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**RIMS Kôkyûroku Bessatsu B68**

# Various aspects of multiple zeta values

edited by Kentaro Ihara

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Research Institute for Mathematical Sciences  
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*RIMS Kôkyûroku Bessatsu B68*

*Various aspects of multiple zeta values*

*July 23 ~26, 2013*

*edited by Kentaro Ihara*

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## Preface

The RIMS workshop “Various aspects of multiple zeta values” was held in Research Institute for Mathematical Sciences (RIMS), Kyoto University from July 23 to July 26, 2013. The workshop was organized by Kentaro Ihara (Kindai University), and was aimed to view recent developments in various aspects of multiple zeta values. In the workshop, there were seventeen talks on recent research in this field. We had fifty participants during the workshop.

This volume of RIMS Kôkyûroku Bessatsu is a collection of articles contributed by speakers of the workshop, and consists of three categories of articles following the views of authors and anonymous referees, namely, Research Articles, Survey Articles and Research Announcements/Proposals. Each paper has been refereed and is in its final form. I would like to express my deep thanks to all authors of the articles and anonymous referees for their great contribution.

The workshop was supported by RIMS and JSPS KAKENHI Grant no. 23340010. We express our gratitude to them and staff in RIMS for their supports.

July, 2017

Kentaro Ihara

## Program

### RIMS 研究集会 多重ゼータ値の諸相 (Various aspects of multiple zeta values)

日時：2013 年 7 月 23 日 (火) 11:00 --- 7 月 26 日 (金) 15:50

場所：京都数理解析研究所 4 階 420 号室 (京都市左京区北白川追分町)

7 月 23 日 (火)

- 11:00---12:00 大野 泰生 (近畿大学)  
多重ゼータ値の関係式入門
- 13:30---14:30 山本 修司 (慶応大学)  
等号付き多重ゼータ値の積分表示に関する注意
- 14:50---15:35 安齋 千隼 (Karlsruher Inst. für Tech.) [\*次の講演と連携しています]  
Analytical Evaluation of Multiple Sums for Loop Integrals
- 15:50---16:35 隅野行成 (東北大学)  
Singularities and MZVs in Feynman Loop Integrals

7 月 24 日 (水)

- 10:00---10:40 小森 靖 (立教大学) [\*次の講演と連携しています]  
Desingularization of complex multiple zeta-functions  
and fundamentals of p-adic multiple L-functions I
- 10:50---11:30 津村 博文 (首都大学東京)  
Desingularization of complex multiple zeta-functions  
and fundamentals of p-adic multiple L-functions II
- 11:50---12:20 中村 隆 (東京理科大学)  
新谷多重ゼータ分布と多次元擬無限分解可能分布
- 14:00---15:00 Li Zhonghua (Tongji University)  
Relations of multiple zeta values :  
from the viewpoint of some special functions
- 15:20---16:20 寺杣 友秀 (東京大学)  
Hypergeometric identities and Brown-Zagier identities

7月25日(木)

- 10:00---11:00 三柴 善範 (九州大学)  
正標数多重ゼータ値とその代数的独立性について
- 11:20---12:20 山下 剛 (Toyota CRDL, Inc./ RIMS)  
p-adic multiple zeta values and motivic Galois groups
- 14:00---15:00 田中 諒 (名古屋大学)  
Hurwitz-Lerch 及び Mordell-Tornheim 型を含む  
一般的な多重ゼータ関数の関数等式
- 15:20---16:20 松本 耕二 (名古屋大学)  
係数のついたオイラー型二重級数の関数等式について  
(YoungJu Choie との共同研究)

7月26日(金)

- 10:00---11:00 町出 智也 (近畿大学)  
3重、4重ゼータ値の対称和に関する公式の一般化について
- 11:15---11:45 田中 立志 (京都産業大学)  
多重ゼータ値の関係式の関係について
- 13:00---14:00 金子 昌信 (九州大学)  
Finite Multiple Zeta Values
- 14:15---15:15 斎藤 新悟 (九州大学)  
Relations among finite multiple zeta values

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