

**RIMS Kôkyûroku Bessatsu B61**

# Microlocal Analysis and Singular Perturbation Theory

edited by Yoshitsugu Takei, Takashi Aoki,

Naofumi Honda, Kiyoomi Kataoka and Tatsuya Koike

**January, 2017**

**Research Institute for Mathematical Sciences  
Kyoto University**

*RIMS Kôkyûroku Bessatsu B61*

*Microlocal Analysis and  
Singular Perturbation Theory*

*October 5 ~ 9, 2015*

*edited by Yoshitsugu Takei, Takashi Aoki, Naofumi Honda,  
Kiyoomi Kataoka and Tatsuya Koike*

*January, 2017*

*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

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## PREFACE

This volume of RIMS Kôkyûroku Bessatsu is the Proceedings of the RIMS Conference

### **“Microlocal Analysis and Singular Perturbation Theory”**

held at the Research Institute for Mathematical Sciences, Kyoto University from October 5 (Monday) through October 9 (Friday) in 2015. The volume contains research papers, announcements and survey articles contributed by invited speakers of the above conference. All papers in the volume have been reviewed and are in their final forms.

The conference was planned to celebrate the eightieth birthday of Professor Hikosaburo Komatsu and the seventieth birthday of Professor Takahiro Kawai. As everyone in the fields knows, Prof. Komatsu and Prof. Kawai have made great contributions to the developments of algebraic analysis, microlocal analysis and singular perturbation theory. We are most obliged to them for all their activities and achievements in these fields. Taking this opportunity, we would like to express our sincerest gratitude to Prof. Komatsu and Prof. Kawai and at the same time we are very pleased to see that they are still very active and enjoying mathematics.

The conference was supported by RIMS. We express our heartiest thanks to the secretary staff of RIMS for their kind assistance.

Kyoto in October, 2016

Yoshitsugu Takei  
(Editor in chief)  
Takashi Aoki  
Naofumi Honda  
Kiyoomi Kataoka  
Tatsuya Koike

# PROGRAM

## Microlocal Analysis and Singular Perturbation Theory

October 5 (Mon) – October 9 (Fri), 2015

Lecture Hall (Room No. 420) of RIMS, Kyoto University

### October 5, Monday

- 13:00 – 13:50 Pierre Schapira (Paris, France)  
Grothendieck topologies for analysis
- 14:00 – 14:50 Kiyoomi Kataoka (Univ. of Tokyo)  
The functor  $\beta_Y(\cdot)$  and mixed problems for  $\mathcal{D}_X$ -modules
- 15:10 – 16:00 Masafumi Yoshino (Hiroshima Univ.)  
Monodromy of some resonant Hamiltonian system
- 16:10 – 17:00 Shinji Sasaki (RIMS, Kyoto Univ.)  
Bifurcation phenomenon of Stokes curves around a double turning point and influence of virtual turning points upon nonadiabatic transition probabilities

### October 6, Tuesday

- 10:00 – 10:50 Setsuro Fujiie (Ritsumeikan Univ.)  
Resonances near an energy-level crossing
- 11:10 – 12:00 Johannes Sjöstrand (Bourgogne, France)  
Non-self-adjoint perturbations of completely integrable Hamiltonians in 2D — rational tori and spectral centipedes  
(joint work with Michael Hitrik)
- 14:00 – 14:50 Reinhard Schäfke (Strasbourg, France)  
Factorisation of fundamental WKB-solutions  
(joint work with Charlotte Hulek)
- 15:10 – 16:00 Keisuke Uchikoshi (National Defense Academy)  
On gravity water waves
- 16:10 – 17:00 Takahiro Kawai (RIMS, Kyoto Univ.) and Naofumi Honda (Hokkaido Univ.)  
An invitation to Sato's postulates in micro-analytic  $S$ -matrix theory



**October 7, Wednesday**

- 10:00 – 10:50 Ovidiu Costin (Ohio State, USA)  
Exact WKB and resurgence
- 11:10 – 12:00 Masaki Kashiwara (RIMS, Kyoto Univ.)  
Riemann-Hilbert problem of irregular holonomic  $\mathscr{D}$ -modules

**October 8, Thursday**

- 10:00 – 10:50 Yuichi Ike (Univ. of Tokyo)  
Hyperbolic localization and Lefschetz fixed point formulas for higher-dimensional fixed point sets
- 11:10 – 12:00 Yves Laurent (Grenoble, France)  
 $b$ -functions and regular holonomic  $D$ -modules
- 14:00 – 14:50 Toshio Oshima (Josai Univ.)  
Linear ordinary differential equations in the complex domain and hypergeometric systems
- 15:10 – 16:00 Toshinori Oaku (Tokyo Woman's Christian Univ.)  
Some algorithmic problems for holonomic distributions
- 16:10 – 17:00 Hikosaburo Komatsu (Univ. of Tokyo)  
History of mathematics of the world due to D.E. Smith

**October 9, Friday**

- 10:00 – 10:50 Tatsuya Koike (Kobe Univ.)  
A remark on the growth order of Borel transform of WKB solutions of one-dimensional Schrödinger equations — Toward a proof of its multisummability
- 11:10 – 12:00 David Sauzin (Pisa, Italy & CNRS, France)  
Nonlinear analysis with endlessly continuable functions  
(joint work with Shingo Kamimoto)
- 14:00 – 14:50 Kohei Iwaki (Nagoya Univ.)  
Topological recursion, quantum curves and Painlevé equations
- 15:10 – 16:00 Takashi Aoki (Kinki Univ.)  
The hypergeometric function and WKB solutions
- 16:10 – 17:00 Yoshitsugu Takei (RIMS, Kyoto Univ.)  
On the exact WKB analysis of discrete Painlevé equations

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