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Report on one month visit to the Yukawa Institute of Theoretical Physics, April-May, 2019

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Abstract. I report on a one month visit to the YITP as a Visiting Fellow supported by the International Research Unit of Advanced Future Studies, from April 14-May 14, 2019. For the first two weeks I participated in a Molecule-type International Workshop on "Frontiers of Lattice QCD and related topics," and for the remaining time worked on a series of projects further developing the three-particle quantization condition.

Keywords: (within 5 words) Lattice QCD, Multihadron physics, Finite volume, Flavor Physics.

Personal Note

I thank Professor and Director Sinya Aoki for the invitation to visit YITP and for his hospitality during my stay. I also thank Professor Tetsuya Onogi of Osaka University, who was on sabbatical leave at the YITP during my visit, for his hospitality. Finally, I thank Director Aoki and the other organizers of the International Molecule-type Workshop on "Frontiers in Lattice QCD and related topics", which was held during the first two weeks of my stay. The program was both interesting and stimulating, with a top-notch array of speakers. Overall, I found the YITP to be an excellent location in which to interact with others and to do focused research. I particularly enjoyed sharing an office with fellow long-term visitor (and long-time collaborator) Professor Gilberto Colangelo from the University of Bern.,

1. Presentation

As noted above, for the first two weeks of my visit there was a workshop at YITP on the area of my research, Lattice Quantum Chromodynamics (LQCD). This brought researchers from all over Japan, and several from Europe and the US, who described cutting-edge work in many directions. One focus was the use of LQCD to study multihadron systems, which is my major area of research at present. On Thursday, April 25 I gave a presentation entitled "Progress on implementing the three-particle quantization condition", in which I summarized a long-term program work that I have done over the last 5 years, and then gave a sampling of recent results. This is work done in collaboration with Tyler Blanton

(University of Washington), Raul Briceno (Old Dominion University and Jefferson Lab), Maxwell Hansen (CERN), and Fernando Romero-Lopez (Valencia).

The aim of this program of research is to relate the spectrum of QCD obtained in finite volumes—something that is possible using the methods of LQCD—to the properties of hadrons in infinite volume. Such a relation is necessary, for example, if LQCD is to be able to study strong-interaction resonances or electroweak decays to multiple hadrons. The methodology is well developed for the case of two particles, and has been applied in many LQCD calculations. The frontier is the three-particle system, and my research has been aimed at developing the necessary theoretical formalism for three particles (a problem in quantum field theory) and learning how to apply it in practice.

My talk explained the general formalism that was now in place for identical scalar particles, and described ongoing efforts to implement this formalism in ever-more-realistic situations. Implementation requires approximations, and the first approximation used was restricting particle interactions to the lowest relative angular momentum, i.e. s-wave, while at the same time requiring the quasi-local three-particle interaction to be independent of particle momenta. We find that he resulting approximation, implemented in (Briceno, 2018), contains enough flexibility to reproduce the expected trimers---three particle bound states---along with their predicted wavefunctions. I then described a more realistic approximation, in which d-wave interactions are consistently included for two and three particles, showing that certain known analytic results were reproduced, and finding that strong d-wave attraction could also lead to a trimer (Blanton, 2019).

The latter is an example of an application of our formalism directly to infinite-volume physics. By determining the spectrum in large volumes we can predict the presence of bound states in a fully relativistic three-particle setting, for which no analytic methods are known.

The final part of my talk concerned current research (discussed more below) in which we have used a new, simpler, method for allowing the possibility of two-particle bound states (dimers) in our calculation. Previously we thought that this required the use of a more complicated formalism (Briceno, 2019) but we have recently realized that our original, simpler formalism (Hansen, 2014). could be adapted to include dimers. The highlight of our new results is the finding that we are able, by going to large enough volumes, to study particle-dimer scattering in detail, mapping out the phase shift for a considerable range of energies. This is because, within this range, there are no nearby free three-particle states. Using this set up, we are able to study a system that is an analog of proton-deuteron scattering, without spin, including the reproduction of the helium-3 bound state. The resulting phase shift is qualitatively similar to that in experiment. This is another example of using our formalism to obtain infinite-volume results.

I discuss this work further below.

Blanton, T., Romero-Lopez, F., and Sharpe, S., Implementing the three-particle quantization condition including higher partial waves, *Journal of High Energy Physics,* **1093**, 106, 2019

Briceno, R., Hansen, M., and Sharpe, S., Numerical Study of the three-body quantization condition in the isotropic approximation, *Physical Review* **D98,** 014506, 2018

Briceno, R., Hansen, M., and Sharpe, S., Three-particle systems with resonant subprocesses in finite volume, *Physical Review* **D99**, 014516, 2019

Hansen, M. and Sharpe, S., Relativistic, model-independent, three-particle quantization condition, *Physical Review* **D90**, 116003, 2014

2. Research activities

2.1. Interactions at the YITP workshop

I had fruitful discussions at the above-mentioned workshop with several members of the HALQCD collaboration, which uses an alternative method based on the Nambu-Bethe-Salpeter wavefunction to extract a two-particle potential. This approach was invented by Professor Aoki and collaborators and has been applied successfully to a wide range of systems, most notably those involving two nucleons. It is an alternative to the general approach that I am following, which is based on the seminal work by Martin Luscher. I think that a detailed understanding of the systematic errors in the two methods is key to knowing their relative merits, and I learned of extensive efforts by HALQCD in this direction.

I also had very useful discussions with Ben Horz from the CALLAT collaboration and Drew Hanlon from the Mainz group. They have been developing methods for more rapid calculation of Wick contractions in multihadron correlators, and are planning on providing results for the three-pion spectrum at near-to-physical quark masses in the near future.

2.2. Three-particle quantization condition including two-particle bound states and resonances

During and particularly after the workshop, I was able to make substantial progress on three ongoing research projects. All concern the three-particle quantization condition and its applications. The first concerns the extension of the method of (Hansen, 2014) to allow two-particle bound states and resonances, as has already been mentioned above. The key new realization is that we have additional flexibility in defining the principal value (PV) prescription used when doing integrals over the three-particle pole. Using this flexibility, we are able to extend the range of applicability of the method. This allows us to study, for example, dimer-particle scattering, with results that I described in part during my presentation (see above).

I wrote a first draft of the theoretical sections of the paper describing this work during my time at the YITP, and held several conference calls discussing this draft and the ongoing work. A poster on this work will be presented in June 2019 by Fernando Romero-Lopez at Lattice 2019.

2.3 Raising the cutoff in the relativistic, three-particle quantization condition

I also continued work while at the YITP on an idea that I had had previously for raising the momentum cutoff in our numerical application of the three-particle quantization condition. This is a rather technical issue, but it is important for applications, as there is some evidence that unphysical solutions to the quantization condition arising from the fact that, in our original formalism, we are forced to use a relatively low cutoff. I have found a method to avoid this limitation, using the flexibility in the PV prescription together with a different choice of boost to the two-particle center of mass frame. This work is under numerical exploration and some progress was made while at the YITP.

2.4. K matrix parametrizations for three-particle scattering amplitudes

The relation of the finite-volume three-particle spectrum to the infinite-volume three-particle scattering amplitude involves two steps: the first is the above-mentioned quantization condition, which involves an intermediate, unphysical three-particle K matrix, while the second is a set of integral equations relating this K matrix to the physical three-particle amplitude. This second step in our relativistic formalism was worked out in (Hansen 2015).

Although not our primary initial interest, this second step provides a parametrization of the infinite-volume amplitude in terms of a real, smooth K matrix. There has been a long history of such three-particle K matrices, and a recent surge of interest as attempts are made to analyse the resonant structure of multiparticle amplitudes measured at Jefferson Lab (JLab) Thus Raul Briceno, Max Hansen and I joined forces with one of the leaders of the JLab theory effort, Adam Sczepaniak, in order to further study the result obtained in (Hansen, 2015), and its relation to other approaches.

During my time at the YITP, I completed a final draft of a paper with this group demonstrating that the result of (Hansen, 2015) satisfies all the requirements of s-channel unitarity. While this result was expected, its demonstration is nontrivial, and the methods used have also allowed me, while at the YITP, to show the relationship of our K matrix to another recent proposal. This may lead to a future publication.

Hansen, M. and Sharpe, S., Expressing the three-particle finite-volume spectrum in terms of the three-to-three scattering amplitude, *Physical Review* **D92**, 114509, 2015

2.5. FLAG activities

I am a member of the Editorial Board (EB) of the Flavor Lattice Averaging Group (FLAG), a worldwide collaboration of about 35 lattice physicists that provides vetted averages of LQCD results of relevance for flavour physics every three years. We have recently submitted the 2019 edition, and the EB is actively working to respond to the referee report, as well as to reconstitute and possibility enlarge FLAG for the next edition. It was thus serendipitous that many members of FLAG were present for some of the time of my visit, allowing extensive discussions with a rapid turnaround. In particular, I was sharing an office with another EB member, Gilberto Colangelo, and had discussions with Advisory Board member Sinya Aoki and strong-coupling working-group member Tetsuya Onogi.