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<th>Title</th>
<th>Characteristic cycle of a rank 1 sheaf on a surface: research announcement (Algebraic Number Theory and Related Topics 2014)</th>
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<tr>
<td>Author(s)</td>
<td>Yatagawa, Yuri</td>
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<tr>
<td>Citation</td>
<td>数理解析研究所講究録別冊 (2017), B64: 201-208</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2017-05</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/243671">http://hdl.handle.net/2433/243671</a></td>
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<tr>
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<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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Characteristic cycle of a rank 1 sheaf on a surface: research announcement

By

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Abstract

This is a research announcement about a study of characteristic cycle of a rank 1 sheaf on a surface on which I am writing a paper. In this announcement, we construct a canonical lifting of Kato's logarithmic characteristic cycle on the cotangent bundle of the surface. As a corollary, an index formula computing the Euler characteristic of the sheaf is yielded by the canonical lifting.

§1. Introduction

This is a research announcement about a study of characteristic cycle of a rank 1 sheaf on a surface on which I am writing a paper. We state without proof a result on the characteristic cycle of a smooth sheaf of rank 1 on a surface of positive characteristic.

Let $X$ be a smooth separated connected scheme of dimension $d$ over an algebraically closed field $k$ of characteristic $p > 0$. Let $\mathcal{F}$ be a constructible complex of $\Lambda$-modules on $X$, where $\Lambda$ is a finite filed of characteristic $\ell \neq p$. A constructible complex $\mathcal{F}$ is a complex of étale sheaves such that $\mathcal{H}^q(\mathcal{F})$ is constructible for any $q$, and equal to 0 except for finitely many $q$.

The characteristic cycle of $\mathcal{F}$ is an analogue of that of a holonomic $\mathcal{D}$-module on a smooth variety of characteristic 0 in the theory of $\mathcal{D}$-modules. It is defined as a $d$-cycle on the cotangent bundle $T^*X$ of $X$. The cotangent bundle $T^*X$ of $X$ is the vector bundle on $X$ corresponding to $\Omega^1_X$. The characteristic cycle of $\mathcal{F}$ satisfies an index
formula computing the Euler characteristic

\[ \chi(X, \mathcal{F}) = \sum_{i=0}^{2d} (-1)^i \dim H^i(X, \mathcal{F}) \]

of \( \mathcal{F} \).

We see a classical example of characteristic cycle. We assume that \( d = 1 \). Let \( U \) be the complement of a divisor \( D \) on \( X \) and \( j: U \to X \) the canonical open immersion. We assume that \( \mathcal{F} \) is the zero extension \( j! \mathcal{G} \) of a smooth sheaf \( \mathcal{G} \) of \( \Lambda \)-modules over \( U \). Let \( T_X^*X \) (resp. \( T_x^*X \)) denote the zero-section of \( T^*X \) (resp. the fiber of \( T^*X \) at a closed point \( x \) of \( X \)). For an integral closed subscheme \( C \) of \( T^*X \), we write \( [C] \) for \( C \) as a prime cycle on \( T^*X \). Then the characteristic cycle \( \text{Char}(\mathcal{F}) \) of \( \mathcal{F} \) is defined by

\[
\text{Char}(\mathcal{F}) = - \left( \text{rank} (\mathcal{G}) [T_X^*X] + \sum_{x \in D} (\text{rank} (\mathcal{G}) + \text{Sw}_x \mathcal{G}) [T_x^*X] \right).
\]

In (1.1), the symbol \( \text{Sw}_x \mathcal{G} \) is an invariant of ramification called the Swan conductor of \( \mathcal{G} \) at \( x \). The Swan conductor of \( \mathcal{G} \) is a non-negative integer and measures the wild ramification of \( \mathcal{G} \). The index formula in this case is the classical Grothendieck-Ogg-Shafarevich formula ([SGA5]). That is, if \( X \) is proper, then

\[ \chi(X, \mathcal{F}) = (\text{Char}(\mathcal{F}), T_X^*X)_{T^*X}, \]

where the right hand side denotes the intersection number in \( T^*X \).

In the general dimensional case, the characteristic cycle of a constructible complex \( \mathcal{F} \) is defined by T. Saito using Beilinson's singular support ([B]) and vanishing cycles in [S4]. The index formula yielded by this characteristic cycle generalizes Deligne and Lau- mon's formula for the Euler characteristic for surfaces ([L] Théorème 1.2.1). However, this characteristic cycle is hard to compute in general.

In the case where \( d = 2 \), let \( U \) be the complement of a divisor \( D \) on \( X \) with simple normal crossings, and \( j: U \to X \) the canonical open immersion. We assume that \( \mathcal{F} \) is the zero extension \( j! \mathcal{G} \) of a smooth sheaf \( \mathcal{G} \) of \( \Lambda \)-modules of rank 1 over \( U \). With this setting, Kato has given another definition of characteristic cycle on the logarithmic cotangent bundle of \( X \) with logarithmic poles \( D \) using ramification theory ([K2]). This characteristic cycle seems easier to compute. We denote it by \( \text{Char}(X, U, \mathcal{G}) \). The index formula computing the Euler characteristic \( \chi(X, \mathcal{F}) \) as the intersection number of this cycle with the zero-section \( T_X^*X(\log D) \subset T^*X(\log D) \) is proved by Kato ([S1]).

We keep the assumption in the last paragraph. The main result in this announcement is a construction of a 2-cycle on the cotangent bundle \( T^*X \) of \( X \) which is a canonical lifting of Kato's characteristic cycle (Theorem 3.2). For the construction of
this canonical lifting, we use Matsuda’s non-logarithmic ramification theory ([M]). Matsuda’s theory is non-logarithmic version of the ramification theory which Kato used. We expect that the canonical lifting is equal to Saito’s characteristic cycle (Conjecture 4.3).

In this announcement, we assume that \( p \neq 2 \) for simplicity. In the \( p = 2 \) case, a new interesting phenomenon arises, which we will discuss in the paper which I am writing.

Throughout this announcement, let \( k \) be an algebraically closed field of characteristic \( p \geq 3 \) and \( X \) a smooth separated connected scheme of dimension 2 over \( k \). We write \( D \) for a divisor on \( X \) with simple normal crossings, and put \( U = X - D \). The symbol \( \Lambda \) denotes a finite field of characteristic \( \ell \neq p \). We consider a smooth sheaf \( \mathcal{G} \) of \( \Lambda \)-modules of rank 1 on \( U \) corresponding to a character \( \chi: \pi^\text{ab}_1(U) \to \Lambda^\times \). We fix an inclusion \( \Lambda^\times \hookrightarrow \mathbb{Q}/\mathbb{Z} \) and identify \( \chi \) with an element of \( H^1(U, \mathbb{Q}/\mathbb{Z}) = \text{Hom}(\pi^\text{ab}_1(U), \mathbb{Q}/\mathbb{Z}) \).

We put \( \mathcal{F} = j_! \mathcal{G} \), where \( j: U \to X \) is the canonical open immersion.

\[ \text{§2. Kato’s logarithmic characteristic cycle} \]

In this section, we recall Kato’s logarithmic characteristic cycle. Kato’s logarithmic characteristic cycle is defined as a 2-cycle on the logarithmic cotangent bundle \( T^*X(\log D) \) of \( X \) with logarithmic poles along \( D \). This cycle satisfies an index formula computing the Euler characteristic.

Let \( \{D_i\}_{i \in I} \) be the irreducible components of \( D \) and let \( \mathfrak{p}_i \) denote the generic point of \( D_i \). The local field at \( \mathfrak{p}_i \) means the complete discrete valuation field \( \text{Frac} \hat{\mathcal{O}}_{X, \mathfrak{p}_i} \), where \( \hat{\mathcal{O}}_{X, \mathfrak{p}_i} \) denotes the completion of the local ring \( \mathcal{O}_{X, \mathfrak{p}_i} \) at \( \mathfrak{p}_i \) by the maximal ideal. We denote the local field at \( \mathfrak{p}_i \) by \( K_i \). Let \( \chi|_{K_i} \) denote the image of \( \chi \) by the composition \( H^1(U, \mathbb{Q}/\mathbb{Z}) \to H^1(k(X), \mathbb{Q}/\mathbb{Z}) \to H^1(K_i, \mathbb{Q}/\mathbb{Z}) \), where \( k(X) \) denotes the function field of \( X \).

We consider the ramification filtration \( \{\text{fil}_n H^1(K_i, \mathbb{Q}/\mathbb{Z})\}_{n \geq 0} \) of \( H^1(K_i, \mathbb{Q}/\mathbb{Z}) \) defined in [K1] Definition (2.1). We define the Swan conductor \( \text{sw}(\chi|_{K_i}) \) to be the minimal number \( n \) such that \( \chi|_{K_i} \in \text{fil}_n H^1(K_i, \mathbb{Q}/\mathbb{Z}) \). We put \( R_X = \sum_{i \in I} \text{sw}(\chi|_{K_i}) D_i \), and call it the Swan conductor divisor of \( \chi \) on \( X \). This is an effective Cartier divisor on \( X \). Let \( Z \) denote the support of \( R_X \). For \( D_i \subset Z \), we define the refined Swan conductor \( \text{rsw}(\chi|_{K_i}) \) to be the image of \( \chi|_{K_i} \) by the map \( \text{gr}_{\text{sw}(\chi|_{K_i})} H^1(K_i, \mathbb{Q}/\mathbb{Z}) \to (\Omega_X^1(\log D)(R_X)|_Z)_{\mathfrak{p}_i} \), defined in [M] Remark 3.2.12.

**Lemma 2.1 ([K2] (3.4.2)).** There exists a unique global section \( \text{rsw}(\chi) \) of the sheaf \( \Omega_X^1(\log D)(R_X)|_Z \) whose germ \( \text{rsw}(\chi)_{\mathfrak{p}_i} \) at any generic point \( \mathfrak{p}_i \) of \( Z \) coincides with the refined Swan conductor \( \text{rsw}(\chi|_{K_i}) \).
Let $T^*X(\log D) = \text{Spec} \mathcal{V}(\Omega^1_X(\log D)^\vee)$ denote the logarithmic cotangent bundle of $X$ with logarithmic poles along $D$. Kato introduced the notion of cleanness ([K2] (3.4.3)). We define a non-negative integer $\text{ord}_x(x, D_i)$ for a point $x$ of $Z$ and an irreducible component $D_i$ of $Z$ containing $x$ by

$$\text{ord}_x(x, D_i) = \max\{n \in \mathbb{Z}_{\geq 0} ; \text{rsw}(\chi)|_{D_i,x} \in m_x^n\Omega^1_X(\log D)(R_{\chi})|_{D_i,x}\}.$$ 

Here $m_x$ is the maximal ideal of the local ring $\mathcal{O}_{X,x}$ at $x$. We say that $(X, U, \mathcal{G})$ is clean at a point $x$ of $X$ if $x \notin Z$ or if $x \in Z$ and $\text{ord}_x(x, D_i) = 0$ for an irreducible component $D_i$ of $Z$ containing $x$. We say that $(X, U, \mathcal{G})$ is clean if $(X, U, \mathcal{G})$ is clean at all points of $X$. He defined a logarithmic characteristic cycle $\text{Char}(X, U, \mathcal{G})$ of $(X, U, \mathcal{G})$ as a 2-cycle of $T^*X(\log D)$ using the refined Swan conductor $\text{rsw}(\chi)$ above as follows ([K2] (3.4.4)).

Let $T^*_X X(\log D)$ be the zero-section of $T^*X(\log D)$, and let $T^*_x X(\log D)$ be the fiber at a closed point of $X$. We define a 2-dimensional integral closed subscheme $L_i$ of $T^*_X X(\log D)$ for $D_i$ contained in $Z$ to be the sub line bundle of $T^*X(\log D) \times X D_i$ associated to the unique locally direct factor of rank 1 of $\Omega^1_X(\log D)|_{D_i}$ containing the image of the multiplication map $\mathcal{O}_X(-R_{\chi})|_{D_i} \to \Omega^1_X(\log D)|_{D_i} ; f \mapsto fr\text{rsw}(\chi)$. For $D_i$ not contained in $Z$, we define $L_i$ by $L_i = \emptyset$.

Then the logarithmic characteristic cycle $\text{Char}(X, U, \mathcal{G})$ is of the form

$$\text{Char}(X, U, \mathcal{G}) = [T^*_X X(\log D)] + \sum_{i \in I} \text{sw}(\chi|_{K_i})[L_i] + \sum_{x \in |D|} s_x[T^*_x X(\log D)],$$

where $|D|$ denotes the set of closed points of $D$. For the definition of $s_x$ in (2.1), we take a composition $f : X' = X_s \to X_{s-1} \to \cdots \to X_0 = \text{Spec} \mathcal{O}_{X,x}$ of blowing-ups at closed points lying over $x$ such that $(X', f^{-1}(U), f^*\mathcal{G})$ is clean ([K2] Theorem 4.1). We put $D' = (f^{-1}(D))_{\text{red}}$. Then $D'$ is a divisor on $X'$ with simple normal crossings. We define $r_x \in \mathbb{Z}$ by $r_x = -(R_{\chi'} - f^*R_{\chi}, R_{\chi'} + D + f^*R_{\chi})$, where $\chi'$ denotes the pull-back of $\chi$ to $H^1(f^{-1}(U), \mathbb{Q}/\mathbb{Z})$ and $R_{\chi'}$ denotes the Swan conductor divisor of $\chi'$ on $X'$ ([K2] Remark 5.7). We define $s_x$ by

$$s_x = \sum_{i \in I, x \in D_i} \text{sw}(\chi|_{K_i})\text{ord}_x(x, D_i) - r_x.$$ 

If $(X, U, \mathcal{G})$ is clean at $x$, the integer $s_x$ is equal to 0 by the definition of $s_x$.

The following theorem follows from [S1] the remark right after the conjecture in the page 168 and the definition of $\text{Char}(X, U, \mathcal{G})$.

**Theorem 2.2 (Index formula).** If $X$ is proper over $k$, we have

$$\chi(X, \mathcal{F}) = (\text{Char}(X, U, \mathcal{G}), T^*_X X(\log D))_{T^*X(\log D)}.$$
§ 3. Construction of a canonical lifting

In this section, we construct a 2-cycle on $T^*X$ which is a canonical lifting of Kato’s characteristic cycle using Matsuda’s non-logarithmic ramification theory ([M]). This is the main result in this announcement (Theorem 3.2). As a corollary, we have an index formula yielded by the canonical lifting. For simplicity, we assume that $(X, U, \mathcal{G})$ is clean. For the general case, we will discuss in the paper which I am writing.

We consider another filtration $\{\text{fil}^n_1 H^1(K_i, \mathbb{Q}/\mathbb{Z})\}_{n \geq 0}$ of $H^1(K_i, \mathbb{Q}/\mathbb{Z})$ ([M] 3.1). We define a conductor $sw'(\chi|_{K_i})$ as the minimal number $n$ such that $\chi|_{K_i} \in \text{fil}^n_1 H^1(K_i, \mathbb{Q}/\mathbb{Z})$. We put $R'_X = Z + \sum_{i \in I} sw'(\chi|_{K_i})D_i$. This is an effective Cartier divisor on $X$. For $D_i \subset Z$, we define the non-logarithmic version $\text{rsw}'(\chi|_{K_i})$ of the refined Swan conductor of $\chi|_{K_i}$ to be the image of $\chi|_{K_i}$ by the map

$$\text{gr}_{\text{rsw}'(\chi|_{K_i})} H^1(K_i, \mathbb{Q}/\mathbb{Z}) \to (\Omega^1_X(R'_X)|_{Z})_{\mathfrak{p}_i}$$

defined in [M] Definition 3.2.5.

Lemma 3.1 ([M] 5.2). There exists a unique global section $\text{rsw}'(\chi)$ of the sheaf $\Omega_X^1(R_X)|_Z$ whose germ $\text{rsw}'(\chi)_{\mathfrak{p}_i}$ at any generic point $\mathfrak{p}_i$ of $Z$ coincides with the refined Swan conductor $\text{rsw}'(\chi|_{K_i})$.

Let $T^*X = \text{Spec} \mathcal{V}(\Omega^1_X)$ denote the cotangent bundle of $X$. We define a 2-cycle $\text{Char}'(X, U, \mathcal{G})$ on $T^*X$, which will be a canonical lifting of Kato’s characteristic cycle, as follows. Let $T^*_X X$ denote the zero section of $T^*X$. Let $T^*_{D_i} X$ denote the conormal bundle of $D_i$ in $X$, and $T^*_X X$ the fiber at a closed point $x$ of $X$. We define a 2-dimensional integral closed subscheme $L'_i$ of $T^*X$ for $D_i$ contained in $Z$ to be the sub line bundle of $T^*_X X \times_X D_i$ associated to the unique locally direct factor of rank 1 of $\Omega^1_X|_{D_i}$ containing the image of the multiplication map

$$\mathcal{O}_X(-R'_X)|_{D_i} \to \Omega^1_X|_{D_i}; \quad f \mapsto \text{frsw}'(\chi).$$

For $D_i$ not contained in $Z$, we define $L'_i = T^*_D X$. We put $R''_X = D + \sum_{i \in I} \text{sw}'(\chi|_{K_i})D_i$. Let $\text{dt}(\chi|_{K_i})$ denote the multiplicity of $D_i$ in $R''_X$.

We construct a 2-cycle $\text{Char}'(X, U, \mathcal{G})$ of the form

$$\text{Char}'(X, U, \mathcal{G}) = [T^*_X X] + \sum_{i \in I} \text{dt}(\chi|_{K_i})[L'_i] + \sum_{x \in |D|} t_x [T^*_x X].$$

We define the integer $t_x$ in (3.1) as follows. For a point $x$ of $Z$ and an irreducible component $D_i$ of $Z$ containing $x$, we define a non-negative integer $\text{ord}'_{\chi}(x, D_i)$ by

$$\text{ord}'_{\chi}(x, D_i) = \max\{n \in \mathbb{Z}_{\geq 0} ; \text{rsw}(\chi)|_{D_i, x} \in m_x^n \Omega^1_X(R'_X)|_{D_i, x}\},$$
where \(m_x\) is the maximal ideal of the local ring \(\mathcal{O}_{X,x}\) at \(x\). Let \(x\) be a closed point of \(D\). We define \(t_x\) by

\[
t_x = \#(T_x) - 1 + \sum_{D_i \in T_x'} sw(\chi_{|K_i})(\text{ord}_{\chi}'(x, D_i) + \sharp(T_x)) + \delta_{sw(\chi_{|K_i})dt(\chi_{|K_i})}(1 - \sharp(T_x)),
\]

where \(T_x = \{D_i \subset D \mid x \in D_i\}\) and \(T_x' = \{D_i \in T_x \mid sw(\chi_{|K_i}) > 0\}\). The symbol \(\delta_{sw(\chi_{|K_i})dt(\chi_{|K_i})}\) is the Kronecker delta.

Let \(\pi: T^*X \to T^*X(\log D)\) be the canonical morphism of vector bundles on \(X\). Let \(SS(X, U, \mathcal{G}) \subset T^*X(\log D)\) denote the support of (2.1). Let \(S/(X, U, \mathcal{G}) \subset T^*X\) denote the support of (3.1). Then it follows that \(SS(X, U, \mathcal{G}) \subset \pi^{-1}(SS(X, U, \mathcal{G}))\). Let \(\pi^1: CH_2(SS(X, U, \mathcal{G})) \to CH_2(\pi^{-1}(SS(X, U, \mathcal{G}))\) be the refined Gysin homomorphism for the l.c.i. morphism \(\pi\) ([F] 6.6). The following theorem is the main result in this announcement.

**Theorem 3.2.** The image of (3.1) in \(CH_2(\pi^{-1}(SS(X, U, \mathcal{G}))\) is equal to the image of (2.1) by \(\pi^1: CH_2(SS(X, U, \mathcal{G})) \to CH_2(\pi^{-1}(SS(X, U, \mathcal{G}))\).

The next corollary follows from Theorem 3.2 since Kato’s characteristic cycle (2.1) satisfies the index formula.

**Corollary 3.3 (Index formula).** If \(X\) is proper over \(k\), we have

\[
\chi(X, \mathcal{F}) = (Char'(X, U, \mathcal{G}), T^*_X X).\]

**§ 4. Saito’s non-logarithmic characteristic cycle and a conjecture**

Saito has given a definition of non-logarithmic characteristic cycle of a constructible complex on a smooth variety of general dimension using Beilinson’s singular support ([B]) in [S4]. This characteristic cycle is characterized by the Milnor formula and satisfies an index formula ([S4]). The index formula for this characteristic cycle is a generalization of Deligne and Laumon’s formula for the Euler characteristic for surfaces ([L] Théorème 1.2.1).

We keep the assumption in §2 and §3. Saito’s non-logarithmic characteristic cycle under this assumption is equal to that defined in [S3] ([S4] Theorem 7.14). In this section, we briefly recall Saito’s non-logarithmic characteristic cycle defined in ([S3]) without giving the detail of the construction. At the end of this section, we state a conjecture on the equality of Saito’s characteristic cycle and the canonical lifting of Kato’s characteristic cycle constructed in the previous section.

We keep the notation in §3. The divisor \(R_{\chi}'\) on \(X\) is shown to be equal to the slope \(R\) of \(\mathcal{G}\) ([S2] Definition 3.1) similarly as in the proof of Théorème 9.10 in [AS].
Saito’s non-logarithmic characteristic cycle $\text{Char}^R(F)$ in this case is a 2-cycle on $T^*X$ of the form

$$
(4.1) \quad \text{Char}^R(F) = [T^*_X X] + \sum_{i \in I} \text{dt}(\chi|_{K_i})[L'_i] + \sum_{x \in |D|} u_x[T^*_X X]
$$

([S2] Definition 3.5, [S3] Definition 3.8, 3.15, and Proposition 3.19). The Milnor formula characterizing this characteristic cycle is Theorem 4.1 below. Let $f: X \to C$ be a flat morphism to a smooth curve $C$ over $k$. Let $df$ denote the section of $T^*X$ defined by the image of a basis of $T^*C \times_C X$ by the canonical morphism $T^*C \times_C X \to T^*X$ induced by $f$. The condition that $f$ is non-characteristic with respect to $F$ is introduced by Saito ([S3] Section 1). This means that the intersection of $df$ and the support of the characteristic cycle is empty.

**Theorem 4.1** (Milnor formula, [S3] Theorem 3.17). Let $f: X \to C$ be a flat morphism to a smooth curve over $k$. Let $x$ be a closed point of $X$. Assume that $f$ is non-characteristic with respect to $F$ in a neighborhood of $x$ possibly except for $x$ and that $D$ is étale over $C$. Then we have

$$
(4.2) \quad -\dim \, \text{tot} \phi_x(F, f) = (\text{Char}^R(F), [df])_{T^*_X X, x},
$$

where $\dim \, \text{tot} \phi_x(F, f)$ denotes the total dimension of the space $\phi_x(F, f)$ of the vanishing cycles at $x$, and the right hand side means the intersection number in the fiber of $T^*X$ at $x$.

Saito’s characteristic cycle satisfies the index formula.

**Theorem 4.2** (Index formula, [S3] Theorem 3.19.). If $X$ is proper over $k$, we have

$$
\chi(X, F) = (\text{Char}^R(F), T^*_X X)_{T^*X, x}.
$$

Finally, we state a conjecture.

**Conjecture 4.3.** Assume that $(X, U, G)$ is clean. Then the characteristic cycles $\text{Char}^R(F)$ and $\text{Char}^*(X, U, G)$ are equal.

In order to prove this conjecture, by (3.1) and (4.1), it is sufficient to prove the equality of $t_x$ in (3.1) and $u_x$ in (4.1) for any closed point $x$ on $D$. The equality of the sum of $t_x$ and that of $u_x$ follows from the index formulas Corollary 3.3 and Theorem 4.2. When the sheaf $G$ is Artin-Schreier, this conjecture is proved using this equality and the fact ([S3] Corollary 3.15) that $u_x$ is determined étale locally.
Acknowledgment

I would like to express my gratitude to the organizers for giving the opportunity for the talk. I would like to thank Professor Takeshi Saito for giving many helpful advices on the draft of this article. I would like to thank the referee for the helpful comments.

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