

# Is Growth Declining in the Service Economy?\*

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## Abstract

This study extends Baumol's (1967) two-sector (manufacturing and services) unbalanced growth model to analyze a situation in which, first, services are used for both final consumption and intermediate inputs in manufacturing production, and second, the productivity of the manufacturing and services sectors endogenously evolves. Using this model, we investigate how the employment share of services and economic growth rate evolve through time. Our results are summarized as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, we obtain an S-shaped relationship between the employment share of services and the economic growth rate: the economic growth rate decreases at first, begins to increase after some time, decreases again, and finally, approaches zero.

*Keywords:* service economy; economic growth; endogenous productivity growth; business services

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# 1 Introduction

Why does the share of services tend to increase over time? What is the relationship between the tendency toward services and economic growth? This study is an attempt to answer these questions.

Here, we define the tendency toward services as an increase in the employment share of the service sector. This tendency toward services is observed broadly in developed economies.<sup>1)</sup>

Baumol (1967) is a pioneering work that examines the relationship between the tendency toward services and economic growth.<sup>2)</sup> He builds a two-sector (manufacturing and services) unbalanced growth model to investigate why the employment share of services increases and the relationship between the tendency toward services and economic growth.<sup>3)</sup> He shows that if the productivity growth of manufacturing is higher than that of services (Baumol's first assumption) and, in addition, if there is a constant ratio of demand for services and demand for manufacturing (Baumol's second assumption), then the employment share of services increases over time, and the economic growth rate continues to decline before finally approaching the rate of productivity growth of services.

After Baumol's seminal work, many studies have been produced on the relationship between the tendency toward services and economic growth.<sup>4)</sup> The present study integrates the elements of Sasaki (2007) and Sasaki (2012), both of which develop Baumol's (1967) argument.<sup>5)</sup> Moreover, this study introduces two generalizations: first, it relaxes Pugno's (2006) assumption that human capital accumulation exhibits constant returns to scale; and second, it relaxes assumption that productivity in service production does not grow. For the second generalization, Duernecker *et al.* (2017) empirically show that some services are

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1) In addition to studies that consider structural change from manufacturing toward services, many studies consider structural change from agriculture toward manufacturing. For example, Imrohoroglu *et al.* (2014) present a model that considers structural change from agriculture toward manufacturing, and empirically investigate factors of structural change in the Turkish economy.

2) For increasing costs of labor-intensive services (e.g., healthcare and education), which are called "cost disease," see Baumol (2012).

3) Kapur (2012) builds a model and investigates a situation in which the service sector is divided into two subsectors: a service sector in which productivity stagnates and a service sector in which productivity grows. Akbulut (2011) investigates the growth of the service sector as an explanation for the increase in women's employment. She develops an economic model that can account for the increase in women's employment and the growth of the service sector at the same time.

4) Duarte and Restuccia (2010) investigate the role of sectoral labor productivity in explaining the process of structural transformation—the secular reallocation of labor across sectors—and the time path of aggregate productivity across countries. Rogerson (2008) presents a three-sector model that captures time allocation between manufacturing, market services, and nonmarket production.

5) For a survey of the literature on the relationship between the tendency toward services and economic growth, see studies cited in Sasaki (2007, 2012).

stagnant while other services exhibit increasing productivity.

To begin with, Sasaki (2007) introduces intermediate service inputs into the Baumol model and investigates how Baumol's results change. In Baumol (1967), services are used entirely for final consumption. In this respect, Oulton (2001) observes that in reality, services are used for intermediate inputs as well as final consumption, and models such a situation. Schettkat and Yocarini (2003) provide data for intermediate services in manufacturing. Using input-output tables, they calculate the share of intermediate services to total manufacturing output. Table 1 shows that intermediate services increase through time in France, Germany, the UK, and the US.<sup>6)</sup> Therefore, considering intermediate business services in manufacturing is important for the analysis of the service economy.<sup>7)</sup>

[Table 1 around here]

Oulton (2001) concludes that as the employment share of services increases, the rate of economic growth also increases. However, in Oulton's model, services are devoted entirely to intermediate inputs into manufacturing, and hence, no services are used for final consumption. Based on this argument, Sasaki (2007) builds a model to capture a situation in which services are used for both intermediate inputs into manufacturing and final consumption. He reaches a conclusion similar to Baumol (1967): if enough time passes, the rate of economic growth declines with the tendency toward services.

Next, Sasaki (2012) introduces endogenous technological progress into the Baumol model and investigates the relationship between the tendency toward services and economic growth. His study is influenced by the work of Pugno (2006), who considers that the consumption of services augments human capital à la Lucas (1988). The consumption of health-care and education services would lead to human capital accumulation. Accordingly, the consumption of services increases the productivity of workers, thereby resulting in an increase in the productivity of both manufacturing and services. Pugno (2006) incorporates this human capital accumulation effect into Baumol's model and shows that if this effect is relatively strong, the employment shift toward services increases, not decreases, the rate of economic growth. Sasaki (2012) considers a learning-by-doing effect in manufacturing as well as Pugno's (2006) human capital accumulation effect. He shows that the employment shift toward services and the rate of economic growth have a U-shaped relationship. That is,

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6) Schettkat and Yocarini (2003) also provide data for manufacturing intermediate inputs in service sectors. They show that manufacturing intermediate inputs in France, Germany, and the US roughly stay constant whereas they increase in the UK. These empirical facts suggest that manufacturing intermediate inputs do not necessarily increase in all countries. Accordingly, our model abstracts manufacturing intermediate inputs in services.

7) Services as intermediate inputs are related to the outsourcing of services. For the relationship between the outsourcing of services and economic growth, see Fixler and Siegel (1999).

if the employment share of services begins to increase from a value of zero, the economic growth rate begins to decline with an increase in the employment share of services, but after some time, it begins to increase with the increase in the employment share of services.

The present study, by integrating the elements of Sasaki (2007) and Sasaki (2012), builds a more general model and investigates the relationship between the employment shift toward services and the economic growth rate.<sup>8)</sup> Specifically, services are used for both final consumption and intermediate inputs, human capital is accumulated through the consumption of services, and the productivities of manufacturing and services increase through time.

As Buera and Kaboski (2009) and Iscan (2010) show, Baumol's productivity bias together with Engel's consumption bias does not exhaust both employment shift and growth deceleration observed in the U.S. This study is an attempt to explain both employment shift and growth deceleration observed in reality by incorporating intermediate services inputs and endogenizing productivity growth in both manufacturing and service sectors.

The main results are as follows. First, if the human capital accumulation function exhibits constant returns to scale with respect to per capita consumption of services, then we obtain a U-shaped relationship between the employment share of services and the economic growth rate. Second, if the human capital accumulation function exhibits decreasing returns to scale with respect to per capita consumption of services, then we obtain an S-shaped relationship between the employment share of services and the economic growth rate: the economic growth rate decreases first, begins to increase after some time, decreases again, and finally, approaches zero.

The remainder of the paper is organized as follows. Section 2 builds our model. Section 3 derives the instantaneous equilibrium. Section 4 obtains long-run growth rates. Section 5 investigates the equilibrium path by using both an analytical method and numerical simulations. Section 6 concludes the paper.

## 2 Model

Consider a closed economy that consists of the manufacturing and service sectors. In the manufacturing sector, manufactured goods are produced with labor inputs and intermediate

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8) We assume that only labor is the primary factor of production, and do not consider capital accumulation because we think that technological progress is the ultimate engine of growth. Since we exclude capital from the model, we do not consider labor-capital substitution, and accordingly, we do not consider how introduction of robots and AI affect economic growth. Moreover, we do not investigate income distribution between wage income and capital income. It is important to investigate how income distribution changes in the service economy. These are limitations of our study and will be left for future research. For an analysis of a service-oriented economy using models that consider capital accumulation, see Kongsamut *et al.* (2001), Klyuev (2005), Bonatti and Felice (2008), and Batabyal and Nijkamp (2013).

service inputs. In the service sector, services are produced with only labor inputs. Consumers consume both manufactured goods and services. Intermediate service inputs can be regarded as outsourcing by manufacturing firms. Accordingly, service firms supply both services for consumers and services for manufacturing firms (i.e., outsourcing).

## 2.1 Firms

We specify the production functions of both sectors as follows<sup>9)</sup>

$$Q_m = A_m \left[ \beta^{\frac{1}{\psi}} (hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}} S^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}, \quad \beta \in (0, 1], \psi > 0, \psi \neq 1, \quad (1)$$

$$Q_s = A_s(hL_s), \quad (2)$$

where  $Q_m$  denotes the output of manufacturing;  $Q_s$  the output of services;  $L_m$  the employment of manufacturing;  $L_s$  the employment of services;  $S$  the intermediate service inputs in manufacturing;  $A_m$  the productivity specific to manufacturing;  $h$  the level of human capital;  $\beta$  a positive parameter; and  $\psi$  the elasticity of substitution between labor inputs and intermediate service inputs. When  $0 < \beta < 1$ , services are used for intermediate inputs into manufacturing. On the other hand, when  $\beta = 1$ , services are not used for intermediate inputs into manufacturing and this case corresponds to the case of Sasaki (2012). Oulton (2001), who considers intermediate business services in manufacturing, assumes that  $\psi > 1$  because intermediate services as represented by business services are considered to be good substitutes for labor inputs. In Section 3, we refer to the size of  $\psi$ .  $A_s$  denotes the productivity specific to services. Human capital is accumulated through workers themselves consuming services, and accordingly, both  $L_m$  and  $L_s$  are multiplied by  $h$ .

Profits of manufacturing firms  $\pi_m$  and service firms  $\pi_s$  are provided as follows

$$\pi_m = p_m Q_m - (w L_m + p_s S), \quad (3)$$

$$\pi_s = p_s Q_s - w L_s, \quad (4)$$

where  $p_m$  denotes the price of manufactured goods and  $p_s$  the price of services. The wage rate is denoted by  $w$ . Suppose that labor is perfectly free to move between the two sectors. Then, the nominal wages in both sectors are equalized.

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9) We abstract manufacturing intermediate inputs into the service sector. As stated in footnote 6), intermediate manufacturing inputs do not increase in all countries. For a growth model that introduces both intermediate service inputs and intermediate manufacturing inputs, see Ghavidel and Sheshkalany (2017).

## 2.2 Consumers

We specify the problem of utility maximization of consumers. Suppose that the representative consumer solves the following optimization problem

$$\max_{c_m, c_s} u = \left[ \alpha^{\frac{1}{\sigma}} c_m^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} (c_s + \gamma)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (5)$$

$$\alpha \in (0, 1), \sigma > 0, \sigma \neq 1, \gamma > 0,$$

$$\text{s.t. } p_m c_m + p_s c_s = w, \quad (6)$$

where  $c_i$  denotes per capita consumption ( $c_i = C_i/L$ ); and  $\sigma$  the elasticity of substitution between the two types of consumption. Following many previous studies on the service economy, we assume  $\sigma < 1$ , which implies that the demand for services is inelastic to price. The parameter  $\alpha$  denotes a positive parameter governing the weight of expenditure for manufacturing; and  $\gamma$  a positive parameter, which governs home productions (Moro, Moslehi, and Tanaka, 2017). Such a nonhomothetic preference is also adopted by Iscan (2010). When  $\gamma = 0$ , the preference is homothetic, and hence, the income elasticities of manufacturing consumption and service consumption are unity. When  $\gamma > 0$ , the preference is nonhomothetic, and hence, the income elasticity of manufacturing demand is less than unity and that of service demand is greater than unity. Introducing a positive  $\gamma$  does not affect the dynamics of the employment share very much. However, it does affect the dynamics of the consumption ratio  $C_s/C_m$ . Baumol (1967) assumes that the consumption ratio remains constant over time. The assumption  $\gamma > 0$  together with  $\sigma < 1$  corresponds to Baumol's Assumption 2, that is, the ratio of demand for services to demand for manufacturing is constant.

## 2.3 Labor and goods markets

Suppose that total labor supply  $L$  is constant. Then, the labor market-clearing condition is given by

$$L_m + L_s = L. \quad (7)$$

The goods market-clearing condition is given by

$$Q_m = C_m, \quad (8)$$

$$Q_s = C_s + S. \quad (9)$$

All of the manufactured goods are used for final consumption. On the other hand, services are used for both final consumption and intermediate inputs.

## 2.4 Productivity growth

Here, we define economic growth used in this study. In our model, there is no capital accumulation and population growth, and hence, productivity growth is the engine of growth. In what follows, we refer to an increase in the total factor productivity (TFP) of the whole economy  $g_{\text{TFP}}$  as economic growth.<sup>10)</sup>

The growth rate of the TFP of the manufacturing sector  $g_{\text{TFP},m}$  is given by

$$g_{\text{TFP},m} = g_{A_m} + \frac{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}}}{\beta^{\frac{1}{\psi}}(hL_m)^{\frac{\psi-1}{\psi}} + (1-\beta)^{\frac{1}{\psi}}S^{\frac{\psi-1}{\psi}}} g_h, \quad (10)$$

where  $g_x \equiv \dot{x}/x$  denotes the growth rate of a variable  $x$ .

On the other hand, the growth rate of the TFP of the service sector  $g_{\text{TFP},s}$  is given by

$$g_{\text{TFP},s} = g_{A_s} + g_h. \quad (11)$$

Let us specify  $g_{A_m}$ ,  $g_{A_s}$ , and  $g_h$  that appear in equations (10) and (11).

First, the productivity specific to  $A_s$  increases exogenously, which is specified as follows

$$A_s(t) = A_{s,0}(1 + \mu\theta t)^{\frac{1}{\theta}}, \quad \theta \geq 0, \mu > 0, \quad (12)$$

where  $A_{s,0}$  denotes the initial level of  $A_s$  and we assume that  $A_{s,0} = 1$  for simplicity; and  $\mu$  and  $\theta$  are positive parameters. In Sasaki (2012),  $A_s$  is assumed to be constant. As explained later, whether  $A_s$  increases considerably affects the results.

When specifying an exogenous and continuous increase in a variable, we usually assume that the variable increases at a constant rate, and hence, we use an exponential function. Instead, based on the study of Groth *et al.* (2010), we use a more flexible specification than an exponential function. From equation (12), the growth rate of  $A_s$  is given by

$$g_{A_s} = \frac{\mu}{1 + \mu\theta t}. \quad (13)$$

From this, we know that if  $\theta > 0$ , the growth rate of  $A_s$  declines over time and  $\lim_{t \rightarrow +\infty} g_{A_s} =$

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10) The expression “TFP” is used to describe the case in which there are some factors of production. In contrast, in our model, labor is the sole primary factor of production, and so, the expression “TFP” may not be appropriate. In this case, the expression “total labor productivity (TLP)” is more appropriate. However, we refer to TFP because it is a more general expression than TLP.

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Equation (12) includes some special cases.

$$A_s = \begin{cases} A_{s,0}e^{\mu t} & \text{if } \theta = 0, \\ A_{s,0}(1 + \mu t) & \text{if } \theta = 1, \\ A_{s,0} & \text{if } \theta \rightarrow +\infty. \end{cases} \quad (14)$$

That is,  $A_s$  becomes an exponential function if  $\theta \rightarrow 0$ , a linear function of time if  $\theta = 1$ , and constant if  $\theta \rightarrow +\infty$ . We use this functional form because we want to consider empirical facts that productivity of services does not increase so rapidly.

Next, we specify the productivity specific to the manufacturing sector,  $A_m$ . We assume that  $A_m$  is an increasing function of the knowledge stock  $K_m$ .

$$A_m = K_m^\phi, \quad \phi > 0, \quad (15)$$

where  $\phi$  denotes the elasticity of  $A_m$  with respect to  $K_m$ . In the manufacturing sector, increasing returns to scale is likely to work because manufacturing production is easy to be mechanized. According to Arrow's (1962) learning by doing effect, we assume that the knowledge stock depends on the production experience that is accumulated until now; we specify the knowledge stock as follows

$$K_m = \exp \left[ \int_{-\infty}^t \frac{L_m(\tau)}{L(\tau)} d\tau \right]. \quad (16)$$

Note that the production experience  $K_m$  is measured by  $L_m/L$ . We use the manufacturing employment share, and not the level of manufacturing employment, to ascertain that the dynamics of the model hold even when the labor force grows. In addition, we use manufacturing employment, and not manufacturing output, for the following reason. In our model, labor is the sole factor of production; then, an increase in output has a one-to-one relationship with an increase in employment. Therefore, for simplicity, we measure production experience by employment of manufacturing, not by output of manufacturing.

Substituting equation (16) into equation (15) and differentiating the resultant expression with respect to time, we obtain

$$\dot{A}_m = \left( \phi \frac{L_m}{L} \right) A_m = \phi \left( 1 - \frac{L_s}{L} \right) A_m. \quad (17)$$

That is,  $A_m$  becomes an increasing (decreasing) function of the employment share of man-

ufacturing (services).<sup>11)</sup> Therefore, if the employment share of the service sector increases, the productivity specific to manufacturing decreases. Note that since productivity specific to the manufacturing sector  $A_m$  is completely endogenized through learning-by-doing and capital accumulation is excluded, the size of the parameter  $\phi$  plays a crucial role in the results when the human capital accumulation function exhibits constant returns to scale. Instead, we can assume that productivity growth in manufacturing is only partially endogenized. For example, we specify  $A_m$  as  $A_m = e^{\phi_1 t} K_m^{\phi_2}$ , where  $\phi_1 \geq 0$  and  $\phi_2 > 0$ . In this case, we obtain  $\dot{A}_m/A_m = \phi_1 + \phi_2(L_m/L)$ . For simplicity, we assume that  $\phi_1 = 0$  and  $\phi_2 = \phi$  in our model. However, if we assumet that  $\phi_1 > 0$  and  $\phi_2 > 0$ , we may obtain different results. Accordingly, some key results of this paper crucially depend on this restrictive assumption, that is, complete endogenization of the growth rate of  $A_m$ . This is the limitation of our model.

Then, we specify the accumulation of human capital  $h$ . According to Pugno (2006), human capital is accumulated through consumption of services.<sup>12)</sup>

$$\dot{h} = \delta c_s^\lambda, \quad \delta > 0, \quad 0 < \lambda \leq 1, \quad (18)$$

where  $\delta$  denotes a positive parameter and captures the efficiency of human capital accumulation. Integrating equation (18) with respect to time, we obtain  $h(t) = \delta \int_{-\infty}^t [c_s(\tau)]^\lambda d\tau$ , which states that total sum of services consumption from the past to the present determines the present level of human capital: accumulated experiences of consumption shape human capital. Note that equation (18) is different from the specifications of Pugno (2006) and Sasaki (2012). They assume that  $\lambda = 1$  whereas we assume that  $0 < \lambda \leq 1$ . The reason is that we investigate a broader situation, which includes a knife-edge case  $\lambda = 1$ . As will be shown later, if we assume that  $0 < \lambda < 1$ , economic growth is not sustainable in the very long run. Instead, we introduce productivity specific to services, that is,  $A_s$  and we consider the case in which  $A_s$  increases through time, which can offset the diminishing effect of  $\dot{h}$ . In both Lucas (1988) and Pugno (2006), they model a situation in which consumers intentionally accumulate human capital through intertemporal utility maximization. In contrast, in this paper, for ease of analysis, we model a situation in which consumers unintentionally accumulate human capital through consumption of services.<sup>13)</sup>

We derive the growth rate of TFP of the whole economy. Note that in our model, services

11) De Vincenti (2007) adopts a specification in which the growth rate of manufacturing productivity is a decreasing function of the employment share of manufacturing.

12) The relationship between health services and economic growth is analyzed in Van Zon and Muysken (2001).

13) Pugno (2006) considers two different cases: in one case, the consumption of services unintentionally leads to human capital accumulation whereas, in the other, agents intentionally consume services to accumulate human capital.

are used for intermediate inputs in manufacturing. In this case, in line with Oulton (2001) and Sasaki (2007), it is appropriate to use Domar aggregation presented by Domar (1961).<sup>14)</sup> Using Domar aggregation, we obtain the growth rate of TFP as follows

$$g_{\text{TFP}} = \frac{p_m Q_m}{\text{GDP}} g_{\text{TFP},m} + \frac{p_s Q_s}{\text{GDP}} g_{\text{TFP},s}. \quad (19)$$

Gross domestic product (GDP) is given by  $p_m C_m + p_s C_s = wL$ . Equation (19) considers that services are used for intermediate inputs, that is,  $Q_s > C_s$ , and hence, the sum of the weights exceeds unity, that is,  $p_m Q_m + p_s Q_s > \text{GDP}$ .

### 3 Instantaneous equilibrium

At some point in time,  $A_m$ ,  $A_s$ , and  $h$  are given. We obtain the instantaneous equilibrium as follows

1. By solving the utility-maximization problem of consumers, we obtain demand functions for manufacturing and services, which depend on  $p_m$ ,  $p_s$ , and  $w$ .
2. By solving the profit-maximization problem of firms, we obtain the optimal ratio of  $L_m$  and  $S$ , which depends on  $p_s$  and  $w$ .
3. By solving the zero-profit conditions of firms, we obtain  $p_m$  and  $p_s$ , which depend on  $A_m$ ,  $A_s$ ,  $h$ , and  $w$ .
4. By solving the goods market-clearing condition, we obtain  $L_m$  and  $L_s$ .

From the utility-maximization problem, we obtain demand functions for manufacturing and services.

$$C_m = \frac{\alpha}{1-\alpha} \left( \frac{p_s}{p_m} \right)^\sigma \left[ \frac{w - \frac{\alpha}{1-\alpha} p_m \left( \frac{p_s}{p_m} \right)^\sigma \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left( \frac{p_s}{p_m} \right)^\sigma} + \gamma \right] L, \quad (20)$$

$$C_s = \frac{w - \frac{\alpha}{1-\alpha} p_m \left( \frac{p_s}{p_m} \right)^\sigma \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left( \frac{p_s}{p_m} \right)^\sigma} \cdot L. \quad (21)$$

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<sup>14)</sup> In addition, for Domar aggregation, see Hulten (1978) and Ten Raa and Schettkat (2001).

From the profit-maximization problem of manufacturing and service firms, that is, the equalization between the marginal rate of substitution and the relative price, we obtain

$$S = \frac{1-\beta}{\beta} A_s^\psi (hL_m). \quad (22)$$

That is, intermediate service inputs are linear with respect to effective labor  $hL_m$ .

From the zero-profit conditions, we obtain

$$p_m = \frac{w}{\beta^{\frac{1}{\psi-1}} A_m h \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{1}{\psi-1}}}, \quad (23)$$

$$p_s = \frac{w}{A_s h}. \quad (24)$$

Hence, the relative price is given by

$$\frac{p_s}{p_m} = \beta^{\frac{1}{\psi-1}} \frac{A_m}{A_s} \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{1}{\psi-1}}. \quad (25)$$

With equation (22), we can express  $Q_m$  as a function of  $L_m$ .

$$Q_m = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}. \quad (26)$$

That is, manufacturing output is linear with respect to manufacturing employment.

The share of intermediate services in manufacturing output is given by

$$\frac{p_s S}{p_m Q_m} = \frac{1}{1 + \frac{\beta}{1-\beta} A_s^{1-\psi}}. \quad (27)$$

When  $A_s$  is constant through time, the share of intermediate services in manufacturing is also constant. When  $A_s$  increases through time, the share of intermediate services in manufacturing increases if  $\psi > 1$  but decreases if  $\psi < 1$ . As shown in Table 1, this share increases through time in reality, and hence, we assume  $\psi > 1$  in the following analysis.

From the goods market-clearing condition  $C_m = Q_m$ , we obtain

$$\frac{\alpha}{1-\alpha} \left(\frac{p_s}{p_m}\right)^\sigma \left[ \frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^\sigma \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^\sigma} + \gamma \right] L = \beta^{\frac{1}{\psi-1}} A_m h L_m \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}. \quad (28)$$

Solving equation (28) for  $L_m/L$ , we obtain the employment share of manufacturing as fol-

lows:<sup>15)</sup>

$$\frac{L_m}{L} = \frac{\frac{\alpha}{1-\alpha} \left(\frac{p_s}{p_m}\right)^\sigma \left[ \frac{w - \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^\sigma \gamma}{p_s + \frac{\alpha}{1-\alpha} p_m \left(\frac{p_s}{p_m}\right)^\sigma} + \gamma \right]}{\beta^{\frac{1}{\psi-1}} A_m h \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)^{\frac{\psi}{\psi-1}}}. \quad (29)$$

Consumption of services is given by  $C_s = Q_s - S$ , which is rewritten in per capita terms as

$$c_s = A_s h \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi-1} h \left(1 - \frac{L_s}{L}\right). \quad (30)$$

From this, the rate of human capital accumulation is given by

$$\frac{\dot{h}}{h} = \delta A_s^\lambda h^{\lambda-1} \left[ \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi-1} \left(1 - \frac{L_s}{L}\right) \right]^\lambda. \quad (31)$$

For the rate of human capital accumulation to be positive, that is,  $\dot{h}/h > 0$ , we need

$$\frac{L_s}{L} > \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{1-\psi}}. \quad (32)$$

The ratio of manufacturing output to GDP and the ratio of service output to GDP are respectively given by

$$\frac{p_m Q_m}{GDP} = \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \left(1 - \frac{L_s}{L}\right), \quad (33)$$

$$\frac{p_s Q_s}{GDP} = \frac{L_s}{L}. \quad (34)$$

These ratios correspond to the Domar weights of manufacturing and services.

The value-added shares of manufacturing and services are as follows:

$$\frac{p_m C_m}{GDP} = \frac{p_m Q_m}{GDP} = \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \left(1 - \frac{L_s}{L}\right), \quad (35)$$

$$\frac{p_s C_s}{GDP} = 1 - \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \left(1 - \frac{L_s}{L}\right). \quad (36)$$

We note that when  $\beta = 1$ , that is, there are no intermediate service inputs, we have the

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15) Note that  $-\alpha/(1-\alpha)$  appears in the numerator of equation (29). Accordingly, if  $\alpha$  is relatively large, then  $L_m/L$  will be negative. Therefore, the size of  $\alpha$  is restricted within a certain range.

following relations:

$$\frac{p_m C_m}{\text{GDP}} = \frac{p_m Q_m}{\text{GDP}} = \frac{L_m}{L}, \quad (37)$$

$$\frac{p_s Q_s}{\text{GDP}} = \frac{p_s C_s}{\text{GDP}} = \frac{L_s}{L}. \quad (38)$$

Moreover, when  $0 < \beta < 1$ , that is, there are intermediate service inputs, we can show the following relationships hold between the value-added shares and the employment shares.

$$\frac{p_m C_m}{\text{GDP}} > \frac{L_m}{L}, \quad (39)$$

$$\frac{p_s C_s}{\text{GDP}} < \frac{L_s}{L}. \quad (40)$$

By observing the US data during the period 1860–2000, Buera and Kaboski (2009) show that there are large deviation between value-added shares and employment shares and state that models of structural change need to explain this deviation. They suggest that the standard model of structural change cannot explain this result without sectoral differences in human capital or factor distortions. For this point, we must note that introduction of intermediate services input into the model can be a remedy for this problem.

The growth rate of the TFP of manufacturing is given by

$$g_{\text{TFP},m} = g_{A_m} + \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{\psi-1}} g_h. \quad (41)$$

Therefore, the growth rate of the TFP for the whole economy is given by

$$g_{\text{TFP}} = \left(1 - \frac{L_s}{L}\right) \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) g_{\text{TFP},m} + \frac{L_s}{L} g_{\text{TFP},s}. \quad (42)$$

## 4 Long-run growth rates

We investigate the growth rate of each sector when the tendency toward services progresses and finally,  $L_s/L = 1$ . In the following analysis, we consider two cases:  $\lambda = 1$  and  $0 < \lambda < 1$ .

### 4.1 Constant returns to scale in human capital accumulation

When the human capital accumulation function exhibits constant returns to scale with respect to consumption of services, that is,  $\lambda = 1$ , each growth rate when  $L_s/L = 1$  is given

by

$$\lim_{L_s/L \rightarrow 1} g_{A_m} = 0, \quad (43)$$

$$\lim_{L_s/L \rightarrow 1} g_h = \delta A_s, \quad (44)$$

$$\lim_{L_s/L \rightarrow 1} g_{TFP,m} = \frac{1}{1 + \frac{1-\beta}{\beta} A_s^{\psi-1}} \delta A_s, \quad (45)$$

$$\lim_{L_s/L \rightarrow 1} g_{TFP,s} = g_{A_s} + \delta A_s, \quad (46)$$

$$\lim_{L_s/L \rightarrow 1} g_{TFP} = g_{TFP,s} = g_{A_s} + \delta A_s. \quad (47)$$

Equations (46) and (47) shows that the growth rate of the TFP for the whole economy is equal to the growth rate of the TFP of the service sector.

These equations show that long-run growth rates depend significantly on the growth of  $A_s$ . When  $\theta \geq 0$ , and hence,  $A_s$  increases through time,  $g_{TFP}$  continues to increase through time. This suggests that the level of TFP continues to increase indefinitely and diverges to infinity within finite time. Accordingly, in the case of  $\lambda = 1$ , as long as  $A_s$  continues to increase through time, irrespective of whether exponentially, the level of TFP becomes infinity, and hence, we cannot permit the growth of  $A_s$ . In contrast, when  $\theta \rightarrow +\infty$ , and hence,  $A_s$  stays constant through time,  $g_{TFP}$  converges to a positive constant value  $g_{TFP} = \delta$ . That is, the economy grows at a positive and constant rate in the long run.

Summarizing these results, we obtain the following proposition.

**Proposition 1.** *When the human capital accumulation function exhibits constant returns to scale with respect to consumption of services, the growth rate of the productivity specific to the service sector has to be zero. In this case, if the tendency toward services completely finishes, that is, the employment share of the service sector approaches unity, the growth rate of TFP for the whole economy approaches  $\delta$  in the long run.*

## 4.2 Decreasing returns to scale in human capital accumulation

When the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services, that is,  $0 < \lambda < 1$ , the growth rate of each sector when  $L_s/L = 1$  is given by

$$\lim_{L_s/L \rightarrow 1} g_{A_m} = 0, \quad (48)$$

$$\lim_{L_s/L \rightarrow 1} g_h = \delta A_s^\lambda h^{\lambda-1}, \quad (49)$$

$$\lim_{L_s/L \rightarrow 1} g_{\text{TFP},m} = \frac{1}{1 + \frac{1-\beta}{\beta}} \delta A_s^\lambda h^{\lambda-1}, \quad (50)$$

$$\lim_{L_s/L \rightarrow 1} g_{\text{TFP},s} = g_{A_s} + \delta A_s^\lambda h^{\lambda-1}, \quad (51)$$

$$\lim_{L_s/L \rightarrow 1} g_{\text{TFP}} = g_{\text{TFP},s} = g_{A_s} + \delta A_s^\lambda h^{\lambda-1}. \quad (52)$$

Suppose that  $A_s$  is constant or increases less than exponentially. Since  $g_h \geq 0$  from equation (49),  $h$  is nondecreasing with respect to time. Moreover, since  $\partial g_h / \partial h \leq 0$  from equation (49), the growth rate of human capital  $g_h$  converges to zero after enough time passes. This means that  $g_{\text{TFP},m}$ ,  $g_{\text{TFP},s}$ , and  $g_{\text{TFP}}$  converge to zero. Therefore, in this case, the growth rate of the TFP for the whole economy converges to zero in the long run.

In contrast, suppose that  $A_s$  increases exponentially. In this case, even when the rate of human capital accumulation is zero,  $g_{\text{TFP}}$  converges to  $\mu$  because  $A_s$  increases at a positive and constant rate  $\mu$ . That is, the economy grows at a positive and constant rate in the long run.

Summarizing these results, we obtain the following proposition.

**Proposition 2.** *Suppose that the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services. When the exogenous productivity specific to the service sector stays constant through time or increases less than exponentially, the growth rate of the TFP for the whole economy converges to zero in the long run if the tendency toward services completely finishes. In contrast, when the exogenous productivity specific to the service sector increases at a constant rate  $\mu$ , the growth rate of the TFP for the whole economy converges to  $\mu$  in the long run if the tendency toward services completely finishes.*

We note that the TFP growth differentials between manufacturing and services are given by as follows:

$$g_{\text{TFP},m} - g_{\text{TFP},s} = \begin{cases} g_{A_m} - g_{A_s} - \underbrace{\frac{g_h}{1 + \frac{\beta}{1-\beta} A_s^{\psi-1}}}_{\Omega} & \text{if } 0 < \beta < 1, \\ g_{A_m} - g_{A_s} & \text{if } \beta = 1. \end{cases} \quad (53)$$

Accordingly, when services are intermediate inputs into manufacturing, it is likely to be  $g_{\text{TFP},m} < g_{\text{TFP},s}$  because the part “ $\Omega$ ” is positive. However, in both cases, we obtain  $g_{\text{TFP},m} = g_{\text{TFP},s}$  in the long run because we have  $g_{A_m} = g_{A_s} = 0$  and the part “ $\Omega$ ” converges to zero as long as  $\psi > 1$ .

## 5 Equilibrium path and numerical simulations

The analysis in Section 4 assumes that the tendency toward services progresses (i.e.,  $L_s/L \rightarrow 1$ ) and that enough time passes (i.e.,  $t \rightarrow +\infty$ ). However, the analysis does not reveal whether the employment share of the service sector increases; moreover, it does not reveal each growth rate along the transitional dynamics path. Accordingly, we investigate the employment share of the service sector and each growth rate along the transitional dynamics path by using both analytical and numerical methods.

### 5.1 Constant returns to scale in human capital accumulation

In this case, from equation (42), the growth rate of the TFP for the whole economy leads to

$$g_{\text{TFP}} = \beta^{-1} \frac{L_m}{L} (g_{A_m} + \beta g_h) + \left(1 - \frac{L_m}{L}\right) g_h. \quad (54)$$

The value of  $g_{\text{TFP}}(0)$ , that is, the growth rate of the TFP for the whole economy when the employment share of services is zero, must be positive.

$$g_{\text{TFP}}(0) = \beta^{-1} [\phi - \delta(1 - \beta)]. \quad (55)$$

From this, we require the following condition

$$\phi - \delta(1 - \beta) > 0. \quad (56)$$

Hereafter, we assume that this condition holds in the case of  $\lambda = 1$ .

Comparing  $g_{\text{TFP}}(0)$  with  $g_{\text{TFP}}(1)$ , which is obtained when  $L_s/L = 1$ , we can see whether an increase in the employment share of services increases or decreases the rate of economic growth in the long run. If  $g_{\text{TFP}}(0) < g_{\text{TFP}}(1)$ , the rate of economic growth increases in the long run. In contrast, if  $g_{\text{TFP}}(0) > g_{\text{TFP}}(1)$ , the rate of economic growth decreases in the long run. From this, we obtain

$$g_{\text{TFP}}(0) \leq g_{\text{TFP}}(1) \iff \phi \leq \delta. \quad (57)$$

With the calculation of  $g_{\text{TFP}}$ , we find that it is a quadratic function of  $L_s/L$ , which is

given by

$$g_{\text{TFP}} = \begin{cases} \frac{\phi}{\beta} \left( \frac{L_s}{L} - \frac{2\phi - \delta}{2\phi} \right)^2 + \delta \left( \frac{4\beta\phi - \delta}{4\beta\phi} \right) & \text{if } 0 < \beta < 1, \\ \phi \left( \frac{L_s}{L} - \frac{2\phi - \delta}{2\phi} \right)^2 + \delta \left( \frac{4\phi - \delta}{4\phi} \right) & \text{if } \beta = 1. \end{cases} \quad (58)$$

This result is almost the same as that of Sasaki (2012). With  $\beta = 1$ , equation (58) is the same as that of Sasaki (2012). Hence, we obtain the following proposition.

**Proposition 3.** *If  $2\phi < \delta$ , the growth rate of TFP increases with the employment shift toward services and converges to  $\delta$ . If  $2\phi > \delta$ , the growth rate of TFP decreases until the employment share of services reaches  $(L_s/L)^* = (2\phi - \delta)/2\phi$ , and from then onward, it increases with the employment shift toward services, finally converging to  $\delta$ .*

This proposition is shown in Figures 1–3.

[Figures 1–3 around here]

When deriving Proposition 3, we assume that the employment share of services increases over time. In our model, productivity growth and employment share are dependent on each other. Therefore, we must analyze the dynamics of employment share and productivity growth simultaneously.

Each sector's employment share, each sector's productivity growth, and the productivity growth of the whole economy depend on  $A_m$  and  $h$ . Moreover,  $\dot{A}_m$  and  $\dot{h}$  depend on  $A_m$  and  $h$ . As a result, if we provide the initial values of  $A_m$  and  $h$ , and if we examine the system of differential equations of  $A_m$  and  $h$ , we can determine the time paths of  $A_m$  and  $h$ . In addition, by using this result, we can obtain the time paths of all variables. However, the differential equations of our model are nonlinear, and hence, analytical solutions are difficult to obtain. Accordingly, we use numerical simulations.

Here, we investigate the following three cases.

$$\text{Case 1 : } \phi < \delta, 2\phi < \delta, \quad (59)$$

$$\text{Case 2 : } \phi < \delta, 2\phi > \delta, \quad (60)$$

$$\text{Case 3 : } \phi > \delta. \quad (61)$$

In Cases 1 and 2, we have  $g(0) < g(1)$ , and in Case 3, we have  $g(0) > g(1)$ .

We set the parameters as in Table 2 and the initial values as  $A_m(0) = h(0) = 1$ . Then, we change the elasticity of substitution  $\sigma$  from 0.1 to 0.9 in intervals of 0.1.

[Table 2 around here]

Figures 4–9 show the results of the numerical simulation. Figures 4, 6, and 8 show the time paths of the employment share of services. Figures 5, 7, and 9 show the time paths of the growth rate of the TFP for the whole economy. In all cases, the employment share of services increases over time. In Cases 2 and 3, the growth rate of TFP first declines, then increases, and finally, converges to  $\delta$ . In Case 1, the growth rate of TFP increases constantly over time, which is similar to the result of Oulton (2001). These results are obtained assuming that  $\theta \rightarrow +\infty$ .

[Figures 4–9 around here]

## 5.2 Decreasing returns to scale in human capital accumulation

Let  $0 < \lambda < 1$  in equation (42). Then, we obtain the growth rate of TFP as follows:

$$g_{\text{TFP}} = \begin{cases} \phi \frac{1-\beta}{\beta} A_s^{\psi-1} \left(1 - \frac{L_s}{L}\right)^2 + \delta A_s^\lambda h^{\lambda-1} \left[ \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \frac{L_s}{L} - \frac{1-\beta}{\beta} A_s^{\psi-1} \right]^\lambda + g_{A_s} \frac{L_s}{L} & \text{if } 0 < \beta < 1, \\ \delta A_s^\lambda h^{\lambda-1} \left(\frac{L_s}{L}\right)^\lambda + g_{A_s} \frac{L_s}{L} & \text{if } \beta = 1. \end{cases} \quad (62)$$

Note that  $A_s$  and  $g_{A_s}$  depend on time, and  $h$  depends on  $L_s/L$ .

In this case, analytical solutions are difficult to obtain, and thus, we resort to numerical simulations.

We set the parameters as in Table 3 and the initial values as  $A_m(0) = h(0) = 1$ . We call this case as Case 4. Then, we change the elasticity of substitution  $\sigma$  from 0.1 to 0.9 in intervals of 0.1.

[Table 3 around here]

[Figures 10–28 around here]

The results of numerical simulation in Case 4 are shown in Figures 10–28 and summarized as follows:

1. The relative price of services increase through time, which is consistent with Baumol's cost disease.
2. The share of intermediate services in manufacturing production increases through time, which is consistent with empirical findings of Schettkat and Yocarini (2003).

3. The employment share of manufacturing decreases through time while that of services increases through time, which is consistent with data for developed countries.
4. The Domar weight of the manufacturing decreases through time while that of services decreases through time, which is consistent with the finding of Oulton (2001).
5. The value-added share of manufacturing decreases through time while that of services increases through time, which is consistent with data for developed countries.
6. The growth rate of  $A_s$  decreases through time.
7. The growth rate of the level of human capital  $h$  increases at first, reaches the maximum, then decreases through time, and finally approaches zero.
8. The growth rate of  $A_m$  decreases through time.
9. The growth rate of TFP in services shows an S-shaped curve, and finally approaches zero.
10. The growth rate of TFP in manufacturing also shows an S-shaped curve, and finally approaches zero. However, the S-shaped curve of manufacturing has a much different shape from the S-shaped curve of services.
11. TFP growth differential between manufacturing and services is positive for some time but it is negative in the long run. This result arises because services are used for intermediate inputs into manufacturing. If no services are used as intermediate inputs, TFP growth differential between manufacturing and services is likely to be positive under the same parameter sets. In either case, however, TFP growth differential approaches zero in the very long run.
12. TFP growth of the whole economy shows an S-shaped curve, and it finally approaches zero.
13. Contribution of TFP growth in manufacturing to TFP growth of the whole economy decreases through time.
14. Contribution of TFP growth in services to TFP growth of the whole economy increases first, reaches the maximum, then, decreases through time, and finally, approaches zero.
15. The instantaneous utility increases through time because the TFPs of both sectors increase. However, the slope of the time path of the utility will flatten in the very long run because the TFP growth rates of both sectors, the two engines of growth in the model, will be zero in the long run.

As stated in equation (52), as long as enough time passes, the growth rate of the TFP for the whole economy approaches zero. However, along with transitional dynamics, the employment share of services and the growth rate of TFP display interesting behavior. In Case 4, the employment share of services monotonically increases whereas the growth rate of TFP first declines, then increases, and finally, declines after some time.

The time paths of the growth rate of TFP in Case 4 can be explained as follows. First, as the employment share of services increases, the learning-by-doing effect of manufacturing weakens, which lowers the economic growth rate. However, as the employment share of services increases further, human capital accumulation proceeds further, and productivity specific to services increases with the passage of time. These contribute to an increase in the growth rate of TFP. Nevertheless, as enough time passes, there is a diminishing positive effect of an increasing service employment share on the growth rate of TFP. In the long run, the growth rate of TFP approaches zero. The phase of the decreasing to increasing effect is similar to the time path obtained in Sasaki (2012) while the phase of the increasing to decreasing effect is similar to the time path obtained in Sasaki (2007).

### 5.3 Summary

Our model introduces intermediate services into Sasaki (2012) and considers decreasing returns to scale in the human capital accumulation function whereas the constant returns to scale function is used in Sasaki (2012). In addition, productivity specific to the service sector  $A_s$  is introduced in our model. We explain which one of these three extensions is crucial to our results.

Table 4 summarizes our results. When  $\lambda = 1$ , that is, the human capital accumulation function exhibits constant returns to scale and  $A_s$  is constant, TFP growth decreases as the employment share of services increases but at some point in time, it begins to increase and converges to a constant value, irrespective of whether services are used for intermediate inputs. When  $\lambda = 1$  and  $A_s$  increases through time, the long-run growth rate of TFP continues to increase and the level of TFP diverges to infinity within finite time.

[Table 4 around here]

Next, we investigate the case in which the human capital accumulation function exhibits decreasing returns to scale. This case is more realistic than the constant returns to scale case.

First, when  $A_s$  is constant, TFP growth continues to decrease and converges to zero as the employment share of services increases, irrespective of whether services are used for intermediate inputs. In the case of decreasing returns to scale, a shift in employment share toward services decreases the growth rate of human capital.

Second, when  $A_s$  increases less than exponentially, the growth rate of TFP first declines, then increases, declines after some time, and converges to zero in the long run, irrespective of whether services are used for intermediate inputs.

Third, when  $A_s$  increases exponentially, TFP growth decreases as the employment share of services increases but at some point in time, it begins to increase and converges to a constant value, irrespective of whether services are used for intermediate inputs. This result is the same as that obtained in the case in which  $\lambda = 1$  and  $A_s$  are constant.

Summarizing these results, we obtain the following proposition.

**Proposition 4.** *Suppose that the human capital accumulation function exhibits decreasing returns to scale with respect to consumption of services. In addition, suppose that the growth rate of the exogenous productivity specific to the service sector diminishes with time. Then, as the employment share of the service sector increases, the growth rate of the TFP for the whole economy decreases first, begins to increase after some time, decreases again, and finally, approaches zero.*

These results show that the results are not affected by whether services are used for intermediate inputs. That is, we obtain similar results when  $0 < \beta < 1$  and when  $\beta = 1$ . However, as stated above, the introduction of intermediate service inputs into manufacturing improves the goodness of fit to the empirical evidence that there exist large differences between employment shares and value-added shares, which is stressed by Buera and Kaboski (2009). Therefore, in this point, the introduction of intermediate service inputs is important.

Moreover, since the result of the case in which  $\lambda = 1$  and  $A_s$  is constant is the same as that of the case in which  $\lambda < 1$  and  $A_s$  increases exponentially, it is crucial for the results that either  $A_s$  or  $h$  increase exponentially. When the human capital accumulation function exhibits decreasing returns to scale, a shift in employment share toward services decreases the growth rate of human capital. This implies that the long-run growth rate of TFP converges to zero unless an increase in  $A_s$  exceeds the decrease of human capital accumulation.

## 5.4 Interpretations

We explain in detail the reason why we obtain the above results. We restrict ourselves to the case in which  $\lambda = 1$  and  $\theta \rightarrow +\infty$  and the case in which  $0 < \lambda < 1$  and  $0 < \theta \ll +\infty$ .

The growth rate of the TFP for the whole economy comprises four components.

$$g_{\text{TFP}} = \underbrace{\frac{p_m Q_m}{\text{GDP}}}_{(A)} g_{\text{TFP},m} + \underbrace{\frac{p_s Q_s}{\text{GDP}}}_{(B)} g_{\text{TFP},s} + \underbrace{\frac{p_m Q_m}{\text{GDP}}}_{(C)} g_{\text{TFP},m} + \underbrace{\frac{p_s Q_s}{\text{GDP}}}_{(D)} g_{\text{TFP},s} \quad (63)$$

$$= \underbrace{\left(1 - \frac{L_s}{L}\right) \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right)}_{(A)} g_{\text{TFP},m} + \underbrace{\frac{L_s}{L}}_{(B)} g_{\text{TFP},s} + \underbrace{\frac{L_s}{L}}_{(C)} g_{\text{TFP},s} + \underbrace{\frac{L_s}{L}}_{(D)} g_{\text{TFP},s}. \quad (64)$$

When  $\lambda = 1$  and  $\theta \rightarrow +\infty$ , (A) and (B) decrease while (C) and (D) increase through time. When the employment share of the service sector is small, the negative effect of decreasing (A) and (B) is large, whereas the positive effect of increasing (C) and (D) is small. Accordingly, the growth rate of TFP decreases. However, as the employment share of the service sector increases, the positive effect of (C) and (D) exceeds the negative effect of (A) and (B), and finally, converges to a positive and constant rate.

The case in which  $0 < \lambda < 1$  and  $0 < \theta \ll +\infty$  is more complicated. Part (A) continues to decrease with time when  $\psi > 1$ .<sup>16)</sup> Part (B) continues to decrease with time when  $\psi > 1$ .<sup>17)</sup> Part (C) continues to increase through time and converges to unity. Part (D) decreases first, then increases, and finally, decreases. Combined with these effects,  $g_{\text{TFP}}$  decreases first, begins to increase after some time, decreases again, and finally, approaches zero.

Figure 29 decomposes the growth rate of the TFP for the whole economy into Parts (A) and (B) and Parts (C) and (D) when  $\sigma = 0.5$  and  $\beta = 0.5$ . We note that for Figure 29, we use  $\beta = 0.5$  instead of  $\beta = 0.95$  for the purpose of explanation. Parts (C) and (D) turn from increasing to decreasing, which is caused by the evolution of the rate of human capital accumulation  $g_h$ .

$$g_h = \underbrace{\delta A_s^\lambda h^{\lambda-1}}_{(a)} \left[ \left(1 + \frac{1-\beta}{\beta} A_s^{\psi-1}\right) \underbrace{\frac{L_s}{L}}_{(b)} - \frac{1-\beta}{\beta} A_s^{\psi-1} \right]^\lambda. \quad (31')$$

Part (a) decreases and finally approaches zero, which has a negative effect on  $g_h$ , whereas Part (b) increases with a rise in  $L_s/L$ , which has a positive effect on  $g_h$ . Therefore, as  $L_s/L$  increases,  $g_h$  increases, begins to decrease after some time, and finally, converges to zero.

[Figure 29 around here]

In Figure 29, the negative effects of (A) and (B) are dominant in Phase I, and hence, the

16) Even when  $\psi < 1$ , Part (A) continues to decrease with time.

17) When  $\psi < 1$ , Part (B) decreases first, then increases, and finally, decreases.

growth rate of TFP decreases. In Phase II, the positive effects of (C) and (D) are dominant, and hence, the growth rate of TFP increases. Finally, in Phase III, the effect of decreasing returns to scale in human capital accumulation is dominant, and the growth rate of TFP decreases, and finally, approaches zero.

We mention the implications of outsourcing. We can regard  $p_s S / (p_m Q_m)$ , that is, the share of intermediate services in manufacturing output, as a measure of outsourcing. In our model, outsourcing increases through time, and this corresponds to the actual data (see Table 1). Then, in our model, as outsourcing increases (Figure 11),  $g_{TFP,m}$  as well as  $g_{TFP}$  decrease and converge to zero in the long run if human capital accumulation exhibits decreasing returns to scale (Figures 21 and 23). This implies that even if manufacturing firms conduct outsourcing to raise their productivity, their productivity growth decreases. Outsourcing expands the service sector (i.e., an increase in  $L_s/L$ ), which decreases the  $g_{A_m}$  that constitutes  $g_{TFP,m}$ . Moreover, continuous increases in  $L_s/L$  make  $g_h$  converge to zero in the long run. Therefore, as outsourcing increases, manufacturing TFP growth decreases and the TFP growth for the whole economy decreases.

## 6 Conclusion

This study extended Baumol's unbalanced growth model to investigate the relationship between the tendency toward services and economic growth. In our model, the productivity growth of both manufacturing and services were determined endogenously, while services were used for intermediate inputs in manufacturing as well as for final consumption.

We showed that the results are affected by the specifications of human capital accumulation and productivity specific to the service sector. If the human capital accumulation function has constant returns to scale with respect to per capita consumption of services, the result is similar to that of Sasaki (2012), who does not consider intermediate service inputs. In this case, we obtain a U-shaped relationship between the employment share of services and economic growth. In contrast, if the human capital accumulation function has decreasing returns to scale with respect to per capita consumption of services, we obtain an S-shaped, that is, a combination of a U-shaped and an inverted U-shaped relationship between the employment share of services and economic growth.

Our model considers both the case of services playing the role of intermediate inputs and the case of consumption of educational and health services leading to human capital accumulation. Hence, the model is more realistic than the previous literature. The main result is that the relationship between the tendency toward services and economic growth is not monotonous. This result has implications for empirical analysis regarding the relationship

between the tendency toward services and economic growth.

Hartwig (2012) empirically tests the three hypotheses about the relationship between the tendency toward services and economic growth: Baumol's (1967) hypothesis—a shift in employment share toward the services sector decreases the rate of economic growth; Kaldor's (1957) hypothesis—the growth rate of per capita real GDP is almost constant despite the presence of structural changes; and Pugno's (2006) hypothesis—a shift in the employment share toward services increases the economic growth rate. Hartwig (2012) concludes that no evidence is found to support Pugno's hypothesis and it is not evident whether structural change is compatible with balanced growth or whether it leads to long-term stagnation.

Our model is able to explain why such ambiguous empirical results are obtained. The relationship between the employment shift toward services and economic growth is not monotonous. Consequently, if we perform a cross-country analysis for countries whose stages of development are diversified, we are likely to obtain ambiguous results. Moreover, if we perform a time-series analysis for a specific country, we are likely to obtain different results depending on the period of data used.

Finally, our results imply that irrespective of whether services are used for intermediate inputs, to achieve sustainable economic growth when the employment share of services continues to increase, it is necessary for the productivity of services to increase sustainably.

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## Tables and Figures

Table 1: Percentage of intermediate services in manufacturing output (Source: Schettkat and Yocarini, 2003)

	France	Germany	UK	US
1968			9.16	
1972	10.82			11.91
1977	11.11			12.65
1978		12.98		
1979			10.22	
1990	16.77	16.73	13.14	13.19

Table 2: List of parameters when human capital accumulation exhibits constant returns to scale ( $\lambda = 1$ )

	$\alpha$	$\beta$	$\gamma$	$\delta$	$\phi$
Case 1	0.5	0.5	1	0.02	0.015
Case 2	0.5	0.9	1	0.03	0.02
Case 3	0.5	0.5	1	0.02	0.04

Table 3: List of parameters when human capital accumulation exhibits decreasing returns to scale ( $0 < \lambda < 1$ )

	$\alpha$	$\beta$	$\gamma$	$\delta$	$\phi$	$\psi$	$\lambda$	$\mu$	$\theta$
Case 4	0.5	0.95	1	0.005	0.02	2	0.9	0.01	0.9

Table 4: Classifications of the time series of TFP growth

	$\lambda = 1$		$0 < \lambda < 1$	
	$\beta = 1$	$0 < \beta < 1$	$\beta = 1$	$0 < \beta < 1$
$A_s$ : constant	$\searrow \nearrow \rightarrow$	$\searrow \nearrow \rightarrow$	$\searrow \rightarrow$	$\searrow \rightarrow$
$A_s$ : less than exponential	explosive	explosive	$\searrow \nearrow \searrow \rightarrow$	$\searrow \nearrow \searrow \rightarrow$
$A_s$ : exponential	explosive	explosive	$\searrow \nearrow \rightarrow$	$\searrow \nearrow \rightarrow$

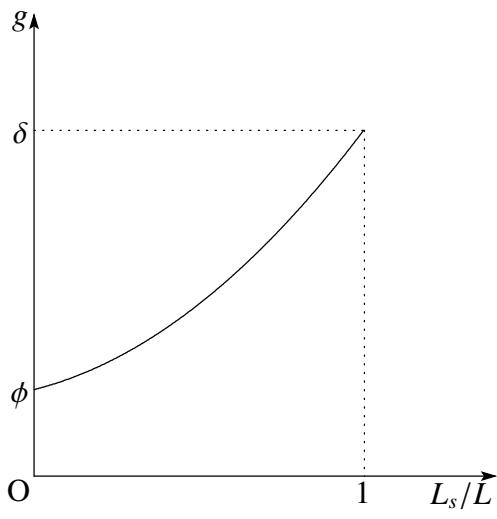


Figure 1: The relationship between  $L_s/L$  and  $g_{\text{TFP}}$  ( $\phi < \delta$  and  $2\phi < \delta$ )

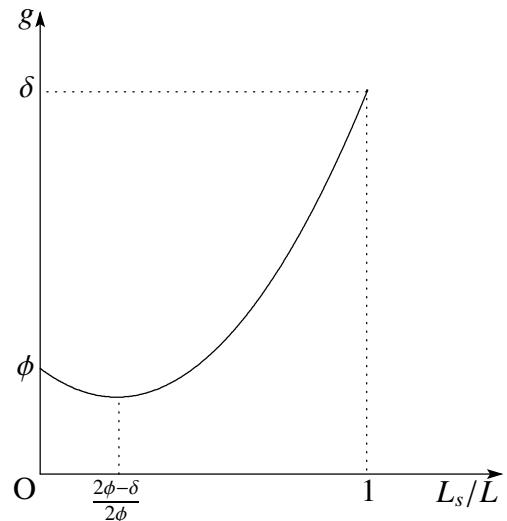


Figure 2: The relationship between  $L_s/L$  and  $g_{\text{TFP}}$  ( $\phi < \delta$  and  $2\phi > \delta$ )

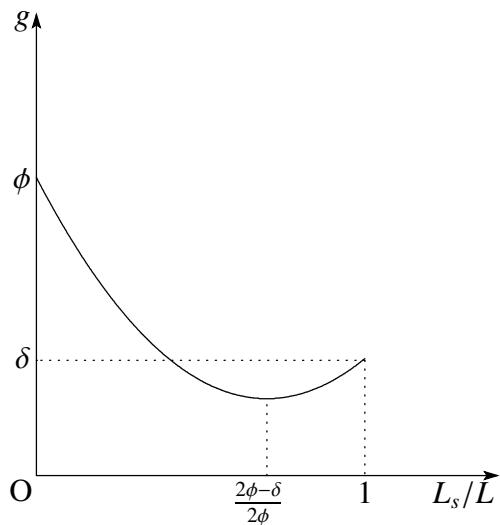


Figure 3: The relationship between  $L_s/L$  and  $g_{\text{TFP}}$  ( $\phi > \delta$ )

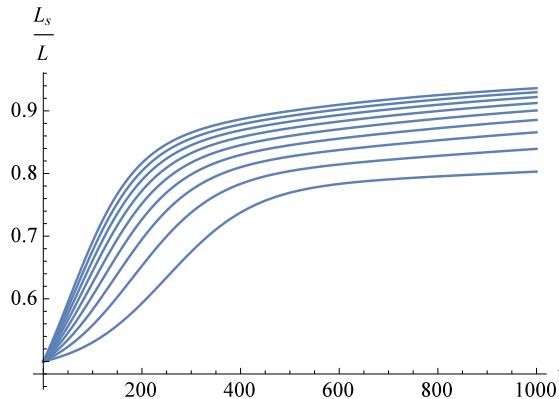


Figure 4: Employment share of services in Case 1

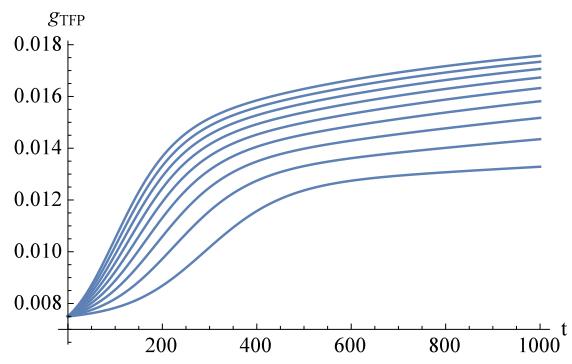


Figure 5: TFP growth rate of the whole economy in Case 1

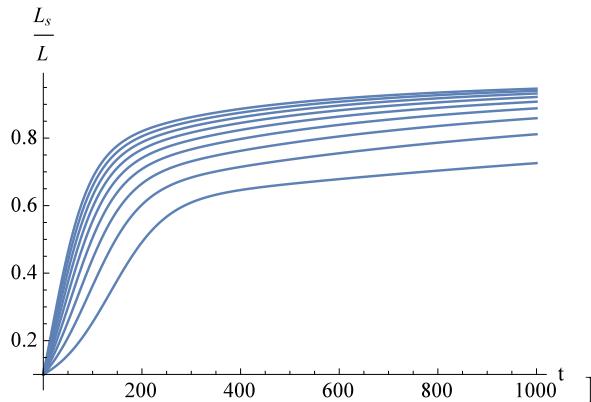


Figure 6: Employment share of services in Case 2

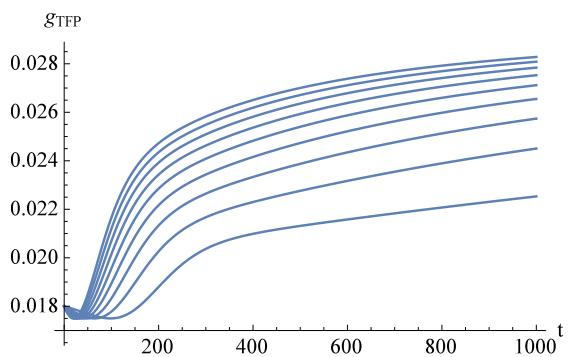


Figure 7: TFP growth rate of the whole economy in Case 2

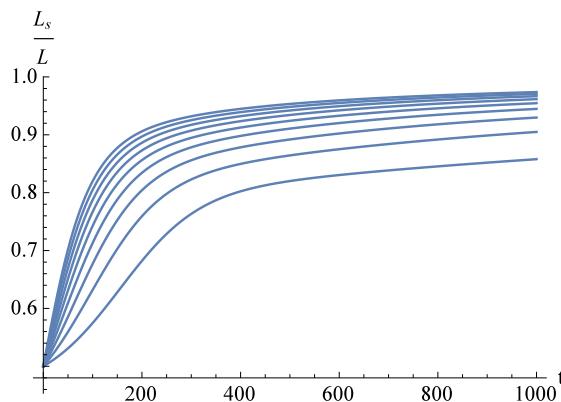


Figure 8: Employment share of services in Case 3

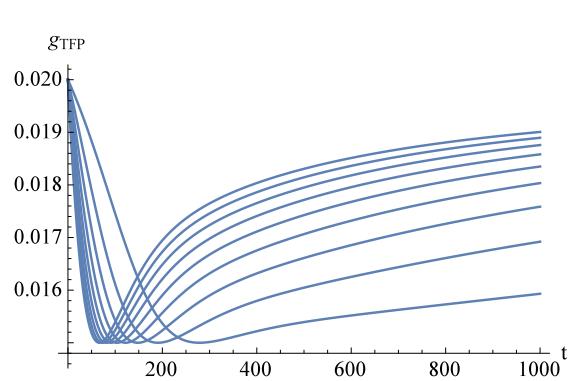


Figure 9: TFP growth rate of the whole economy in Case 3

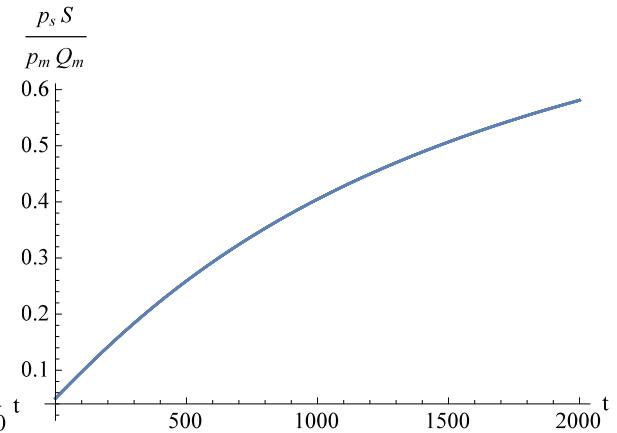
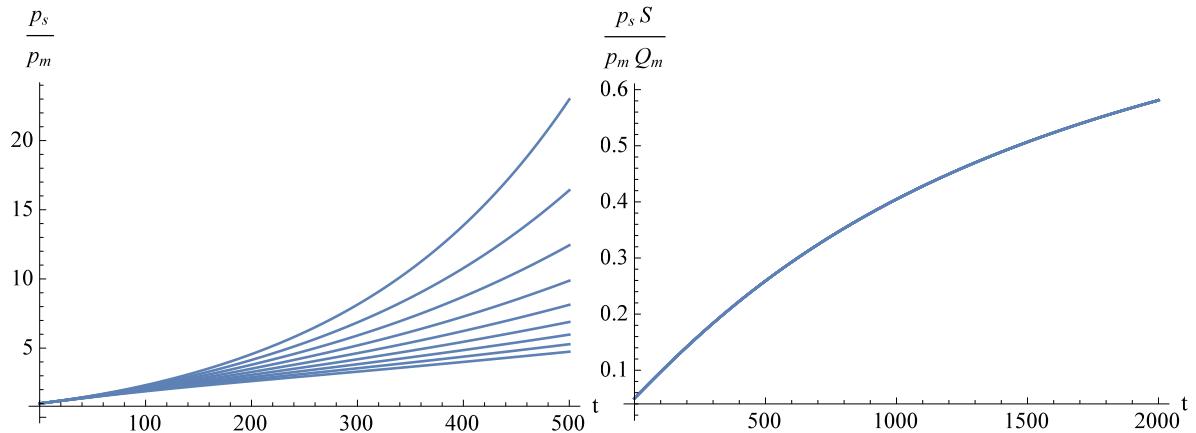


Figure 10: Relative price of services in Case 4

Figure 11: Share of intermediate services in manufacturing production in Case 4

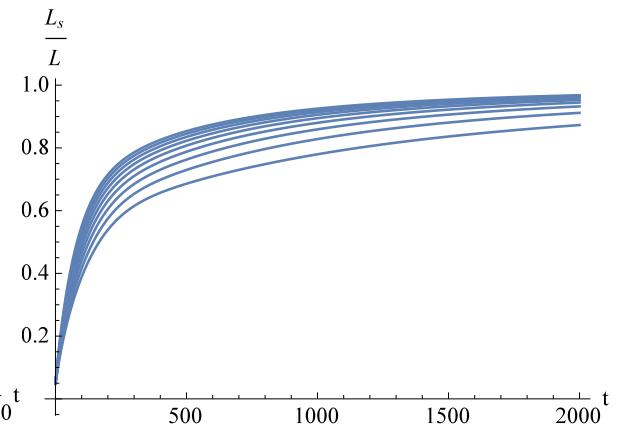
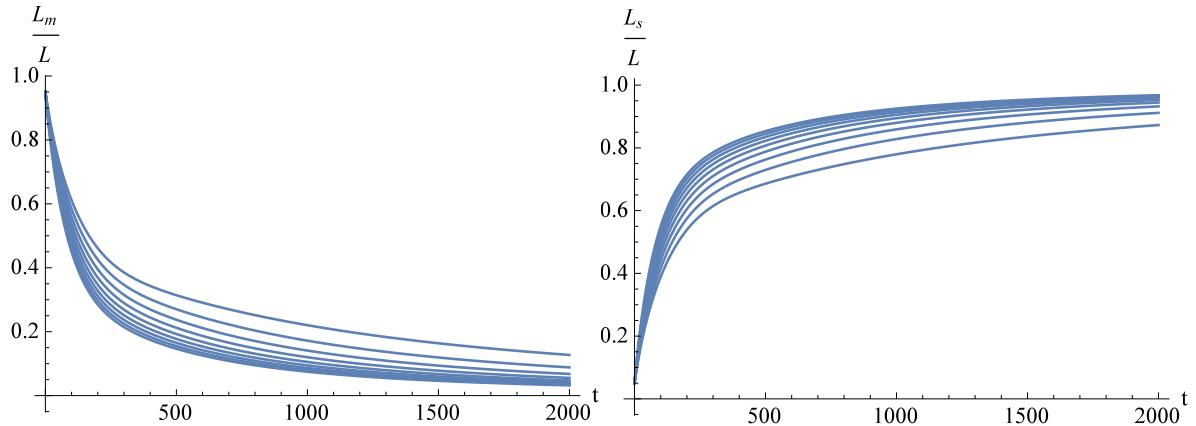
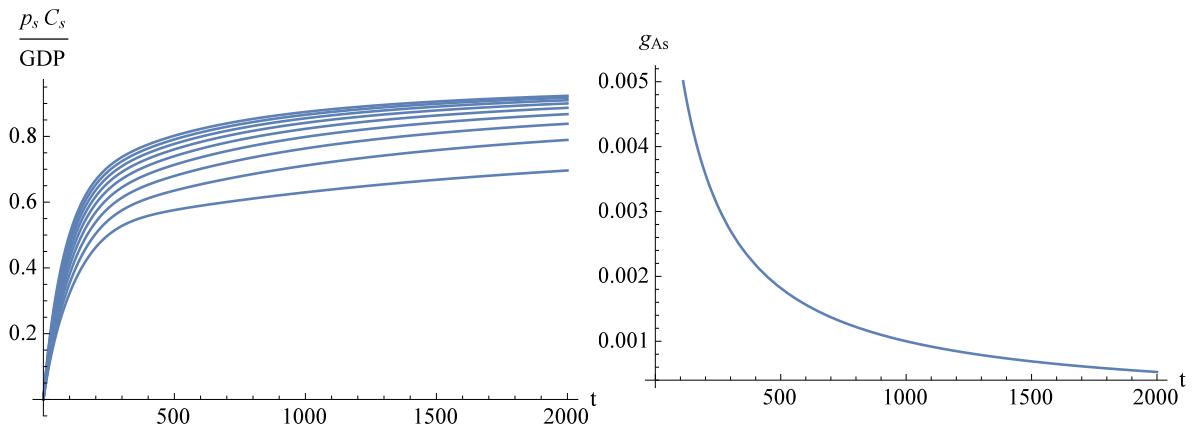
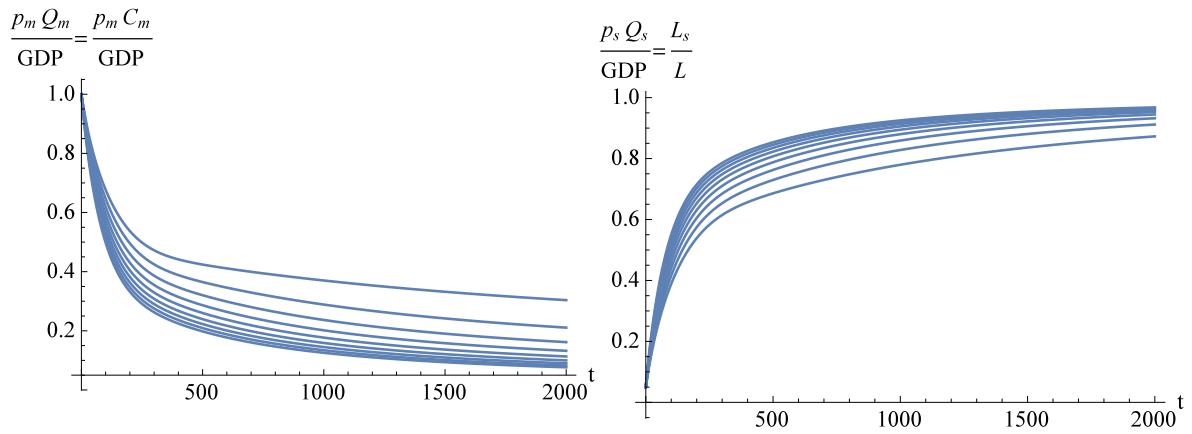


Figure 12: Employment share of manufacturing in Case 4

Figure 13: Employment share of services in Case 4



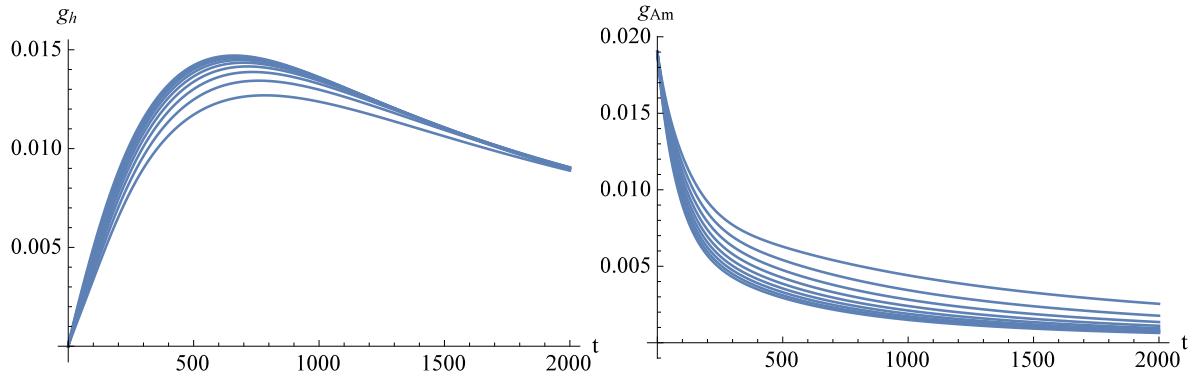


Figure 18: Human capital growth rate in Case 4

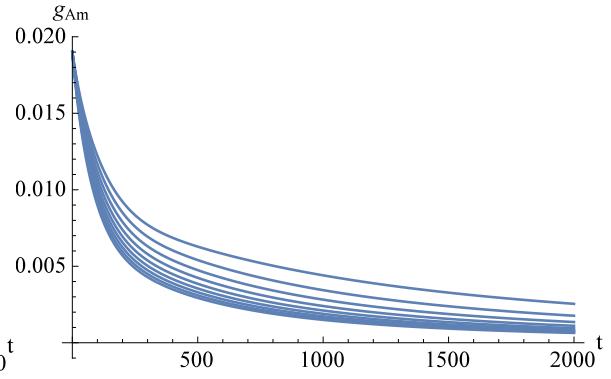


Figure 19: Productivity growth rate specific to manufacturing in Case 4

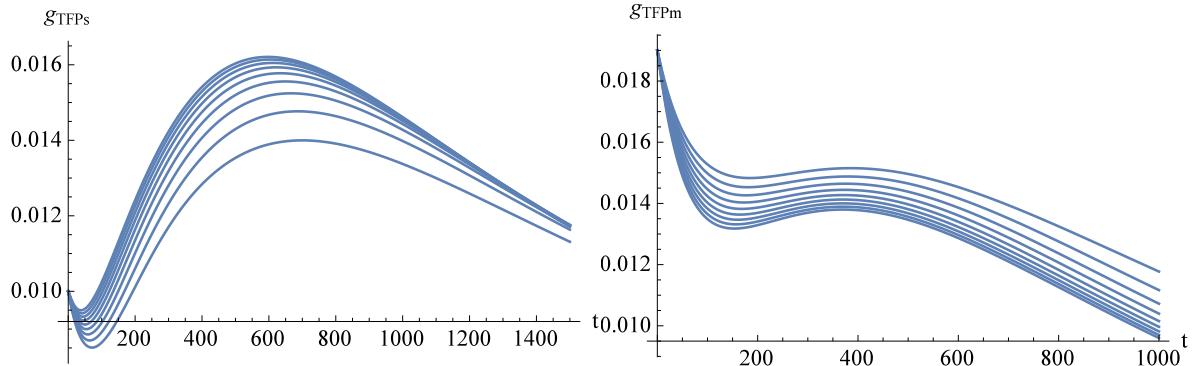


Figure 20: TFP growth rate in services in Case 4

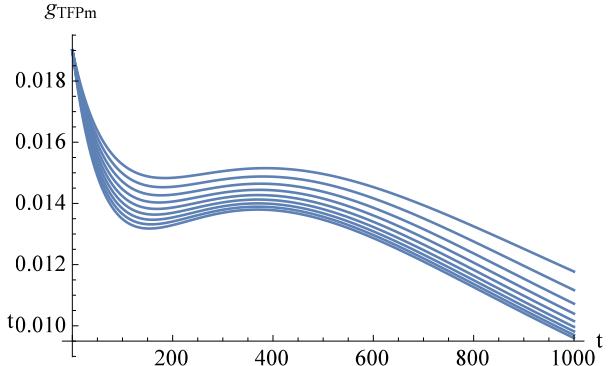


Figure 21: TFP growth rate in manufacturing in Case 4

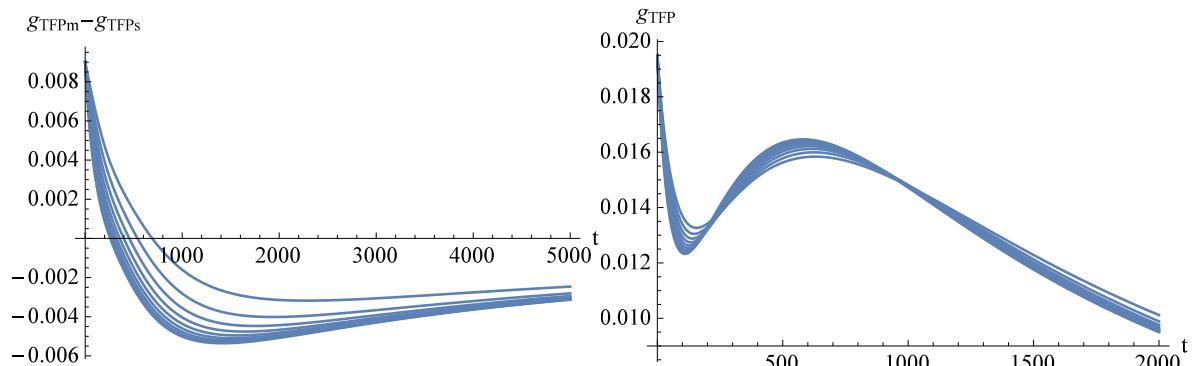


Figure 22: TFP growth rates differential between manufacturing and services in Case 4

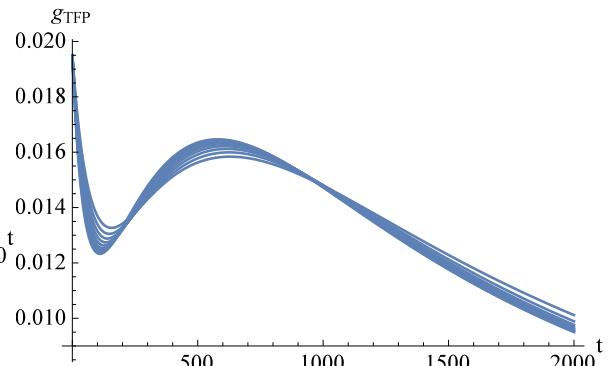


Figure 23: TFP growth rate of the whole economy in Case 4

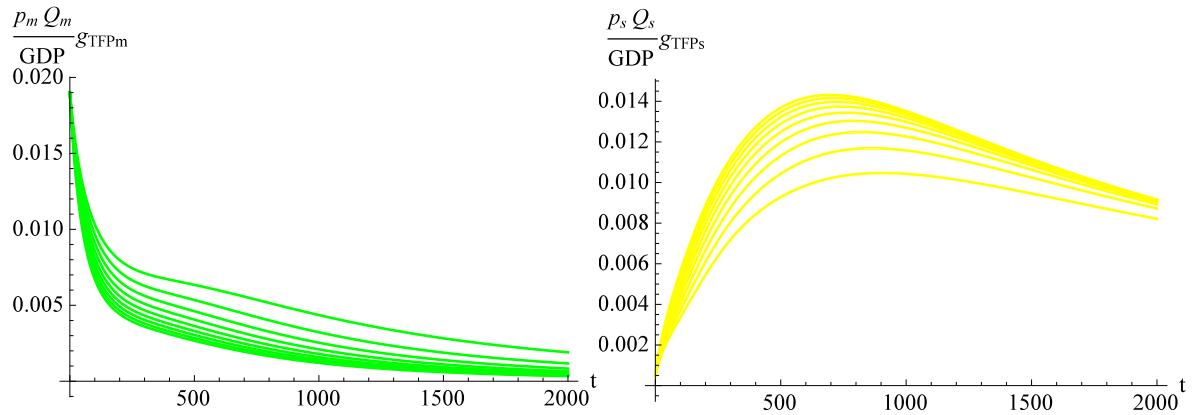


Figure 24: Contribution of TFP growth in manufacturing to TFP growth of the whole economy in Case 4  
 Figure 25: Contribution of TFP growth in services to TFP growth of the whole economy in Case 4

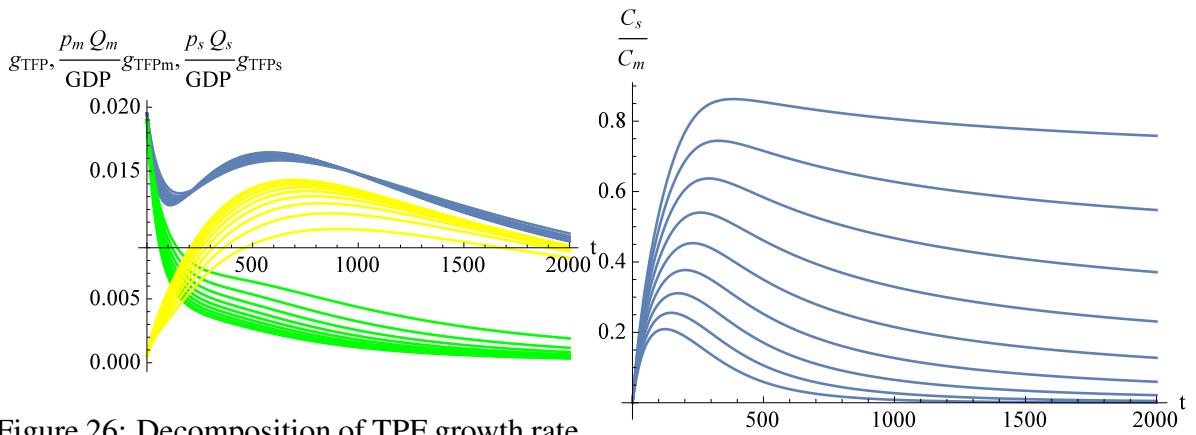


Figure 26: Decomposition of TPF growth rate of the whole economy into contributions of manufacturing and services in Case 4

Figure 27: Consumption ratio of services to manufacturing in Case 4

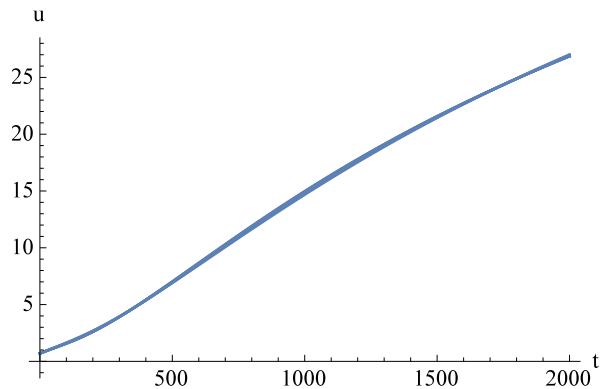


Figure 28: Instantaneous utility in Case 4

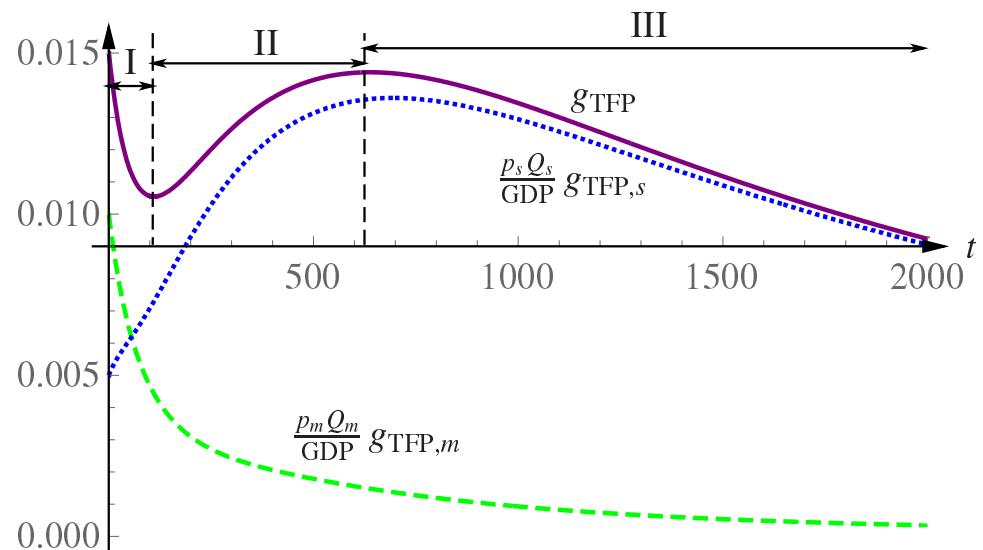


Figure 29: Decomposition of TFP growth in Case 4 when  $\sigma = 0.5$  and  $\beta = 0.5$