



Fate of spin polarization in a relativistic fluid: An entropy-current analysis



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ABSTRACT

We derive relativistic hydrodynamic equations with a dynamical spin degree of freedom on the basis of an entropy-current analysis. The first and second laws of local thermodynamics constrain possible structures of the constitutive relations including a spin current and the antisymmetric part of the (canonical) energy-momentum tensor. Solving the obtained hydrodynamic equations within the linear-mode analysis, we find spin-diffusion modes, indicating that spin density is damped out after a characteristic time scale controlled by transport coefficients introduced in the antisymmetric part of the energy-momentum tensor in the entropy-current analysis. This is a consequence of mutual convertibility between spin and orbital angular momentum.

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1. Introduction

Recent experimental developments opened a new avenue to study spin-dependent observables. For example, in relativistic heavy-ion collisions, the spin polarization of Λ hyperons has been measured [1]. The result suggests that a fraction of quark spin (and perhaps gluon spin) in the hyperons is aligned along a particular direction, and in turn implies that a quark-gluon plasma (QGP) carries significant magnitudes of angular momentum and/or vorticity which cause the spin alignments. On the other hand, dynamics of spin is also a hot subject in condensed matter physics, especially in the field of spintronics, where generation of a spin current—a flow of spin angular momentum—is one of the key issues (see Ref. [2] for a review).

To describe macroscopic dynamics of spin, it is desirable to generalize hydrodynamics to a spinful fluid. In the field of spintronics, a nonrelativistic framework of spin hydrodynamics has been utilized for describing the spin-current generation in the presence of a coupling between spin and vorticity in elastic materials [3,4] as well as in liquid metals [5,6]. A similar framework was also used to describe the so-called micropolar fluids [7,8]. More recently, “ideal” relativistic hydrodynamics with spin was proposed in the context of the QGP [9] (See also Refs. [10–12] for recent attempts to apply a Lagrangian description of the spinful relativistic fluid), in which spin is regarded as a conserved quantity in the leading-order of a gradient expansion. However, the aforementioned nonrelativistic formulation has shown that spin is not conserved even at the leading order.

In this Letter, we elaborate a relativistic framework to resolve the disharmony between the nonrelativistic and relativistic formulations and to describe the spin-dependent spacetime evolution of relativistic fluids like the QGP. We employ a phenomenological entropy-current

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analysis, which was originally used to formulate relativistic viscous hydrodynamics [13], and has been recently applied to, e.g., derivation of dissipative corrections in magnetohydrodynamics (MHD) [14–17], and of the chiral magnetic/vortical effect in anomalous hydrodynamics [18] and chiral MHD [17]. We shall show that spin density is an inherently dissipative quantity due to mutual conversion between spin and orbital angular momentum even in the leading-order spinful relativistic fluid. This is a consequence of the fact that spin itself is not a conserved quantity. It is, therefore, crucial to include dissipative terms appearing in the first-order derivative corrections to the energy-momentum tensor as we will discuss. In the rest of this Letter, we first discuss the entropy-current analysis. Then, we reinforce our observation of the spin dissipation by solving the obtained system of equations with respect to linear perturbations applied to spin density and other hydrodynamic variables.

2. Phenomenological derivation of spin hydrodynamics

Phenomenological derivation of hydrodynamics is based on the conservation laws [13]. In the present case, one should consider the conservation laws of the total angular momentum $J^{\mu\alpha\beta}$ as well as the energy-momentum $\Theta^{\mu\nu}$, which are, respectively, expressed as

$$\partial_\mu \Theta^{\mu\nu} = 0, \quad (1a)$$

$$\partial_\mu J^{\mu\alpha\beta} = 0. \quad (1b)$$

The total angular momentum has two contributions from orbital angular momentum and intrinsic spin. Microscopically, they arise as two distinct components in the Noether current for the Lorentz symmetry as $J^{\mu\alpha\beta} = (x^\alpha \Theta^{\mu\beta} - x^\beta \Theta^{\mu\alpha}) + \Sigma^{\mu\alpha\beta}$, where $\Sigma^{\mu\alpha\beta} = -\Sigma^{\mu\beta\alpha}$. $\Sigma^{\mu\alpha\beta}$ arises from the invariance with respect to the representation of the Lorentz group acting on a field under consideration, and is naturally identified with an internal spin degree of freedom. On the other hand, the orbital angular momentum in the parentheses comes from the coordinate transformation of the argument of the field. $\Theta^{\mu\nu}$ in this expression is called the *canonical* energy-momentum tensor, which has both symmetric and antisymmetric components: $\Theta^{\mu\nu} \equiv \Theta_{(s)}^{\mu\nu} + \Theta_{(a)}^{\mu\nu}$. Note that $\partial_\nu \Theta^{\mu\nu} \neq 0$ in general.

The dynamical variables near local thermal equilibrium are assumed to satisfy the first law of thermodynamics generalized with finite spin density $S^{\mu\nu}$ ¹:

$$\begin{aligned} Ts &= e + p - \omega_{\mu\nu} S^{\mu\nu}, & Tds &= de - \omega_{\mu\nu} dS^{\mu\nu}, \\ TDs &= De - \omega_{\mu\nu} DS^{\mu\nu}, \end{aligned} \quad (2)$$

where T , s , e , and p denote the local temperature, entropy density, energy density, and pressure, respectively. In this Letter, we consider a neutral fluid in the absence of conserved charges. Here, we introduced a “spin potential” $\omega_{\mu\nu}$ which is conjugate to the spin density $S^{\mu\nu}$. Note that $\omega_{\mu\nu}$ has no a priori relation to the fluid velocity u^μ at this stage since we introduced the spin density as a new independent degree of freedom. We also defined the derivative operator $D \equiv u^\mu \partial_\mu$, which corresponds to the Lagrange derivative in fluid mechanics.

One may organize the constitutive relations on the basis of a derivative expansion:

$$\Theta^{\mu\nu} = eu^\mu u^\nu + p\Delta^{\mu\nu} + \Theta_{(1)}^{\mu\nu}, \quad (3a)$$

$$\Sigma^{\mu\alpha\beta} = u^\mu S^{\alpha\beta} + \Sigma_{(1)}^{\mu\alpha\beta}. \quad (3b)$$

Here we employ the mostly plus signature of the Minkowski metric $\eta^{\mu\nu} \equiv \text{diag}(-1, +1, +1, +1)$. Thus, the normalization of the fluid velocity and the transverse projection operator read $u^\mu u_\mu = -1$ and $\Delta^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu$, respectively. Here, the spin density $S^{\mu\nu}$ is assumed to satisfy the antisymmetric property $S^{\mu\nu} = -S^{\nu\mu}$ so that it has the same number of components as the total angular momentum has. Accordingly, we have $\omega_{\mu\nu} = -\omega_{\nu\mu}$. We introduced the first-order derivative correction to the energy-momentum tensor $\Theta_{(1)}^{\mu\nu}$ and the spin current $\Sigma_{(1)}^{\mu\alpha\beta} (= -\Sigma_{(1)}^{\mu\beta\alpha})$. As mentioned earlier, the former contains both symmetric and antisymmetric components, whose roles will be elaborated below. More discussions about the tensor decomposition in Eq. (3) can be found in Appendix A.

Based on the above relations, we now analyze the entropy conservation/production. In the lowest order in the gradient, we identify the entropy current as

$$s_{(0)}^\mu = su^\mu. \quad (4)$$

Therefore, by using the thermodynamic relations (2), we find

$$\partial_\mu (su^\mu) = Ds + s\theta = \beta(De - \omega_{\alpha\beta} DS^{\alpha\beta}) + s\theta, \quad (5)$$

where $\theta = \partial_\mu u^\mu$. To eliminate the Lagrange derivatives, we use equations of motion for e and $S^{\alpha\beta}$ which are obtained by substituting the constitutive equations (3) into the conservation laws (1) and contracting Eq. (1a) with u_ν :

$$De = -(e + p)\theta + u_\nu \partial_\mu \Theta_{(1)}^{\mu\nu}, \quad (6a)$$

$$DS^{\alpha\beta} = -S^{\alpha\beta}\theta - 2\Theta_{(1a)}^{\alpha\beta} - \partial_\mu \Sigma_{(1)}^{\mu\alpha\beta}. \quad (6b)$$

¹ Note that the second and third equations have slightly different physical meanings. The differentiation d in the second equation describes a transition between two different local equilibrium states, and determines physical properties of the local thermodynamic functions (e.g., the entropy density). On the other hand, the third equation defines the spacetime dynamics of the entropy density.

We use the equations of motion at the lowest-order, that is, Eq. (6a) with $\Theta_{(1)}^{\mu\nu} = 0$ and Eq. (6b) with $\Sigma_{(1)}^{\mu\alpha\beta} = 0$. One should, however, maintain the first-order term $\Theta_{(1a)}^{\alpha\beta}$ in the antisymmetric part of the energy-momentum tensor since this term is one of the lowest-order terms in Eq. (6b). Inserting these equations into Eq. (5), the divergence of the entropy current can be evaluated as

$$\partial_\mu s_{(0)}^\mu = \beta\theta\{sT - (e + p - \omega_{\alpha\beta}S^{\alpha\beta})\} + 2\beta\omega_{\alpha\beta}\Theta_{(1a)}^{\alpha\beta} = 2\beta\omega_{\alpha\beta}\Theta_{(1a)}^{\alpha\beta}, \quad (7)$$

where we used Eq. (2) to reach the second line. This expression indicates that the lowest-order hydrodynamic equations of motion (6) do not conserve the lowest-order entropy current (4). This is in contrast to the case of a fluid without spin, where the lowest-order equations of motion correspond to ideal hydrodynamics and describe reversible fluid dynamics. The entropy production implies that spin density is inherently a non-conserved quantity, which dissipates in a finite time scale. Indeed, the conservation law of the total angular momentum (1) can be cast into a “non-conservation” law of the spin current

$$\partial_\mu \Sigma^{\mu\alpha\beta} = -2\Theta_{(a)}^{\alpha\beta}. \quad (8)$$

The right-hand side comes from the orbital angular momentum, which acts as a source/absorption term for the spin current. This is of course a natural relation indicating that spin and orbital angular momentum are converted to each other. The lowest-order entropy current is conserved only when $\Theta_{(a)}^{\alpha\beta} = 0$, i.e., when spin and orbital angular momentum are separately conserved.² Since the spin current is inherently dissipative, there is no counterpart of ideal hydrodynamics that could be called ideal spin hydrodynamics. In other words, spin density is not a strict hydrodynamic variable that survives in the long time scale like a conserved charge associated with a symmetry. Nevertheless, when the relaxation time of spin density is longer than the mean-free-time of microscopic scattering processes, hydrodynamic description of the transient spin dynamics is expected to work (see also discussions in Ref. [19]). Such a situation should be characterized by small values of transport coefficients for spin dissipation, which we will introduce shortly in our formalism. It is, therefore, important to investigate the dissipative corrections.

For this purpose, we keep all the first-order terms in the equations of motion (6). The divergence of the entropy current (5) now reads

$$\partial_\mu (su^\mu) = \beta u_\nu \partial_\mu \Theta_{(1s)}^{\mu\nu} + (\beta u_\nu \partial_\mu \Theta_{(1a)}^{\mu\nu} + 2\beta\omega_{\mu\nu}\Theta_{(1a)}^{\mu\nu}) + \beta\omega_{\alpha\beta}\partial_\mu \Sigma_{(1)}^{\mu\alpha\beta}. \quad (9)$$

To proceed, we need to elaborate the counting scheme of the derivative expansion. In Eq. (9), we regard the temperature (and the energy density) as a zeroth-order quantity. This suggests a natural assignment $\omega^{\mu\nu} = \mathcal{O}(\partial^1)$ so that the two terms between the brackets fall in the same order in the gradient. Here, we employ this counting scheme, and will accordingly drop the last term which is an order higher as compared to the others. Then, one can further arrange this expression as

$$\partial_\mu (su^\mu + s_{(1)}^\mu) = -\Theta_{(1s)}^{\mu\nu}\partial_\mu\beta_\nu - \Theta_{(1a)}^{\mu\nu}(\partial_\mu\beta_\nu - 2\beta\omega_{\mu\nu}), \quad (10)$$

where we defined $\beta^\mu \equiv \beta u^\mu$ and identified the first-order correction to the entropy current as $s_{(1)}^\mu = -\Theta_{(1s)}^{\mu\nu}\beta_\nu$. The semipositivity of this entropy production, as required by the second law of thermodynamics, can be ensured for any hydrodynamic configuration when each term on the right-hand side has a semipositive bilinear form [13]. In turn, this constrains possible tensor structures of the first-order derivative corrections up to scalar coefficients introduced as transport coefficients.

In passing, we note that one may also employ another counting scheme with $\omega^{\mu\nu} = \mathcal{O}(\partial^0)$. In this counting, the spin potential $\omega^{\mu\nu}$ modifies the constitutive relations even at the zeroth order and provides a preferred orientation specified by $\omega^\mu = (1/2)\epsilon^{\mu\nu\rho\sigma}u_\nu\omega_{\rho\sigma}$ like a “strong” magnetic field $B^\mu = \mathcal{O}(\partial^0)$ in magnetohydrodynamics [14–17]. This counting scheme suggests an interesting extension of the present work.

One may write the most general tensor structure of the first-order corrections as

$$\Theta_{(1s)}^{\mu\nu} = 2h^{(\mu}u^{\nu)} + \tau^{\mu\nu}, \quad (11a)$$

$$\Theta_{(1a)}^{\mu\nu} = 2q^{[\mu}u^{\nu]} + \phi^{\mu\nu}, \quad (11b)$$

where $\{h^\mu, \tau^{\mu\nu}, q^\mu, \phi^{\mu\nu}\} = \mathcal{O}(\partial^1)$, $\tau^{\mu\nu} = \tau^{\nu\mu}$, $\phi^{\mu\nu} = -\phi^{\nu\mu}$, and $h^\mu u_\mu = q^\mu u_\mu = \tau^{\mu\nu}u_\nu = \phi^{\mu\nu}u_\nu = 0$. We use the shorthand notations $X^{(\mu\nu)} \equiv (X^{\mu\nu} + X^{\nu\mu})/2$, $X^{[\mu\nu]} \equiv (X^{\mu\nu} - X^{\nu\mu})/2$, $X^{(\mu\nu)} \equiv (X^{\mu\nu} + X^{\nu\mu})/2 - X^\sigma_\sigma \Delta^{\mu\nu}/3$. Then, the second law of thermodynamics with spin is guaranteed if we identify the first-order corrections as

$$h^\mu = -\kappa(Du^\mu + \beta\partial_\perp^\mu T), \quad (12a)$$

$$\tau^{\mu\nu} = -2\eta\partial_\perp^{(\mu}u^{\nu)} - \zeta\theta\Delta^{\mu\nu}, \quad (12b)$$

$$q^\mu = -\lambda(-Du^\mu + \beta\partial_\perp^\mu T - 4\omega^{\mu\nu}u_\nu), \quad (12c)$$

$$\phi^{\mu\nu} = -2\gamma(\partial_\perp^{[\mu}u^{\nu]} - 2\Delta_\rho^\mu\Delta_\lambda^\nu\omega^{\rho\lambda}), \quad (12d)$$

with $\partial_\perp^\mu \equiv \Delta^{\mu\nu}\partial_\nu$ and $\kappa, \eta, \zeta, \lambda, \gamma \geq 0$. κ, η , and ζ are the well-known heat conductivity, shear and bulk viscous coefficients, respectively. λ and γ are new transport coefficients in relativistic spin hydrodynamics. Notice that, in the leading order of Eq. (8), the antisymmetric part $\Theta_{(1a)}^{\alpha\beta}$ provides a four-dimensional torque acting on the evolution of the Lorentz generator $\Sigma^{0\alpha\beta}$. Therefore, the spacial projection $\phi^{\mu\nu} = \Delta_\alpha^\mu\Delta_\beta^\nu\Theta_{(1a)}^{\alpha\beta}$ gives rise to an antisymmetric stress which diminishes the intrinsic angular momentum of the fluid cell. On the other

² It should be stressed that the tensors in Eq. (8) are in the canonical forms, only in which the tensor $\Sigma^{\mu\alpha\beta}$ corresponds to the internal spin degree of freedom. Therefore, one cannot conclude separate conservation of the spin component, e.g., with a Belinfante-improved symmetric energy-momentum tensor.

hand, the temporal projection $q^\mu = u_\alpha \Delta_\beta^\mu \Theta_{(1a)}^{\alpha\beta}$ boosts the fluid cell. For these reasons, we call γ the *rotational viscosity* [20] and λ the *boost heat conductivity*. The latter is a relativistic effect and does not have a nonrelativistic counterpart. In global equilibrium, the entropy production should cease, so that $\delta^{[\mu} \beta^{\nu]} = 2\beta\omega^{\mu\nu}$ from Eq. (10). This implies that the spin potential $\omega^{\mu\nu}$ is no longer an independent variable and is completely determined by the thermal vorticity [21,19].

Using the leading-order equation of motion for u^μ , i.e., $(e+p)Du^\mu = -\partial_\perp^\mu p + \mathcal{O}(\partial^2)$, we can eliminate Du^μ in h^μ and q^μ as

$$h^\mu = -\kappa \left[\frac{-\partial_\perp^\mu p}{e+p} + \beta \partial_\perp^\mu T + \mathcal{O}(\partial^2) \right] = \mathcal{O}(\partial^2), \quad (13a)$$

$$q^\mu = -\lambda \left[\frac{2\partial_\perp^\mu p}{e+p} - 4\omega^{\mu\nu} u_\nu \right] + \mathcal{O}(\partial^2). \quad (13b)$$

The heat current h^μ is beyond the first order and can be neglected within the present working accuracy as a consequence of the spacetime translational symmetries.³ In addition, we note that q^μ cannot be eliminated by a frame choice at $\mathcal{O}(\partial^1)$. In fact, the variation of q^μ under a redefinition of the fluid velocity $u^\mu \rightarrow u^\mu + \delta u^\mu$ with $\delta u^\mu = \mathcal{O}(\partial^1)$ reads $\delta q^\mu = \delta(\Delta_{[\alpha}^\mu u_{\beta]})\Theta_{(a)}^{\alpha\beta}$. Since $\Theta^{\alpha\beta}$ is assumed to be invariant [23, 24], this variation should be at most a second-order quantity $\delta q^\mu = \mathcal{O}(\partial^2)$, and is negligible as compared to q^μ in the original frame.

3. Linear mode analysis

Relativistic hydrodynamic equations with spin are obtained by plugging the constitutive relations (12) and (13) into the conservation laws (1). Below, we consider linear perturbations on top of global thermal equilibrium, and solve the hydrodynamic equations to discuss the dynamic evolution of the (non-)hydrodynamic modes in a spinful relativistic fluid. Namely, we consider perturbations given by

$$\begin{aligned} e(x) &= e_0 + \delta e(x), & p(x) &= p_0 + \delta p(x), \\ v^i(x) &= 0 + \delta v^i(x), & S^{\mu\nu}(x) &= 0 + \delta S^{\mu\nu}(x), \\ \omega^{\mu\nu}(x) &= 0 + \delta \omega^{\mu\nu}(x), \end{aligned} \quad (14)$$

where v^i is the three-velocity of a fluid and $u^\mu = (1, \delta v^i) + \mathcal{O}((\delta v)^2)$. Here, we concentrate on one of the global equilibrium configurations where the background fluid velocity and the spin density are vanishing. In general, finite thermal vorticity can survive in global equilibrium [21] and so is the spin potential $\omega^{\mu\nu}$ in another counting scheme mentioned earlier. This situation would serve as another starting point of the linear-mode analysis.

Linearizing the hydrodynamic equations with respect to the perturbations (14), we obtain

$$0 = \partial_0 \delta e + \partial_i \delta \pi^i - 2(c_s^2 \lambda' \partial_i \delta^i \delta e + D_b \partial^i \delta S^{0i}), \quad (15a)$$

$$0 = (\partial_0 \delta \pi^i + c_s^2 \partial^i \delta e) - \gamma_\parallel \partial^i \partial_j \delta \pi^j - (\gamma_\perp + \gamma') (\delta_j^i \nabla^2 - \partial^i \partial_j) \delta \pi^j + D_s \partial_j \delta S^{ji}, \quad (15b)$$

$$0 = \partial_0 \delta S^{ij} + 2\{D_s \delta S^{ij} - \gamma' (\partial^i \delta \pi^j - \partial^j \delta \pi^i)\}, \quad (15c)$$

$$0 = \partial_0 \delta S^{0i} + 2(c_s^2 \lambda' \partial^i \delta e + D_b \delta S^{0i}), \quad (15d)$$

where $\delta \pi^i \equiv \delta \Theta^{0i} = (e_0 + p_0) \delta v^i + \lambda c_s^2 \partial^i \delta e + D_b \delta S^{0i}$, and summation over repeated (spatial) indices are assumed. We also introduced the constants as

$$\begin{aligned} c_s^2 &\equiv \frac{\partial p}{\partial e}, & \chi_s &\equiv \frac{\partial S^{ij}}{\partial \omega^{ij}}, & D_s &\equiv \frac{4\gamma}{\chi_s}, & \gamma' &\equiv \frac{\gamma}{e_0 + p_0}, \\ \chi_b &\equiv \frac{\partial S^{i0}}{\partial \omega^{i0}}, & D_b &\equiv \frac{4\lambda}{\chi_b}, & \lambda' &\equiv \frac{2\lambda}{e_0 + p_0}, \\ \gamma_\parallel &\equiv \frac{1}{e_0 + p_0} \left(\zeta + \frac{4}{3} \eta \right), & \gamma_\perp &\equiv \frac{\eta}{e_0 + p_0}. \end{aligned} \quad (16)$$

The eigenmodes of the linearized hydrodynamic equations (15a)–(15d) can be obtained straightforwardly. We put the detailed calculation in Appendix B. The dispersion relations of those modes read

$$\omega = -2iD_s, \quad (17a)$$

$$\omega = -2iD_b, \quad (17b)$$

$$\omega = \begin{cases} -2iD_s - i\gamma' k_z^2 + \mathcal{O}(k_z^4), \\ -i\gamma_\perp k_z^2 + \mathcal{O}(k_z^4), \end{cases} \quad (17c)$$

³ This statement is valid up to an ambiguity in the definition of u^μ known as the “frame choice” (see, e.g., Refs. [13,22–25]). However, in the current case for a neutral fluid, there are no other natural frame choices motivated by conserved currents.

$$\omega = \begin{cases} \pm c_s k_z - i \frac{\gamma_{\parallel}}{2} k_z^2 + \mathcal{O}(k_z^3), \\ -2i D_b - 2i c_s^2 \lambda' k_z^2 + \mathcal{O}(k_z^4). \end{cases} \quad (17d)$$

Note that one may take the momentum in the z -direction $\mathbf{k} = (0, 0, k_z)$ without loss of generality according to the rotational symmetry of the system. Here, we expanded the dispersion relations with respect to the wave number k_z up to $\mathcal{O}(k_z^2)$. There are two duplicates of the second and third solutions because of the residual rotational symmetry around \mathbf{k} . In total, there are ten modes, which are composed of six massive modes and four massless modes—two longitudinal and two transverse hydrodynamic modes. As in usual hydrodynamics, the massless modes are the shear mode and the sound mode, which are affected by the viscous corrections. The gaps in the former six modes arise as a consequence of the non-conservative nature of the six spin degrees of freedom $S^{\mu\nu}$. Therefore, we conclude that even if finite spin density is presented in a relativistic fluid, it will be damped out after characteristic time scales $\tau_s \equiv 1/D_s$ and $\tau_b \equiv 1/D_b$.⁴

When τ_s and τ_b take large enough values compared with the typical microscopic time scale in a problem, spin hydrodynamics may work with the spin density being a quasi-conserved quantity. This means that such slow dynamics is captured by our new transport coefficients, the rotational viscosity γ and boost heat conductivity λ [cf., Eq. (16)]. We again emphasize that there is no symmetry which guarantees separate conservation of the spin component out of the total angular momentum. The absence of a symmetry is the reason why there is no rigorous notion of “ideal spin hydrodynamics” no matter how large the spin lifetime is, and why possible presence of a slow spin variable depends on details of a system via the transport coefficients. Namely, the slowness of spin dissipation requires specific reasons why spin rotation in each microscopic collision process is suppressed. One possible reason would be suppression of spin interactions by the mass of constituent particles.

4. Summary and outlook

We have derived the relativistic hydrodynamic equations with a dynamical spin degree of freedom on the basis of the phenomenological entropy-current analysis. The resulting constitutive relations acquire the spin current and the antisymmetric part of the energy-momentum tensor as well as the usual symmetric part. We identified two new important transport coefficients in the antisymmetric part of the energy-momentum tensor that control the relaxation time of spin density.

We have also solved the derived spin hydrodynamic equations within the linear-mode analysis, and found four massless hydrodynamic modes—two longitudinal propagating modes and two transverse diffusive modes—and six non-hydrodynamic modes corresponding to the six non-conserved degrees of freedom in the spin density $S^{\mu\nu}$.

There are several interesting directions which we can pursue in future: (1) We can extend the present linear-mode analysis to the case with a finite thermal vorticity. Since such a background configuration breaks, e.g., the parity symmetry, there may appear mode mixing between (non-)hydrodynamic modes. Similarly, introducing an external or dynamical magnetic field may also lead to mode mixing or the appearance of new modes; (2) The dissipative spin hydrodynamic equations should be derived from underlying microscopic theories, allowing for a comparison with the result presented in this Letter. Established frameworks include the low-energy effective theory from the local Gibbs distribution [26–28], the kinetic theory based on the Wigner function formalism [29,30], and the multi-moment formalism within the Boltzmann transport theory [31] (see references therein).

Finally, we note that the spin hydrodynamic equations can be applied, for example, to describe the QGP created in relativistic heavy-ion collisions and to the spintronics of emergent relativistic quasiparticles in condensed matter physics. For these applications, it is important to establish Kubo formulas and quantify the new transport coefficients from underlying microscopic theories, and to extend the present framework to the second order to form a causal and numerically stable system of equations. We leave these directions as future works.

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Appendix A. General tensor decomposition

We here discuss the tensor decomposition in Eq. (3). Making a projection with respect to u^μ , we can always write

$$\Theta^{\mu\nu} = e u^\mu u^\nu + p \Delta^{\mu\nu} + \tilde{\Theta}^{\mu\nu}, \quad (A.1a)$$

$$\Sigma^{\mu\alpha\beta} = u^\mu S^{\alpha\beta} + \tilde{\Sigma}^{\mu\alpha\beta}, \quad (A.1b)$$

$$s^\mu = s u^\mu + \tilde{s}^\mu, \quad (A.1c)$$

in such a way that $\tilde{\Theta}^{\mu\nu} u_\mu u_\nu = \tilde{\Sigma}^{\mu\alpha\beta} u_\mu = \tilde{s}^\mu u_\mu = 0$. Then, the corresponding divergence of the entropy current reads

⁴ Note that τ_s and τ_b are different from each other due to the absence of the Lorentz symmetry.

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