak{f}} \int_X d\mu \left\{ \sum_{v=r+1}^\infty \sum_{x \in S_r} n_\xi(x) |S_r| C_1 p_0(v - r) \right\}$$
$$\le 2C_{\mathfrak{f}} C_1 |S_r| \sum_{v=r+1}^\infty \left\{ \sum_{|x|=v} \rho(x) \right\} p_0(v - r).$$

Since $\sum_{|x|=v} \rho(x) = O(v^{d-1+\kappa})$, $p_0(v-r) = O(v^{-d-\alpha})$ and $\alpha > \kappa$, we check that the RHS of (3.4) is finite. Combining this and (3.2) we conclude $\mathscr{E}(\mathfrak{f}, \mathfrak{f}) < \infty$. Since $\mathfrak{f}$ is bounded, it is proved that $\mathfrak{f} \in L^2(X, \mu)$. Hence we can see $\mathfrak{f} \in \mathscr{D}_\infty$. Thus the proof is completed. □

For the reader's convenient we give the definition of quasi-regular Dirichlet form.

Definition 3.2. A symmetric Dirichlet form $(\mathfrak{E}, \mathfrak{D}(\mathfrak{E}))$ on $L^2(X, m)$ is called quasi-regular if $(\mathfrak{E}, \mathfrak{D}(\mathfrak{E}))$ satisfies the following:

(Q.1) There exists an $\mathfrak{E}$-nest consisting of compact sets.

(Q.2) There exists an $|| \cdot ||_1$-dence subset of $\mathfrak{D}(\mathfrak{E})$ whose elements have $\mathfrak{E}$-continuous m-versions. Here $||u||_1^2 = ||u||^2_{L^2(X,m)} + \mathfrak{E}(u, u)$.

(Q.3) There exist $u_n \in \mathfrak{D}(\mathfrak{E})$, $n \in \mathbb{N}$, having $\mathfrak{E}$-continuous m-versions $\widetilde{u}_n$, and an $\mathfrak{E}$-exceptional set N such that $\{\widetilde{u}_n\}$ separates the points of $X - N$, i.e. for every pair (s_1, s_2) of distinct points of $X - N$, there exists a function $\widetilde{u}_n$ which satisfies $\widetilde{u}_n(s_1) \neq \widetilde{u}_n(s_2)$.

Proof of Theorem 2.1. The condition (Q.1) is showed using by the slight modified Lemmas of the proof of [2, Theorem 2.1]. Since $\mathfrak{D}_\infty \subset C(X)$ and $\mathfrak{D}_\infty$ is dence in $\mathfrak{D}$, (Q.2) is clear. Let

$$U_r^n = \{u \in C(S_r^n);\ u \text{ is permutation invariant}\}.$$

We regard elements of U_r^n as functions on $S_r^n/\sim$. Here $\sim$ is the equivalence relation generated by permutations. For each $n, r \in \mathbb{N}$ let $\{u_{r,m}^n\}_{m\in\mathbb{N}}$ be a sequence in U_r^n that separates the points of $S_r^n/\sim$. We can choose $\{u_{r,m}^n\}_{m\in\mathbb{N}}$ so as $n < u_{r,m}^n(x) \leq n+1$ for all $x \in S_r^n$. Let $\mathfrak{u}_{r,m}^n \in \mathscr{B}_r$ be such that $\mathfrak{u}_{r,m}^n(\xi) = 0$ for $\xi \notin X_r^n$, and $\mathfrak{u}_{r,m}^n(\xi) = u_{r,m}^n(\boldsymbol{x}_{r,n}(\xi))$ for $\xi \in X_r^n$, where $\boldsymbol{x}_{r,n}(\xi)$ is a S_r^n-coordinate of ξ. We set $\mathfrak{u}_{r,m}^0 \equiv 0$ when $n = 0$. Then $\mathscr{U} = \{\mathfrak{u}_{r,m}^n\}_{n,r,m\in\mathbb{N}}$ separates the points of X. From Lemma 3.1 we show that $\mathscr{U}$ is a sequence in $\mathscr{D}_\infty$. Hence we obtain (Q.3). Thus the proof is completed. □

§ 4. Results for the exclusion case

In this section we discuss the exclusion case. In the exclusion case the interacting particle systems are constructed more simply. Let S be a subset in $\mathbb{Z}^d$ such that $0 \in S$. A distance on S is denoted by d. We set $X_{\text{ex}} = \{0, 1\}^S$. For $\xi \in X_{\text{ex}}$, $\xi(x)$ denotes a value of ξ at $x \in S$. We endow X_{ex} with the product topology. Then X_{ex} is the Polish space and by Tychonoff's theorem X_{ex} is a compact set with the topology. We call X_{ex} the exclusive configuration space over S. In the following we call the exclusive configuration the configuration to simplify. We regard $\xi(x)$ as the occupation by the particle at x i.e. if $\xi(x)=1$ then the site x is occupied, if $\xi(x)=0$ then the site x is empty. Let $\operatorname{supp}\xi=\{x\in S;\xi(x)=1\}$. Firstly we introduce bilinear form to describe our infinite particle system. We define $\pi_r : X_{\mathrm{ex}} \to X_{\mathrm{ex}}$ by

$$\pi_r(\xi)=\begin{cases}\xi(x) & \text{if } x\in S_r,\\ 0 & \text{if } x\notin S_r.\end{cases}$$

Let $\mathscr{B}_r^{\mathrm{ex}}=\{\mathfrak{f}: X_{\mathrm{ex}}\to\mathbb{R};\ \mathfrak{f}\text{ is }\sigma[\pi_r]\text{-measurable}\}$. We set $\mathscr{D}_\circ^{\mathrm{ex}}=\bigcup_{r=1}^\infty\mathscr{B}_r^{\mathrm{ex}}$.Let $C(X_{\mathrm{ex}})$ be a set of all continuous functions on X_{ex}. It is easily seen that $\mathscr{D}_\circ^{\mathrm{ex}}\subset C(X_{\mathrm{ex}})$. We note $\mathfrak{f}$ is bounded for all $\mathfrak{f}\in\mathscr{D}_\circ^{\mathrm{ex}}$ by definition. For $\mathfrak{f},\mathfrak{g}\in\mathscr{D}_\circ^{\mathrm{ex}}$ we set $\mathbb{D}_{\mathrm{ex}}[\mathfrak{f},\mathfrak{g}]: X_{\mathrm{ex}}\to\mathbb{R}$ by

$$\mathbb{D}_{\mathrm{ex}}[\mathfrak{f},\mathfrak{g}](\xi)=\frac{1}{2}\sum_{x\in\operatorname{supp}\xi}\sum_{y\in S}\nabla^{xy}f(\xi)\nabla^{xy}g(\xi)p(x,y),$$

where $\nabla^{xy}f(\xi)=f(\xi^{xy})-f(\xi)$ with

$$\xi^{xy}(u)=\begin{cases}\xi(y), & \text{if } u=x,\\ \xi(x), & \text{if } u=y,\\ \xi(u), & \text{if } u\neq x,y.\end{cases}$$

Here $p: S^2\to[0,\infty)$ satisfies that $p(x,y)$ depends only on the distance between x and y and $\sum_{y\in\mathbb{Z}^d}p(0,y)<\infty$. We set

$$\mathscr{E}_{\mathrm{ex}}(\mathfrak{f},\mathfrak{g})=\int_{X_{\mathrm{ex}}}\mathbb{D}_{\mathrm{ex}}[\mathfrak{f},\mathfrak{g}](\xi)d\mu,$$
$$\mathscr{D}_\infty^{\mathrm{ex}}=\{\mathfrak{f}\in\mathscr{D}_\circ^{\mathrm{ex}}\cap L^2(X_{\mathrm{ex}},\mu);\mathscr{E}_{\mathrm{ex}}(\mathfrak{f},\mathfrak{f})<\infty\}.$$

We assume:

(F.1) $\qquad (\mathscr{E}_{\mathrm{ex}},\mathscr{D}_\infty^{\mathrm{ex}})$ is closable on $L^2(X_{\mathrm{ex}},\mu)$,

By (F.1) we denote by $(\mathscr{E}_{\mathrm{ex}},\mathscr{D}_{\mathrm{ex}})$ the closure of $((\mathscr{E}_{\mathrm{ex}},\mathscr{D}_\infty^{\mathrm{ex}}),L^2(X_{\mathrm{ex}},\mu))$. Now we state an proposition to construct interacting particle systems for the exclusion case. Since X_{ex} is a compact set then we can prove the following arguments.

Proposition 4.1. *Suppose that (F.1) holds. Then $(\mathscr{E}_{\mathrm{ex}},\mathscr{D}_{\mathrm{ex}})$ is a regular Dirichlet form on $L^2(X_{\mathrm{ex}},\mu)$.*

Corollary 4.2. *Suppose that (F.1) holds. Then there exists a Hunt process $\{\mathbb{P}_\xi\}_{\xi\in X_{\mathrm{ex}}}$ associated with $((\mathscr{E}_{\mathrm{ex}},\mathscr{D}_{\mathrm{ex}}),L^2(X_{\mathrm{ex}},\mu))$. Moreover $\{\mathbb{P}_\xi\}_{\xi\in X_{\mathrm{ex}}}$ is reversible with invariant measure μ.*

§ 5. Sufficient condition of closability

In this section we give a sufficient condition of the closability for the exclusion case. Firstly we introduce a Hamiltonian on a bounded set $\boldsymbol{x} \subset S_r$ as follows. For measurable functions $\Phi : S \to \mathbb{R} \cup \{\infty\}$ and $\Psi : S \times S \to \mathbb{R} \cup \{\infty\}$ with $\Psi(x, y) = \Psi(y, x)$, let

$$\mathcal{H}_r^{\Phi,\Psi}(\boldsymbol{x}) = \sum_{x^i \in \boldsymbol{x}} \Phi(x^i) + \sum_{x^i, x^j \in \boldsymbol{x}, i<j} \Psi(x^i, x^j), \quad \text{where } \boldsymbol{x} = \{x^i\} \subset S_r.$$

We assume $\Phi < \infty$ a.e. to avoid triviality. For two measure ν_1, ν_2 on a measurable space $(\Omega, \mathcal{B})$ we write $\nu_1 \le \nu_2$ if $\nu_1(A) \le \nu_2(A)$ for all $A \in \mathcal{B}$. We say a sequence of finite Radon measures $\{\nu^N\}$ on a Polish space Ω convergence weakly to a finite Radon measure ν if $\lim_{N\to\infty} \int f d\nu^N = \int f d\nu$ for all $f \in C_b(\Omega)$. Next, we introduce the quasi-Gibbs measure. For an increasing sequence $\{b_r\}$ of natural numbers, we write $B_r = S_{b_r}$ for all $r \in \mathbb{N}$.

Definition 5.1. A probability measure μ is said to be a (Φ, Ψ)-quasi Gibbs measure if there exists an increasing sequence $\{b_r\}$ of natural numbers and measures $\{\mu_{r,k}\}$ such that, for each $r \in \mathbb{N}$, $\mu_{r,k}$ and $\mu_r := \mu \circ \pi_{B_r}$ satisfy

$$\mu_{r,k} \le \mu_{r,k+1} \text{ for all } k, \quad \lim_{k\to\infty} \mu_{r,k} = \mu_r \quad \text{weakly},$$

and that, for all $r, k \in \mathbb{N}$, for $\mu_{r,k}$-a.e. $\xi \in X_{\mathrm{ex}}$ and for $\boldsymbol{x} \subset B_r$,

$$C_5^{-1} e^{-\mathcal{H}(\boldsymbol{x})} \le \mu_{r,k,\xi}(\boldsymbol{x}) \le C_5 e^{-\mathcal{H}(\boldsymbol{x})}.$$

Here $\mathcal{H}(\boldsymbol{x}) = \mathcal{H}_{b_r}^{\Phi,\Psi}(\boldsymbol{x})$, $C_5 = C_5(r, k, \pi_{B_r^c}(\xi))$ is positive constant and $\mu_{r,k,\xi}$ is the conditional probability measure of $\mu_{r,k}$ defined by

$$\mu_{r,k,\xi}(\boldsymbol{x}) = \mu_{r,k}\left(\mathrm{supp}\{\pi_{B_r}(\eta)\} = \boldsymbol{x} \,|\, \pi_{B_r^c}(\eta) = \pi_{B_r^c}(\xi)\right).$$

We remark that (Φ, Ψ)-canonical Gibbs measures are (Φ, Ψ)-quasi Gibbs measures. The converse can not be true.

We assume

(QG.1) μ is a (Φ, Ψ)-quasi Gibbs measure,

(QG.2) $\Gamma := \{s; \Psi(0, s) = \infty\}$ is a compact set.

Theorem 5.2. *Assume (QG.1)–(QG.2). Then* $((\mathscr{E}_{\mathrm{ex}}, \mathscr{D}_\infty^{\mathrm{ex}}), L^2(X_{\mathrm{ex}}, \mu))$ *is closable.*

Proof. The proof of Theorem 5.2 is the same as the one of [2, Theorem 2.5]. □

Example 5.3. We give examples of quasi-Gibbs measures.
(i) All canonical Gibbs measures are also quasi-Gibbs measures. Here we give a typical example of these measures, which are given by a potential with polynomial decay. Let $S = \mathbb{Z}^d$ and $\alpha > d$. We set self potential $\Phi(x) = 0$ and interaction potential $\Psi(x, y) = |x - y|^{-\alpha}$. It is known that for their potentials the correspond random point field μ_{pol} is a canonical Gibbs measure, thus also is a quasi-Gibbs measure. Hence we can apply our main result for μ_{pol}. Then we can construct a μ_{pol}-reversible Hunt process $\{\mathbb{P}_\xi^{\mathrm{pol}}\}_{\xi \in X_{\mathrm{ex}}}$ associated with $((\mathscr{E}^{\mathrm{pol}}, \mathscr{D}^{\mathrm{pol}}), L^2(X_{\mathrm{ex}}, \mu_{\mathrm{pol}}))$.
(ii) Let $a \in \{2, 3, \dots\}$ and $\rho := 1/a$. We define discrete type Sine_2 random point field $\mu_{\mathrm{dys},2}^\rho$. The random point field is defined as a determinantal point field associated with the kernel K_{dys}^ρ given by

$$K_{\mathrm{dys}}^\rho(x, y) = \frac{\sin(\rho\pi(y - x))}{\pi(y - x)}.$$

It is known that $\mu_{\mathrm{dys},2}^\rho$ can not be a canonical Gibbs measure. On the other hand by using the similar way to prove [10, Theorem 2.2] for $\beta = 2$, we can prove that $\mu_{\mathrm{dys},2}^\rho$ is a quasi-Gibbs measure for self potential $\Phi(x) = 0$ and interaction potential $\Psi(x, y) = -2\log|x - y|$. Here we need to modify a method of a finite-particle approximation. In our situation we take a finite-particle approximation associated the circular ensembles $\{\check{\nu}^N\}$ on $\mathbb{Z}^{an_N}$ given by

$$\check{\nu}^N(x^1, \dots, x^{n_N}) = \frac{1}{Z}\prod_{i=1}^{n_N} \mathbf{1}_{\{x^i \in \mathbb{T}_N\}} \prod_{i,j=1, i<j}^{n_N} |e^{2\pi\sqrt{-1}x^i/n_N} - e^{2\pi\sqrt{-1}x^j/n_N}|^2,$$

where Z is the normalization, $n_N = 2^{4N}$ and $\mathbb{T}_N = [-an_N + 1, an_N] \cap \mathbb{Z}$. Then correlation functions $\{\rho_n\}$ are associated with a kernel $K_{2,\rho}^N$ given by

$$K_{2,\rho}^N(x, y) = \frac{\rho}{n_N}\frac{\sin(\rho\pi(x - y))}{\sin(\rho\pi(x - y)/n_N)}.$$

By using this finite approximation we can prove the quasi-Gibbs property of $\mu_{\mathrm{dys},2}^\rho$. Hence we can apply our main result for $\mu_{\mathrm{dys},2}^\rho$. Then we can construct a $\mu_{\mathrm{dys},2}^\rho$-reversible Hunt process $\{\mathbb{P}_\xi^{\mathrm{dys},2,\rho}\}_{\xi \in X_{\mathrm{ex}}}$ associated with $((\mathscr{E}^{\mathrm{dys},2,\rho}, \mathscr{D}^{\mathrm{dys},2,\rho}), L^2(X_{\mathrm{ex}}, \mu_{\mathrm{dys},2}^\rho))$.

§ 6. noncolliding RW as a determinantal process

Let $\xi_{a\mathbb{Z}}$ be a configuration whose support is $\{ak\}_{k\in\mathbb{Z}}$ for $a \in \{2, 3, \dots\}$. We consider the noncolliding infinite particle systems of continuous-time random walks on $\mathbb{Z}$, denoted by $\mathsf{X}(t)$, starting at $\xi_{a\mathbb{Z}}$. In [1, Theorem 5.6] we showed the relaxation phenomena for the noncolliding system.

Theorem 6.1 ([1]). *For each $a \in \{2,3,\dots\}$, the noncolliding system of continuous-time random walks $(\mathsf{X}(t), t \in [0,\infty), \mathbb{P}_{\xi_{a\mathbb{Z}}})$ starting from $\xi_{a\mathbb{Z}}$ shows a relaxation phenomenon to the stationary process $(\mathsf{X}(t), t \in [0,\infty), \mathbb{P}_\rho)$ with $\rho = 1/a$. The stationary process $(\mathsf{X}(t), t \in [0,\infty), \mathbb{P}_\rho)$ is reversible with respect to $\mu^\rho_{\mathrm{dys},2}$ and is determinantal with the correlation kernel given by*

$$\mathbf{K}_\rho(t-s, y-x) = \begin{cases} \displaystyle\int_0^\rho du \cos(u\pi(y-x)) e^{-(t-s)\cos u\pi}, & \text{if } s < t, \\ \displaystyle\frac{\sin(\rho\pi(y-x))}{\pi(y-x)}, & \text{if } s = t, \\ \displaystyle -\int_\rho^1 du \cos(u\pi(y-x)) e^{-(t-s)\cos u\pi}, & \text{if } s > t. \end{cases}$$

We conjecture that this process is equivalent to $\mu^\rho_{\mathrm{dys},2}$-reversible process $\{\mathbb{P}^{\mathrm{dys},2,\rho}_\xi\}_{\xi \in X_{\mathrm{ex}}}$ constructed in Example 5.3 in some sense. We will discuss the equivalence of these processes in a forthcoming paper.

§ 7. Examples of L^2-generator

In this section we give examples of the L^2-generator for the processes constructed in the previous section.

Example 7.1. Let μ be a Gibbs measure with a self potential Φ and a interaction potential Ψ. Then we have

$$\frac{\rho^1(y)}{\rho^1(x)} \frac{d\mu_y}{d\mu_x}(\xi \setminus x) = \exp\left\{ -\Phi(y) + \Phi(x) - \sum_i \left\{ \Psi(s^i - y) - \Psi(s^i - x) \right\} \right\},$$

for $x, y \in S$ where $\xi \setminus x \in X_{\mathrm{ex}}$ is defined by

$$(\xi \setminus x)(z) = \begin{cases} 0 & z = x, \\ \xi(z) & \text{otherwise}, \end{cases}$$

for $\xi \in X_{\mathrm{ex}}$ and $x \in \operatorname{supp}\xi$, and we write $\operatorname{supp}(\xi \setminus x) = \{s^i\}_{i=1}^\infty$. Thus the associated L^2-generator L is given by

$$Lf(\xi) = \sum_{x \in S} \xi(x) \sum_{y \in \mathbb{Z}^d} c(\xi, x; y)[f(\xi^{xy}) - f(\xi)],$$

where

$$c(\xi, x; y) = p(x,y) \left\{ 1 + \exp\left\{ -\Phi(y) + \Phi(x) - \sum_i \left\{ \Psi(s^i - y) - \Psi(s^i - x) \right\} \right\} \right\}, \tag{7.1}$$

if $\xi(x) = 1$. Here we give a example for a canonical Gibbs measure associated with a potential with polynomial decay. We consider the process $\{\mathbb{P}_\xi^{\mathrm{pol}}\}_{\xi\in X_{\mathrm{ex}}}$ associated with the canonical Gibbs measure μ_{pol} defined above. In this case, from (7.1) we have

$$c(\xi, x; y) = p(x, y)\left\{1 + \exp\left\{-\sum_i |s^i - y|^{-a} + \sum_i |s^i - x|^{-a}\right\}\right\}.$$

Remark. We consider that our result can be more interesting for a quasi-Gibbs state which is not a Gibbs state. This is because for random point fields on continuum space it is known interesting phenomenon, for instance μ and μ_x are mutually singular but μ_x and μ_y are mutually absolutely continuous for the Ginibre random point field. Here the Ginibre random random point field μ_{gin} is defined as a determinantal point field on $\mathbb{C}$ associated with the kernel K_{gin} given by $K_{\mathrm{gin}}(z_1, z_2) = \frac{1}{\pi}\exp\left(-\frac{|z_1|^2}{2} - \frac{|z_2|^2}{2} + z_1 \cdot \overline{z}_2\right)$ where $z_1, z_2 \in \mathbb{C}$ and $\overline{z}$ denotes the complex conjugate of $z \in \mathbb{C}$. It is proved by Osada in [10, Theorem 2.3] that μ_{gin} is a quasi-Gibbs measure with self potential $\Phi(x) = 0$ and interaction potential $\Psi(x, y) = -2\log|x - y|$. In addition it is proved by Osada and Shirai in [11, Theorem 1.3] that

$$\frac{d\mu_y}{d\mu_x}(\xi \setminus x) = \lim_{r\to\infty} \prod_{|s^i|<r} \frac{|y - s^i|^2}{|x - s^i|^2}.$$

Even though the discrete counterparts of these results have not proved yet, we consider those properties hold in discrete setting.

§ 8. Glauber dynamics

We can construct Glauber dynamics by the same way on the present paper. Of course if we take an invariant measure μ from Gibbs measures we can consider an equilibrium Glauber dynamics. Indeed to consider the dynamics we use the absolute continuity of the conditional probability (1.1) with respect to the Gibbs measure. In this case L^2-generator L_{Gla} of the equilibrium Glauber dynamics is given by

$$L_{\mathrm{Gla}} f(\xi) = \sum_{x\in S}(f(\xi \cdot x) - f(\xi))\rho^1(x)\frac{d\mu_x}{d\mu}(\xi) + \sum_{x\in S}\xi(x)(f(\xi \setminus x) - f(\xi)).$$

Here $\xi \cdot x \in X_{\mathrm{ex}}$ is defined by

$$\xi \cdot x = \begin{cases} 1 & z = x, \\ \xi(z) & \text{otherwise}, \end{cases}$$

for $\xi \in X_{\mathrm{ex}}$ and $x \notin \operatorname{supp}\xi$. This generator is associated with the bilinear form

$$\mathfrak{E}_{\mathrm{Gla}}(f, g) = \int_{X_{\mathrm{ex}}} d\mu(\xi)\sum_{x\in S}\xi(x)(f(\xi \setminus x) - f(\xi))(g(\xi \setminus x) - g(\xi)).$$

Under the condition (F.1) we can show that the closure of $(\mathfrak{E}_{\rm Gla}, \mathscr{D}_\infty^{\rm ex})$ is a regular Dirichlet form by the same argument. Whereas for μ singular to its conditional probability measure μ_x such as continuum Ginibre random point field the operator $L_{\rm Gla}$ can not be defined and the associated Glauber dynamics could not exist. However in such situations we can consider Glauber dynamics in the sence of Strook-Zegarlinski [13]. Strook-Zegarlinski type Glauber dynamics is defined by the following. For $k \in \mathbb{Z}$ and a subset $A \subset S$ such that a number of points in A is equal to k. For $\eta \in X_{\rm ex}$ we set $\eta_A = \pi_A(\eta)$ and $\eta_{A^c} = \pi_{A^c}(\eta)$. Then we have $\eta = \eta_A + \eta_{A^c}$. For $f, g \in \mathscr{D}_\circ^{\rm ex}$, $\eta \in X_{\rm ex}$ we set

$$\mathfrak{E}_{\rm loc}^A(f,g)[\eta] = \int_{\{0,1\}^A} (f(\zeta + \eta_{A^c}) - f(\eta))(g(\zeta + \eta_{A^c}) - g(\eta)) d\mu_{\eta_{A^c}}^{\eta_A}(\zeta).$$

Here we set

$$\mu_{\eta_{A^c}}^{\eta_A}(\zeta) = \mu\left(\zeta - \eta |\, \zeta(x) = 1 \text{ if } \eta(x) = 1,\ \zeta_{A^c} = \eta_{A^c}\right),$$

where

$$\zeta - \eta = \begin{cases} \zeta(x) & \text{if } \eta(x) = 0 \\ 0 & \text{if } \eta(x) = 1 \end{cases}.$$

We define a bilinear form $\mathfrak{E}_k$ by the following.

$$\mathfrak{E}_k(f,g) = \int_{\{0,1\}^S} d\mu(\eta) \sum_{\substack{A \subset \mathbb{Z}^d \\ \sharp A \le k}} \mathfrak{E}_{\rm loc}^A(f,g)[\eta],$$

where $\sharp A$ denote the number of points in A. We define restricted bilinear form $\mathfrak{E}_A(f,g)$ by the following.

$$\mathfrak{E}_A(f,g) = \int_{\{0,1\}^S} d\mu(\eta) \mathfrak{E}_{\rm loc}^A(f,g)[\eta].$$

Then the closability of $((\mathscr{E}_k, \mathscr{D}_\infty^{\rm ex}))$ on $L^2(X_{\rm ex}, \mu)$ is proved by the similar way as the one of [2, Theorem 2.5]. Here we note

$$\mathfrak{E}_k = \sum_{\substack{A \subset \mathbb{Z}^d \\ \sharp A \le k}} \mathfrak{E}_A(f,g).$$

Then under the assumption (QG.1) and (QG.2), we can show the closability of $((\mathscr{E}_k, \mathscr{D}_\infty^{\rm ex}))$ on $L^2(X_{\rm ex}, \mu)$ by the similar argument of the proof of [2, Theorem 4.6]. Hence under the assumption (QG.1) and (QG.2), we can prove that $((\mathscr{E}_k, \mathscr{D}_\infty^{\rm ex}))$ is closable on $L^2(X_{\rm ex}, \mu)$. We denote by $(\mathfrak{E}_k, \mathfrak{D}_k)$ the closure of $((\mathscr{E}_k, \mathscr{D}_\infty^{\rm ex}), L^2(X_{\rm ex}, \mu))$. Since $X_{\rm ex}$ is a compact set we can consider that $(\mathscr{E}_k, \mathscr{D}_k)$ is a regular Dirichlet form on $L^2(X_{\rm ex}, \mu)$.

However we do not know whether a process associated with $(\mathscr{E}_k, \mathscr{D}_k)$ is not trivial. Related these consideration we have interesting phenomenon, for instance the geometric rigidity of Ginibre random point field. For the random point field we conjecture that a process associated with $\mathfrak{E}_1$ are not well-defined due to the geometric rigidity. Moreover we also conjecture that for dynamics associated with $\mathfrak{E}_A$ a number of particle in A is fixed by the rigidity.

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