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<th>項目</th>
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<tr>
<td>Title</td>
<td>期のトロピカルCalabi-Yauhypersurfaces</td>
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<tr>
<td>Author(s)</td>
<td>Yamamoto, Yuto</td>
</tr>
<tr>
<td>Citation</td>
<td>代数幾何学シンポジウム記録 2019: 121-121</td>
</tr>
<tr>
<td>Issue Date</td>
<td>2019</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/2433/245738">http://hdl.handle.net/2433/245738</a></td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Textversion</td>
<td>publisher</td>
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1. Background

- $K := \mathbb{C}\{t\}$: the convergent Laurent series field
- $f = \sum_{m} k_m x^m \in K[x_1^0, \ldots, x_n^0]$ 

**Definition**

- The **tropicalization** of $f$ is the piecewise affine function $\text{trop}(f) : \mathbb{R}^{d+1} \to \mathbb{R}$ defined by $\text{trop}(f)(X) := \max_{m} \{\text{val}(k_m) + m \cdot X\}$.
- The **tropical hypersurface** $V(\text{trop}(f)) \subset \mathbb{R}^{d+1}$ is the corner locus of $\text{trop}(f)$.

Conjecture (Gross–Wilson, Kontsevich–Soibelman)

Maximally degenerating families of Calabi–Yau manifolds with Ricci-flat Kähler metrics converge to $d$-spheres with integral affine structures with singularities in the Gromov–Hausdorff topology.

In Gross–Siebert program, an integral affine manifold with singularities $B$ is constructed as the dual intersection complex of a toric degeneration [2]. In the case of hypersurfaces, it coincides with the central part of a tropical Calabi–Yau hypersurface $V(\text{trop}(f))$ [1].

2. Goal

To describe the asymptotics of Hodge structure of a degenerating family of Calabi–Yau hypersurfaces by use of the tropical Calabi–Yau hypersurface $V(\text{trop}(f)) = \mathcal{B}$.

3. Radiance obstructions

- $\mathcal{B}$: an integral affine manifold with singularities
- $\iota : \mathcal{B} \hookrightarrow \mathcal{B}$: the smooth part
- $\mathcal{T}_\mathcal{B}$: the sheaf of integral tangent vectors on $\mathcal{B}$
- $\mathcal{T}_0 := \mathcal{T}_\mathcal{B} \otimes \mathbb{Q}$ for $\mathcal{Q} = \mathbb{R}, \mathbb{C}$
- $\{U_i\}$: a sufficiently fine open covering of $\mathcal{B}$
- $\{s_i \in \Gamma(U_i \cap \mathcal{B}, T^d_{\mathcal{B}})\}$

**Definition** (Goldman–Hirsch ’84)

The **radiance obstruction** $c_B \in H^i(B, T\mathcal{B})$ is defined by $c_B(U_i, U_j) := s_j - s_i$ for each 1-simplex $(U_i, U_j)$ of $\{U_i\}$.

4. Main results

- $\Delta \subset M_\mathcal{B}, \hat{\Delta} \subset N_\mathcal{B}$: reflexive polytopes dual to each other
- $f = \sum_{m} k_m x^m \in K[x_1^0, \ldots, x_n^0]$
- $\mathcal{B}$: an integral affine $d$-sphere obtained by contracting the tropical hypersurface $V(\text{trop}(f))$
- $H^i(B, \Lambda^d \mathcal{B}) := \bigoplus_{n=0}^i H^i(B, \Lambda^n \mathcal{B})$

- $\Sigma \subset M_\mathcal{B}$: a subdivision of the normal fan of $\hat{\Delta}$ that gives rise to a crepant resolution
- $X_\Sigma$ : the complex toric variety associated with $\Sigma$
- $D_\rho$ : the toric divisor on $X_\Sigma$ corresponding to $\rho \in \Sigma(1)$
- $\mathcal{Y} \subset X_\Sigma$ : an anti-canonical hypersurface
- $H^{2i}_{\text{amb}}(\mathcal{Y}, \mathbb{Z}) := \text{Im} \left[ \{ r : H^i(X_\Sigma, \mathbb{Z}) \to H^i(\mathcal{Y}, \mathbb{Z}) \} \right]$
- $H^*_{\text{amb}}(\mathcal{Y}, \mathbb{Z}) := \bigoplus_{i=0}^d H^{2i}_{\text{amb}}(\mathcal{Y}, \mathbb{Z})$

**Theorem 1 (Y.)**

1. There is an injective graded ring homomorphism $\psi : H^*_{\text{amb}}(\mathcal{Y}, \mathbb{Z}) \hookrightarrow H^*_{\text{amb}}(B, \Lambda^d \mathcal{B})$.

2. The radiance obstruction $c_B$ is given by
$$c_B = \sum_{\rho \in \Sigma(1)} \left[ \text{val}(\rho) - h(0) \right] \psi(D_\rho).$$

**Definition**

The **tropical period** of $\mathcal{B}$ is the following polarized logarithmic Hodge structure $(\mathcal{B}, \mathcal{Q}, F)$ on the standard log point $(0)$:

- the locally constant sheaf $\mathcal{H}_\mathcal{B}$ on $(0)_{\text{log}} \cong \mathcal{S}$ whose stalk is isomorphic to $H^*_{\text{amb}}(B, \Lambda^d \mathcal{B})$ and the monodromy is given by the cup product with $\exp(-2\pi \sqrt{-1} \text{deg})$,
- the $(-1)^i$-symmetric pairing $Q : H^i_{\text{amb}}(B, \Lambda^d \mathcal{B}) \times H^i_{\text{amb}}(B, \Lambda^d \mathcal{B}) \to H^d(B, \Lambda^d \mathcal{B})$.

**Theorem 2 (Y.)**

The restriction of this logarithmic VPH to $(0)$ is isomorphic to the tropical period of $\mathcal{B}$.

References


