The Kyoto Economic Review 73(2): 87–107 (December 2004)

On the Pricing of Corporate Value under Information Asymmetry

Takashi Shibata[1]

[1] *International Graduate School of Social Sciences, Yokohama National University. E-mail: tshibata @poplar.ocn.ne.jp*

This paper examines the corporate value of a decentralized firm in the presence of principal-agent conflicts due to information asymmetries. When owners delegate the management to managers, contracts must be designed to provide incentive for managers to truthfully reveal private information. Using a contingent claims approach, we demonstrate that an underlying option value of the firm can be decomposed into two components: a manager's option and an owner's option. The value of a decentralized firm is lower than that of an owner-managed firm. In particular, the implied manager's decisions in a decentralized firm differ significantly from those in an owner-managed firm.

Keywords: option pricing, principal-agent conflicts, information asymmetries
JEL Clssification Numbers: D82, G13, G32

1. Introduction

The purpose of this paper is to calculate the corporate value of a decentralized firm (i.e., the value of the firm under separation of ownership and control) by using the contingent claims approach to corporate valuation initiated by Merton (1974)[1)]. We incorporate principal-agent conflicts under a decentralized firm into a contingent claims approach to the corporate valuation model.

The standard corporate valuation model calculates the value of the firm as a contingent claim written on the underlying uncertainty. Since we can regard the bankruptcy decision as a financial option, bankruptcy is analogous to an American put option written on the underlying uncertainty. The existing literature gives us a link (mode of analysis) between a statistical model describing default and an

[1)] This method is sometimes referred to as the option-theoretic approach, since it is directly inspired by the Black-Scholes-Merton methodology for valuation of financial options. See, e.g., Brennan and Schwatz (1984), Leland (1994), Leland and Toft (1996), and Mella-Barrel and Perraudin (1997) for details of the contingent claims approach on corporate valuation.

economic-pricing model (see, e.g., Chiarella (2002) and Kijima (2002) for details about option pricing theory). In particular, we can examine the optimal capital structure, the timing of bankruptcy, and the default probability. An excellent overview of this approach is summarized in Dixit and Pindyck (1994) and Bielecki and Rutkowski (2002).

In the standard corporate valuation model using the contingent claims approach, there are no principal-agent conflicts between an owner and a manager, because the firm is assumed to be managed by the owner. In most modern corporations, however, owners delegate the corporate operations to managers, taking advantage of managers' special skills and expertise. In the situation of separation of ownership and management, there is likely to be hidden information due to information asymmetries between the owner and the manager. It is often assumed that a portion of corporate value is privately observed by the manager (agent), while it is not observed by the owner (principal). This information asymmetry leads to what is called principal-agent conflicts. An excellent overview of the literature on information asymmetry can be found in Fudenberg and Tirole (1991), Mas-Collel et al. (1995) and Salanié (1997).

Although information asymmetries are extensively examined in microeconomics, to the best of our knowledge, they have not been examined in a corporate valuation model using contingent claims pricing. When management decisions are delegated to managers under information asymmetries, information asymmetries lead to principal-agent conflicts. Thus, the owner's problem is to design an optimal contract to provide incentives for managers to truthfully reveal their private information. What is of great interest is to derive the optimal contract under information asymmetry, and to calculate the corporate value using the contingent claims approach under principal-agent conflicts.

The principal-agent setting leads to a decomposition of the underlying option into two components: a "manager's option" and an "owner's option". Importantly, there is a conflict between the interests of the owner and those of the manager, i.e., there is a conflict between a manager's option and an owner's option value. In such principal-agent conflicts, the manager attempts to increase his option value by using private information. This action of the manager, at the same time, decreases the owner's option value due to principal-agent conflicts. The contracts must be designed to provide incentive for managers to truthfully reveal private information and preserve the value of the owner's option.

In this paper, in the presence of principal-agent conflicts due to information asymmetry, we calculate the corporate value and derive the optimal contracts. This paper demonstrates that the corporate value with optimal contracts is significantly different from that implied by the first-best (full-information) solution. The corporate value in a decentralized firm is lower than that in an owner-managed firm. The result comes from the fact that managers display greater inertia in their bankruptcy behavior, in that they bankrupt later than implied by the first-best solution.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model. Section 3 simplifies the optimization problem and solves for the optimal contracts. In Section 4, we analyze the implications of the model in terms of the stock price's reaction, the agency cost due to the principal-agent problem, and the comparative statics (sensitivity analysis) of the optimal contracts with respect to the key parameter (volatility). Section 5 concludes. Appendices contain the proofs and the solutions of the optimal contracts.

2. Model

In this section, we begin with a description of the model. We then, as a benchmark, provide the solution and the corporate value in the first-best no-principal-agent (full-information) setting. Finally, we consider the principal-agent optimization problem under separation of ownership and control.

2.1. Setup

Throughout our analysis, we suppose that capital markets are frictionless, agents are risk neutral and can borrow and lend freely at a constant interest rate, r. The assumption of risk neutrality represents little loss of generality. If agents are risk averse, the analysis may be developed under risk neutral rather than actual probabilities (see Harrison and Kreps (1979)).

The owner of a firm (principal) has an option to hire a manager (agent) to operate the company. We assume that the owner delegates the corporate operation to a manager because the manager has more skills and expertise. For simplicity, we assume in addition that the firm finances the capital only with pure equity. This model is similar to the one developed by Grenadier and Wang (2005), which we here apply to the contingent claim approach to corporate valuation[2].

Consider a manager hired by an owner of a firm that produces a unit of output, which it sells for a price, X_t. We assume that X_t follows a geometric Brownian motion under a risk neutral measure, $\mathbb{P}$, i.e.[3],

$$\frac{\mathrm{d}X_t}{X_t} = \mu \mathrm{d}t + \sigma \mathrm{d}z_t, \quad X_0 = x \in \mathbb{R}_{++}, \tag{1}$$

where $(z_t)_{t \in \mathbb{R}_+}$ denotes the standard Brownian motion under the risk neutral measure, $\mathbb{P}$, and where the mean growth rate μ as well as the volatility σ are positive constants. For convergence, we assume that $\mu < r$. While in production, the firm incurs costs per period of $w \in \mathbb{R}_{++}$, its net earnings flow is $X_t - w$.

Here, the corporate value consists of two sources. One portion is observable and contractible to both the owner and the manager, while the other portion is privately observed only by the manager. Let $W(x)$ represent the observable component with an income flow $x - w$ where $X_t = x$, and θ represent the value of the privately observed component. Thus, the sum of values is the corporate value, $W(x) + \theta$. The

---

[2] Grenadier and Wang (2004) examines the investment timing in a real options approach.
[3] Formally, define the filtered probability space as $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in \mathbb{R}_+}, \mathbb{P})$.

assumption of information asymmetry represents the fact that the manager has more skills and expertise[4].

The private component of corporate value, θ, may take one of two possible values: θ_1 or θ_2 with $\theta_1 > \theta_2$. We denote $\Delta\theta := \theta_1 - \theta_2$. We may regard a draw of θ_1 as a "higher quality" private component and a draw of θ_2 as a "lower quality" private component. The probability of drawing θ_1 equals p, an exogenous variable. Immediately after making a contract with the owner at time zero, the manager observes whether the private component is of "higher quality" or "lower quality".

Now we assume that bankruptcy occurs when the value of the observable component first hits some constant γ, because this value is observed by both the owner and manager. So, the value of the firm at bankruptcy turns to be $\gamma + \theta$. From now on, we refer to $\gamma + \theta$ as the scrapping value. By assumption, one portion of the bankruptcy level, γ, is known by both the owner and manager, while the other portion, θ, is privately observed only by the manager. Although the owner cannot observe the true value of θ, he does observe the amount transferred to himself at the time of bankruptcy to be handed over by the manager[5]. While the manager could attempt to hand over θ_2 when the true value is θ_1, it will be seen in equilibrium that the amount transferred to the owner at the time of bankruptcy will always be the true value. Although the owner cannot contract on the private component of the value, θ, he can contract on the observable component of the value, X_t. Contingent on the level of X_t at bankruptcy, the owner designs the optimal compensation paid to the manager.

The assumption that a portion of the value is privately observed only by one (e.g., manager) and not observed by the other (e.g., owner) is quite common in the information asymmetry literature. This information asymmetry invites a host of principal-agent issues. An excellent overview of the information asymmetry approach is found in Mas-Collel et al. (1995) and Salanié (1997).

In summary, the owner faces an optimization problem with information asymmetry (the owner does not observe the true realization of θ). The owner needs to provide compensation incentive to induce the manager to reveal his type voluntarily and truthfully, by choosing the equilibrium bankruptcy strategy and supplying the corresponding unobservable component of the firm value.

2.2. First-Best Benchmark (Full-Information Setting)

It is useful to begin our analysis by looking at the optimal contracting problem

---

[4] For example, two components of corporate value, the publicly observed portion and the privately observed portion, can be regarded as tangible assets, and intangible assets, respectively. Since the manager takes advantage of more skills and expertise in the corporate operation, compared to the owner, the manager can evaluate intangible assets more accurately, while it may be difficult for the owner to evaluate these precisely. This condition leads to information asymmetry between the owner and the manager. Concretely speaking, since the owner has less knowledge about technology, it is more difficult for the owner to evaluate accurately the patent on the technology.

[5] In our model, since we assume that the firm finances the capital only with pure equity, it may be noted that the owner is the only residual claimant at bankruptcy.

when θ is publicly observable by both the owner and the manager. Equivalently, this first-best (full-information) solution can be achieved in a principal-agent setting, provided that θ is both publicly observable and contractible. Let $V(x;\theta)$ denote the value of the firm in a situation where θ is a known parameter. Since $V(x;\theta)$ is the value of the firm with an income flow $x-w$ where $X_t = x$, the financial market equilibrium under risk neutrality requires that

$$\frac{1}{2}\sigma^2 x^2 V''(x;\theta) + \mu x V'(x;\theta) - rV(x;\theta) + (x-w) = 0, \quad x > x^*(\theta), \tag{2}$$

where $x^*(\theta)$ denotes the bankruptcy trigger under the realized value of θ. The boundary conditions serve to ensure that an optimal bankruptcy strategy is chosen:

$$V(x^*(\theta);\theta) = \gamma + \theta, \quad V'(x^*(\theta);\theta) = 0, \quad \lim_{x\to\infty} V(x;\theta) = \frac{x}{r-\mu} - \frac{w}{r}.$$

Here, the first condition is the value-matching condition. It means that at the moment when bankruptcy occurs, the scrapping value of the firm is $V(x^*(\theta);\theta) = \gamma + \theta$. The second condition is the smooth-pasting or high-contact condition. This condition ensures that the trigger $x^*(\theta)$ is chosen so as to maximize the value of $V(x;\theta)$. The third condition is the no-bubbles condition. In the absence of bubbles, as $x \to \infty$, $V(x;\theta)$ must approach the expected discounted integral of future income flows:

$$\mathbb{E}^x\Big[\int_t^\infty e^{-r(s-t)}(X_s - w)ds \mid \mathcal{F}\Big] = \frac{x}{r-\mu} - \frac{w}{r},$$

where $\mathbb{E}^x[\cdot \mid \mathcal{F}_t]$ denotes the time t conditional expectation operator given that $X_t = x$.

As we show in Appendix A.1, solving the ordinary differential equation for $V(x;\theta)$, one obtains:

Lemma 2.1 (Solution of the Ordinary Differential Equation): *The value of the firm at the time zero*, $V(x;\theta)$, *and the trigger*, $x^*(\theta)$, *are equal to*:

$$V(x;\theta) = \begin{cases} \dfrac{x}{r-\mu} - \dfrac{w}{r} + \Big[(\gamma+\theta) - \dfrac{x^*(\theta)}{r-\mu} + \dfrac{w}{r}\Big]\Big(\dfrac{x}{x^*(\theta)}\Big)^{\beta}, & x > x^*(\theta), \\ \gamma + \theta, & x \le x^*(\theta) \end{cases} \tag{3}$$

and

$$x^*(\theta) = -\frac{\beta}{1-\beta}\Big\{\frac{w}{r} + (\gamma+\theta)\Big\}(r-\mu), \tag{4}$$

where β *is the negative root of* $Q(y) = 0$, *where* $Q(y) = y(y-1)\frac{\sigma^2}{2} + y\mu - r$, *i.e.*,

$$\beta = \frac{1}{\sigma^2}\left(-\Big(\mu - \frac{\sigma^2}{2}\Big) - \sqrt{\Big(\mu - \frac{\sigma^2}{2}\Big)^2 + 2r\sigma^2}\right) < 0. \tag{5}$$

The upper function in equation (3) has a simple intuitive interpretation; the first two terms correspond to the discounted value of all future operating profits, the third term represents an option value for each θ_1 and θ_2, respectively, when bankruptcy occurs. On the other hand, the lower function in equation (3) is the scrapping value at bankruptcy.

Since the realized value of θ can be either θ_1 or θ_2, we denote the first-best bankruptcy triggers by $x_1^* := x^*(\theta_1)$ and $x_2^* := x^*(\theta_2)$, respectively. We state a lemma in the first-best (full-information) setting.

Lemma 2.2 (First-Best Bankruptcy Trigger): *The first-best bankruptcy trigger for the realized state θ_1 is strictly bigger than that for θ_2, i.e., $x_1^* > x_2^*$*

Lemma 2.2 implies that bankruptcy trigger for the firm having the state θ_1 is bigger than for the firm having the state θ_2. In general, the firm having the state θ_2 will display greater inertia in its bankruptcy behavior than the firm having the state θ_1.

We denote the *ex-ante* value of the firm in the first-best no-principal-agent (full-information) setting by $\pi^*(x)$. Then, we can obtain the following result.

Lemma 2.3 (First-Best Corporate Value): *Since the value of the firm in the first-best no-principal-agent setting is defined by*

$$\pi^*(x) = pV(x;\theta_1) + (1-p)V(x;\theta_2),$$

$\pi^*(x)$ is equal to:

$$\pi^*(x) = \frac{x}{r-\mu} - \frac{w}{r} + p\left\{\gamma + \theta_1 - \frac{x_1^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_1^*}\right)^\beta$$
$$+(1-p)\left\{\gamma + \theta_2 - \frac{x_2^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_2^*}\right)^\beta, \tag{6}$$

where $x \geq x_1^*$.

Figure 1 plots the values, $V(x;\theta)$, $\pi^*(x)$, and the trigger $x^*(\theta)$ obtained by Lemmas 2.1 to 2.3. The asymptotic values to which $V(x;\theta)$ tends as $x \to \infty$ in the absence of bubbles is shown as the dashed line.

Next, we examine the comparative statics (sensitivity analysis) with respect to the volatility σ. As we show in Appendix A.1, one can obtain the following result.

Lemma 2.4 (Comparative Statics): *In the first-best no-principal-agent setting, the trigger $x^*(\theta)$ is increasing with the volatility σ for all $\theta \in \{\theta_1, \theta_2\}$. Moreover, the value of the firm, $\pi^*(x)$, is decreasing with the volatility σ.*

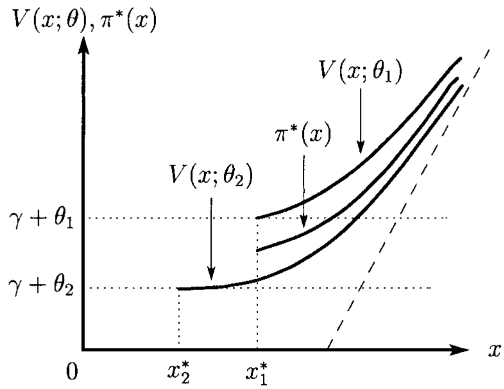

Figure 1 Values and triggers in the first-best (full-information) situation. The values, $V(x;\theta)$, and $\pi^*(x)$ are shown as functions of the output price, x. The bankruptcy trigger $x^*(\theta)$ is determined as the optimal level of $V(x;\theta)$ for $\theta \in \{\theta_1, \theta_2\}$. The asymptotic value to which $V(x;\theta)$ and $\pi^*(x)$ tend as $x \to \infty$ in the absence of bubbles is shown as the dashed line.

Lemma 2.4 implies that uncertainty accelerates bankruptcy. In general, the greater the uncertainty, the less inertia in their bankruptcy behavior managers display. This result is exactly the same as the one in the standard option-pricing literature.

2.3. A Principal-Agent Setting

The owner offers the manager a contract at time zero that commits the owner to pay the manager's compensation (wage) at the time of bankruptcy. Renegotiation is not allowed. The commitment leads to an increase in the owner's value at the time of bankruptcy. The payment can be made contingent on the observable component of the corporate value at the time of bankruptcy. Thus, in principle, for any realized value $\tilde{x}$ of X_t obtained at the time of bankruptcy, a contracted compensation $k(\tilde{x})$ can be specified, provided that $k(\tilde{x}) \geq 0$. The contract will endogenously provide incentives to ensure that the manager declares bankruptcy in accordance with the owner's rational expectations and delivers the true scrapping value of the firm to the owner at the time of bankruptcy. Thus, it is noted that although $k(\tilde{x})$ is regarded as a wage or compensation payment for the manager, $k(\tilde{x})$ can be also regarded as the opportunity cost incurred by the owner in order to induce that the amount transferred to the owner at the time of bankruptcy will always be the true value[6].

The principal-agent setting leads to a decomposition of the underlying option into two options: an owner's option and a manager's option. The owner's option

[6] In general, when bankruptcy occurs, managers never get a compensation. In this model, however, the manager has an information advantage relative to the owner. Since we regard the compensation for the manager as information rent incurred by the owner, we assume that the manager get the compensation at the time of bankruptcy.

has a payoff function of $V(\tilde{x};\theta)-k(\tilde{x})$, and the manager's option has a payoff function of $k(\tilde{x})$. Obviously, the sum of these payoff functions equals the payoff of the underlying option. The manager's option is a traditional American put option, since the manager chooses the exercise time to maximize the value of his option. However, in this optimal contracting setting, it is the owner who sets the contract parameters that induce the manager to follow an exercise policy that maximizes the value of the owner's option.

Since there are only two possible value of θ, for any $k(\tilde{x})$ scheduled, there can be at most two wage/bankruptcy trigger pairs that will be chosen by the manager. Thus, the contract need only include two wage/bankruptcy trigger pairs from which the manager can choose: one that will be chosen by the manager when he observes θ_1, and one chosen by the manager when he observes θ_2. Therefore, the owner will offer a contract that promises a wage of k_1 if the manager declares bankruptcy at x_1 and a wage of k_2 if the manager declares bankruptcy at x_2. The revelation principle will ensure that a manager who privately observes θ_1 will declare bankruptcy at the trigger x_1, and a manager who privately observes θ_2 will declare bankruptcy at the trigger x_2[7].

The owner has a scrapping value of $\gamma+\theta_1-k_1$ if $\theta=\theta_1$ at the time of bankruptcy, and $\gamma+\theta_2-k_2$ if $\theta=\theta_2$. Thus, the value of the owner's option, $\pi^o(x;k_1,k_2,x_1,x_2)$, can be written as:

$$\begin{aligned}\pi^o(x;k_1,k_2,x_1,x_2)=&\frac{x}{r-\mu}-\frac{w}{r}+p\left\{(\gamma+\theta_1)-\frac{x_1}{r-\mu}+\frac{w}{r}-k_1\right\}\left(\frac{x}{x_1}\right)^{\beta}\\&+(1-p)\left\{(\gamma+\theta_2)-\frac{x_2}{r-\mu}+\frac{w}{r}-k_2\right\}\left(\frac{x}{x_2}\right)^{\beta}.\end{aligned}\tag{7}$$

The deviation is exactly the same as in the proof of Lemmas 2.1 and 2.3.

The manager's option has a payoff function of k_1 if $\theta=\theta_1$ and k_2 if $\theta=\theta_2$. As we show in Appendix A.1, the value of manager's option, $\pi^m(x;k_1,k_2,x_1,x_2)$, can be written as:

$$\pi^m(x;k_1,k_2,x_1,x_2)=p\left(\frac{x}{x_1}\right)^{\beta}k_1+(1-p)\left(\frac{x}{x_2}\right)^{\beta}k_2.\tag{8}$$

It is important to note that the payoff of the manager has the property of the single crossing condition. The single crossing condition means that the manager's marginal rate of substitution between $x(\theta)$ and $k(\theta)$ is monotone with respect to state θ. It is necessary for $(x(\theta),k(\theta))$ to be implementable[8].

For notational simplicity, we will drop the parameters and simply write the

---

[7] The contract is modeled as a mechanism, $\mathcal{M}=\left\{x(\tilde{\theta}),k(\tilde{\theta});\tilde{\theta}\in\{\theta_1,\theta_2\}\right\}$, which may be contingent on a reported $\tilde{\theta}$. Since the revelation principle ensures that the manager truthfully reveals a true θ as private information, we will make no distinction between a reported $\tilde{\theta}$ and a true θ. Thus we will drop the suffix "tilde" on the reported $\tilde{\theta}$ and simply write the reported type as θ.

[8] See, e.g., Fudenberg and Tirole (1991) for details of the single crossing condition.

owner's and manager's option value as $\pi^o(x)$, and $\pi^m(x)$, respectively.

In principal-agent optimal setting, the owner sets the contract pairs in order to induce the manager to engage in truth-telling action at the bankruptcy trigger. In order to accomplish these objectives, the owner must attempt to design two types of constraints: the incentive compatibility and individual rationality constraints. These constraints are common in the literature on information asymmetry. For example, entirely analogous conditions appear in Fudenberg and Tirole (1991), Mas-Collel et al. (1995), and Salanié (1997).

The incentive compatibility constraint ensures that the manager will exercise in accordance with the owner's expectations. Specifically, the manager having a θ_1-type privately observed component will declare bankruptcy at the trigger x_1, and the manager having θ_2 will declare bankruptcy at the trigger x_2. To provide such a timing incentive, the manager must not have any incentive to divert value. These conditions ensure that this value diversion does not occur. The incentive compatibility constraints in this model are as follows:

$$\left(\frac{x}{x_1}\right)^{\beta} k_1 \geq \left(\frac{x}{x_2}\right)^{\beta} (k_2 + \Delta\theta), \tag{9}$$

$$\left(\frac{x}{x_1}\right)^{\beta} (k_1 - \Delta\theta) \leq \left(\frac{x}{x_2}\right)^{\beta} k_2. \tag{10}$$

Constraints (9) and (10) are the incentive compatibility constraints for the manager in state θ_1 and θ_2, respectively. Consider, for example, constraint (9). The manager's payoff in state θ_1 is $(x/x_1)^{\beta} k_1$ if he tells the truth, but it is $(x/x_1)^{\beta}(k_2 + \Delta\theta)$ if he instead claims that it is state θ_2. Thus, he will tell the truth if (9) is satisfied. Constraint (10) follows similarly. Constraint (10) will be shown not to bind, so only constraint (9) is relevant to our discussion.

On the other hand, the individual rationality (or participation) constraints in this model are as follows:

$$k_1 \geq 0, \tag{11}$$

$$k_2 \geq 0. \tag{12}$$

Note that non-negative k_1 and k_2 insures that the manager makes an agreement about employment. For example, if $w_2 < 0$, then the manager would rather refuse the contract on learning that $\theta = \theta_2$. Thus, we assume a non-negative wage.

Therefore, the owner's problem can be summarized as the maximization of its objective function, subject to the four inequality constraints (9) to (12). Fortunately, we will find in the next section that the problem can be simplified in that we can reduce the number of constraints to only one.

3. Model Solution

In this section, we provide the solution to the optimal contracting problem described in the previous section: maximizing the owner's value function subject to the four inequality constraints (9) to (12).

3.1. A Simplified Statement

Our concern is with the best pair of contracts in the principal-agent setting, which is obtained by solving the following optimization problem:

$$\max_{x_1,x_2,k_1,k_2} \; \frac{x}{r-\mu}-\frac{w}{r}+p\left\{(\gamma+\theta_1)-\frac{x_1}{r-\mu}+\frac{w}{r}-k_1\right\}\left(\frac{x}{x_1}\right)^{\beta} \\ +(1-p)\left\{(\gamma+\theta_2)-\frac{x_2}{r-\mu}+\frac{w}{r}-k_2\right\}\left(\frac{x}{x_2}\right)^{\beta}, \tag{13}$$

subject to four constraints (9) to (12).

We now proceed to characterize the solution to problem (13) through a series of steps. Proofs of these procedures are shown in Appendix A.2 (summarized in Proposition A.1). This argument is exactly similar to that in Grenadier and Wang (2005). Proposition A.1 implies that constraints (10) and (11) are not binding.

Following the steps given in Appendix A.2, we can simplify the optimization problem (13), and show that we can determine the optimal contracts by solving the following optimization problem:

$$\max_{x_1,x_2,k_1} \; \frac{x}{r-\mu}-\frac{w}{r}+p\left\{(\gamma+\theta_1)-\frac{x_1}{r-\mu}+\frac{w}{r}-k_1\right\}\left(\frac{x}{x_1}\right)^{\beta} \\ +(1-p)\left\{(\gamma+\theta_2)-\frac{x_2}{r-\mu}+\frac{w}{r}\right\}\left(\frac{x}{x_2}\right)^{\beta}, \tag{14}$$

subject to only one constraint:

$$\left(\frac{x}{x_1}\right)^{\beta}k_1 \geq \left(\frac{x}{x_2}\right)^{\beta}\Delta\theta. \tag{15}$$

It is important to note that we can simply substitute the optimal contract $k_2 = 0$ into the problem.

In summary, we now have a simplified optimization problem for the owner. Equation (14) is the owner's option value. Constraint (15) is the simplified *ex-post* incentive constraint for the manager having the state θ_1.

3.2. Optimal Contracts

In this subsection, we provide the solution to the optimal contracting problem: maximizing (14) subject to only one inequality constraint (15).

Before we provide the explicit solutions, we define θ_3 by:

$$\theta_3 = \theta_2 - \frac{p}{1-p}\Delta\theta, \tag{16}$$

and we define x_3^* by:

$$x_3^* := \frac{-\beta}{1-\beta}\left\{\frac{w}{r} + (\gamma + \theta_3)\right\}(r-\mu). \tag{17}$$

Thus, the triggers are ordered by $x_1^* > x_2^* > x_3^*$ because of $\theta_1 > \theta_2 > \theta_3$.

Solving the owner's optimization problem (14) subject to only one constraint (15), we obtain the following results. The proofs detailing the solutions are provided in Appendix A.3.

Proposition 3.1 (Optimal Contracts in the Principal-Agent Setting): *The optimal contracts* (x_1, x_2, k_1, k_2) *are as follows*:

$$x_1 = x_1^*, \quad x_2 = x_3^*, \quad k_1 = \left(\frac{x_1^*}{x_3^*}\right)^{\beta}\Delta\theta, \quad k_2 = 0.$$

Proposition 3.1 implies that the manager declares bankruptcy at the first time that x hits x_1^* if $\theta = \theta_1$, and that x hits x_3^* if $\theta = \theta_2$. Moreover, the owner pays k_1 to manager if bankruptcy occurs at x_1^*, and pays nothing if bankruptcy occurs at x_3^*.

The first property of the solution is that the manager of state θ_1 will declare bankruptcy at the first-best trigger x_1^*. Intuitively, for any manager's option value that satisfies constraint (15), the owner will always prefer to choose the first-best bankruptcy trigger, x_1^*, and vary wage k_1 to achieve the same level of bankruptcy trigger.

The second property of the solution is that the manager of state θ_2 will not declare bankruptcy at the first-best trigger x_2^*. As we shall now see, it is less costly for the owner to distort x_2 away from x_2^* than to distort x_1 away from x_1^* in order to provide the appropriate incentives to the manager. Intuitively, the necessity of ensuring that the manager of state θ_1 does not imitate the one of state θ_2 leads the manager of state θ_2 to display a greater "option to wait" than the first-best solution. In order to dissuade the manager of state θ_1 from declaring bankruptcy at x_2, the contract must sufficiently decrease x_2 below x_2^*.

Remark 3.1. *In the principal-agent setting, the bankruptcy trigger for a manager of state* θ_2 *is lower than that in the first-best no-principal-agent setting, i.e.,* $x_3^* = x_2 < x_2^*$.

The third property of the solution is that the owner sets the optimal wage k_1 according to the level of triggers. The wage k_1 is the present value of information rent paid to the manager in order to provide the incentive to truthfully reveal private information. Also, as $\Delta\theta$ increases, the manager has a greater incentive to divert this difference in value. Thus, increased $\Delta\theta$ increases the optimal wage k_1.

The last property of the solution is that the owner keeps the optimal wage k_2 zero. The intuition is straightforward. Giving the manager of state θ_2 positive rent implies higher rent for the manager of state θ_1 in order to induce the manager to engage in truth-telling at the time of bankruptcy. In order to minimize these rents (subject to manager's incentive and participation constraints), it is optimal for the owner to keep k_2 zero. In addition, by using these contracts pairs, we can obtain the following results.

Proposition 3.2 (Second-Best Corporate Value): *In the principal-agent setting, the owner's and manager's option value,* $\pi^o(x)$, *and* $\pi^m(x)$, *respectively, can be written as*:

$$\pi^o(x) = \frac{x}{r-\mu} - \frac{w}{r} + p\left\{\gamma + \theta_1 - \frac{x_1^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_1^*}\right)^\beta$$
$$+(1-p)\left\{\gamma + \left(\theta_2 - \frac{p}{1-p}\Delta\theta\right) - \frac{x_3^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_3^*}\right)^\beta, \tag{18}$$

and

$$\pi^m(x) = p\left(\frac{x}{x_3^*}\right)^\beta \Delta\theta. \tag{19}$$

Let $\pi^{**}(x)$ *denote the value of the firm in the principal-agent setting. Then, the value of the firm,* $\pi^{**}(x) = \pi^o(x) + \pi^m(x)$, *can be written as*:

$$\pi^{**}(x) = \frac{x}{r-\mu} - \frac{w}{r} + p\left\{\gamma + \theta_1 - \frac{x_1^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_1^*}\right)^\beta$$
$$+(1-p)\left\{\gamma + \theta_2 - \frac{x_3^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_3^*}\right)^\beta. \tag{20}$$

It is interesting to note that the solution for the owner's option value is equivalent to the first-best solution in which one substitutes x_3^* for x_2^*.

We can generalize our model by allowing for a multiple-point distribution for θ, an admissible continuous distribution for θ (e.g., Mæland (2001), and Grenadier and Wang (2005) examine the real options model with admissible continuous distribution). Subtle technical issues arise when we allow for θ to be continuous. However, the basic outcome and intuition remain valid.

4. Model Implications

In this section, we analyze several of the more important implications of the model. First, Subsection 4.1 examines the stock price reaction to bankruptcy. We shall demonstrate that the stock price will move by a discrete jump due to the information released at trigger x_1^* because of $x_1^* > x_3^*$. Bankruptcy at x_1^* signals good news about the manager's privately observed component and the stock price jumps upwards; No-bankruptcy at x_1^* signals bad news about the manager's privately observed component and the stock price jumps downward. Second, Subsection 4.2

considers the agency cost compared with the first-best outcome. Since the timing of bankruptcy differs from that of the first-best situation, the principal-agent problem results in an agency cost. Third, Subsection 4.3 considers the comparative statics of contract pairs with respect to several key parameters of the model.

4.1. Stock Price Reaction to Bankruptcy

In this subsection, we analyze the stock price (equity value) reaction to the information released via the manager's bankruptcy decision. The manager's bankruptcy decision will signal to the owner the true value of θ, and the stock price will reflect this information revelation. That is, while in the model we have made the wage in the incentive contract contingent on the manager's bankruptcy decision, the wage can also be made contingent on the stock price.

The stock price (equity value) is equal to the value of the owner's option value. Prior to the point at which the process $(X_t)_{t\in\mathbb{R}_+}$ reaches the trigger x_1^*, the owner does not know the true value of θ: the owner believes that $\theta = \theta_1$ with probability p and $\theta = \theta_2$ with probability $1 - p$.

Once the process hits the trigger x_1^*, the manager's privately observed value is fully revealed. If bankruptcy occurs, the manager reveals to the owner that the manager's privately observed value is high. Thus, the stock price instantly jumps upward to:

$$\pi^o(x_1^*;\theta_1) = V(x_1^*;\theta_1) - \left(\frac{x_1^*}{x_1^*}\right)^{\beta} k_1 = \gamma + \theta_1 - k_1. \tag{21}$$

If bankruptcy does not occur at x_1^*, then the owner infers that the manager's privately observed component of the value is low. Then, the stock price instantly jumps downward to:

$$\pi^o(x;\theta_2) = V(x;\theta_2) = \frac{x}{r-\mu} - \frac{w}{r} + \left\{\gamma + \theta_2 - \frac{x_3^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_3^*}\right)^{\beta}, \quad x_3^* \le x \le x_1^*. \tag{22}$$

It is noted that the stock price (the owner's option value) given in (22) is defined on the interval $[x_3^*, x_1^*]$.

Figure 2 plots the stock price (equity value) as a function of the output price, x. For all $x > x_1^*$, $\pi^o(x)$ is given in (18). It follows that prior to the point where the process reaches the trigger x_1^*, the manager does not reveal to the owner that the value of θ is high or low. On the other hand, for all $x \le x_1^*$, $\pi^o(x)$ is given in (21) if bankruptcy occurs at the trigger x_1^*, and $\pi^o(x)$ is given in (22) if bankruptcy does not occur at the trigger x_1^*. The asymptotic value to which $V(x;\theta)$ and $\pi^*(x)$ tend as $x \to \infty$ in the absence of bubbles is shown as the dashed line.

Thus, the manager's bankruptcy decision will signal to the owner the true value of θ, and the stock price will jump upward or downward. That is, the stock price is discontinuous at x_1^*.

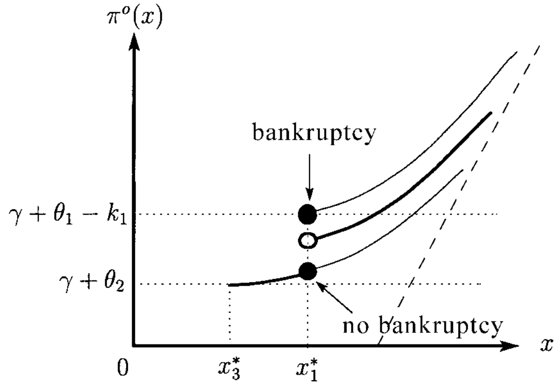

Figure 2 Stock price reaction to bankruptcy. The stock price (equity value), $\pi^o(x)$, is equal to the value of the owner's option value. Prior to the point where the process reaches the trigger x_1^*, the manager does not reveal to the owner that the value of θ is high or low. Thus, for all x over x_1^*, the stock price equals the value of the owner's option given (18). For all $x \le x_1^*$, $\pi^o(x)$ is given in (21) if bankruptcy occurs at the trigger x_1^*, and $\pi^o(x)$ is given in (22) if bankruptcy does not occur at the interval $[x_3^*, x_1^*]$.

4.2. Agency Cost

Although the owner chooses the value-maximizing contact to provide an incentive for the manager to truthfully reveal private information, the principal-agent problem ultimately still proves costly. In an owner-managed firm, bankruptcy occurs at the first-best stopping time. However, in a decentralized firm, there will be an agency cost due to the firm's suboptimal strategy. We obtain the following proposition.

Proposition 4.1. *The value of the firm in the principal-agent setting,* $\pi^{**}(x)$, *is strictly lower than that in the first-best no-principal-agent setting,* $\pi^*(x)$, *i.e.,* $\pi^{**}(x) < \pi^*(x)$.

Importantly, the principal-agent problem leads to a decrease in the value of the firm. This is due to the fact that in the principal-agent setting, the trigger $x_2 = x_3^*$ is lower than the first-best trigger x_2^*, in order to dissuade the manager of state θ_1 from mimicking the one of state θ_2. We refer to the difference between the first-best option value and the suboptimal option value. Thus, we can define the agency cost due to principal-agent issues as C, where $C = \pi^*(x) - \pi^{**}(x)$. Simplifying, we have:

$$C = (1-p)\left[\left\{\gamma + \theta_2 - \frac{x_2^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_2^*}\right)^{\beta} - \left\{\gamma + \theta_2 - \frac{x_3^*}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_3^*}\right)^{\beta}\right]. \quad (23)$$

Note that the agency cost is strictly positive, $C > 0$, because $x_3^* \neq x_2^*$ where,

$$x_2^* = \underset{y}{\operatorname{argmax}} \left\{ y + \theta_2 - \frac{y}{r-\mu} + \frac{w}{r} \right\} \left(\frac{x}{y} \right)^{\beta}. \tag{24}$$

The agency cost is driven by the distance of the trigger x_3^* from x_2^*. Note that as $\Delta\theta$ decreases, x_3^* converges into x_2^*, and the agency cost compared with the first-best value also converges to zero. Importantly, the agency cost arises *not* from the delegation of the corporate operation *but* from information asymmetry between the owner and manager.

4.3. Sensitivity to Volatility

To get to a deeper understanding of the insights of the model, we now perturb some of the key parameter of the model and analyze their impacts on the optimal contracts pairs and the value of the firm. We begin with the sensitivity of the optimal contracts pairs (k_1, $x_1 = x_1^*, x_2 = x_3^*$) with respect to the volatility σ.

Lemma 4.1. *In the principal-agent* (*information asymmetry*) *setting*, *as the volatility* σ *is increasing*, *the wage* k_1 *is decreasing. Moreover*, *as the volatility* σ *is increasing, the triggers*, x_1^* *and* x_3^*, *are increasing.*

Note that an increase in the volatility σ enables the owner to increase the triggers in the principal-agent (information asymmetry) setting. That is, this result is exactly the same as the one in the first-best no-principal-agent (full-information) setting.

We then examine the comparative statics of the owner's and manager's option values with respect to the volatility σ.

Lemma 4.2. *In the principal-agent* (*information asymmetry*) *setting*, *an increase in the volatility* σ *decreases the owner's option value* $\pi^o(x)$, *while it has an ambiguous effect on the manager's option value* $\pi^m(x)$.

It is important to note that an increase in the volatility may possibly give rise to what is called "asset substitution". If the state of the underlying price is relatively low, in that,

$$\log\left(\frac{x}{x_3^*}\right) < \frac{1}{1-\beta}, \tag{25}$$

then an increase in the volatility increases the manager's options value. Therefore, if (25) is satisfied, an increase in the volatility decreases the owner's value, while it increases the manager's value. These results imply that an increase in the volatility shifts wealth from the owner to the manager. This possibility to transfer wealth is known as "asset substitution". Naturally, since the sum of these two values is the corporate value, whether this sum is increasing or decreasing in σ is an interesting question. The result is as follows:

Lemma 4.3. *In the principal-agent* (*information asymmetry*) *setting*, *an increase in the volatility*, σ, *has an ambiguous effect on the value of the firm*, $\pi^{**}(x)$.

There are positive or negative effects of the corporate value with respect to the volatility σ. Note that this result is different from the one in the first-best no-principal-agent (full-information) setting.

5. Conclusion

This paper extends the corporate valuation model to account for information asymmetries between the owner and the manager. Information asymmetries lead to principal-agent conflicts. When management decisions are delegated to managers under information asymmetries, employment contracts must be designed to provide incentives for managers to truthfully reveal their private information. This paper presents a model of optimal contracting in a continuous-time principal-agent setting in which there are information asymmetries. The implied behavior at the time of bankruptcy differs significantly from that of the first-best no-principal-agent solution. In particular, there will be greater inertia in bankruptcy, as the model predicts that the manager will have a more valuable option to wait than the owner. The value of the firm in a decentralized firm is lower than that in an owner-managed firm.

Some extensions of the model would be interesting. First, the model could be generalized to include debt in capital structure. As shown by Mella-Barral and Perraudin (1997), the force of debt greatly alters the bankruptcy behavior implied by standard corporate valuation models. Second, the model could also be generalized to include the manager's action. This richer setting would force the owner to design optimal contracts with additional features.

Appendices

A.1. Proof of Lemmas and Propositions

Proof of Lemma 2.1. Since equation (2) is an ordinary differential equation of the Euler type, we can obtain the solution (3) by using a suitable transformation of variables. Also, the solution (4) can be derived by two boundary conditions; the value-matching and smooth-pasting conditions. □

Proof of Lemma 2.4. Differentiating the trigger with respect to σ yields

$$\frac{\mathrm{d}x^*(\theta)}{\mathrm{d}\sigma} = \frac{\partial x^*(\theta)}{\partial \beta}\frac{\partial \beta}{\partial \sigma} = \left(\frac{-1}{(1-\beta)^2}\left\{\frac{w}{r} + (\gamma+\theta)\right\}(r-\mu)\right)\frac{\partial \beta}{\partial \sigma} > 0,$$

where we have used the fact that $\frac{\partial \beta}{\partial \sigma} < 0$. As for this statement, differentiating

$Q(y) := y(y-1)\frac{\sigma^2}{2} + \mu y - r = 0$ with respect to σ gives

$$\frac{\partial Q}{\partial \beta}\frac{\partial \beta}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0.$$

Since $\frac{\partial Q}{\partial \beta} > 0$ and $\frac{\partial Q}{\partial \sigma} > 0$, we obtain $\frac{\partial \beta}{\partial \sigma} < 0$.

As for the latter statement, differentiating the value with respect to σ gives

$$\begin{aligned}\frac{\mathrm{d}V(x;\theta)}{\mathrm{d}\sigma} &= \left(\frac{\partial V(x;\theta)}{\partial \beta} + \frac{\partial V(x;\theta)}{\partial x^*(\theta)}\frac{\partial x^*(\theta)}{\partial \beta}\right)\frac{\partial \beta}{\partial \sigma} \\ &= \left(\frac{x}{x^*(\theta)}\right)^{\beta}\left(\gamma + \theta - \frac{x^*(\theta)}{r-\mu} + \frac{w}{r}\right)\log\left(\frac{x}{x^*(\theta)}\right)\frac{\partial \beta}{\partial \sigma} \\ &< 0,\end{aligned}$$

where the deviation from first to second equation follows from the envelope theorem (i.e., $\frac{\partial V(x;\theta)}{\partial x^*(\theta)} = 0$), and where the negative sign condition of the last equation follows from the fact that $(\gamma + \theta - \frac{x^*(\theta)}{r-\mu} + \frac{w}{r}) > 0$, and $\frac{\partial \beta}{\partial \sigma} < 0$. See, e.g., Mas-Collel et al. (1995) for details about the envelope theorem. □

Proof of Manager's Option Value. We calculate the present value of one dollar received at the first moment time that a specified trigger $\tilde{x}$ is reached. Denote this present value operator by the discount function $D(x;\tilde{x})$. This is simply the solution to the differential equation:

$$\frac{1}{2}\sigma^2 x^2 D''(x;\tilde{x}) + \mu x D'(x;\tilde{x}) - rD(x;\tilde{x}) = 0,$$

subject to the boundary condition that $D(\tilde{x};\tilde{x}) = 1$, and $D(0;\tilde{x}) = 0$. The solution can be written as:

$$D(x;\tilde{x}) = \left(\frac{x}{\tilde{x}}\right)^{\beta}, \quad x > \tilde{x}.$$

Hence we obtain the manager's option value (8). □

Proof of Lemma 4.1. Differentiating the wage k_1 with respect to σ yields

$$\begin{aligned}\frac{\mathrm{d}k_1}{\mathrm{d}\sigma} &= \left(\frac{\partial k_1}{\partial \beta} + \frac{\partial k_1}{\partial x_1^*}\frac{\partial x_1^*}{\partial \beta} + \frac{\partial k_1}{\partial x_3^*}\frac{\partial x_3^*}{\partial \beta}\right)\frac{\partial \beta}{\partial \sigma} \\ &= \left(\frac{x_1^*}{x_3^*}\right)^{\beta}\Delta\theta\left\{\log\left(\frac{x_1^*}{x_3^*}\right) + \beta\left((x_1^*)^{-1}\frac{\partial x_1^*}{\partial \beta} - (x_3^*)^{-1}\frac{\partial x_3^*}{\partial \beta}\right)\right\}\frac{\partial \beta}{\partial \sigma} \\ &= \left(\frac{x_1^*}{x_3^*}\right)^{\beta}\Delta\theta\left\{\log\left(\frac{x_1^*}{x_3^*}\right)\right\}\frac{\partial \beta}{\partial \sigma} < 0,\end{aligned}$$

where the step from second to third equation follows from the fact that $\frac{\partial x_i^*}{\partial \beta} = \frac{-x_i^*}{\beta(\beta-1)}$.

Since $\log(x_1^*/x_3^*) > 0$ because of $x_1^* > x_3^*$, we obtain the above negative sign. □

Proof of Lemma 4.2. Differentiating the manager's option value given by (19) with respect to σ yields

$$\frac{\mathrm{d}\pi^m}{\mathrm{d}\sigma} = \left(\frac{\partial\pi^m}{\partial\beta} + \frac{\partial\pi^m}{\partial x_3^*}\frac{\partial x_3^*}{\partial\beta}\right)\frac{\partial\beta}{\partial\sigma} \quad = p\left(\frac{x}{x_3^*}\right)^{\beta}\Delta\theta\left\{\log\left(\frac{x}{x_3^*}\right) + (-\beta)(x_3^*)^{-1}\left(\frac{\partial x_3^*}{\partial\beta}\right)\right\}\left(\frac{\partial\beta}{\partial\sigma}\right).$$

The first term is postive, while the second term is negative. An increase in σ in the right hand side has an ambiguous effect on the manager's option value. Since $\frac{\partial x_3^*}{\partial\beta} = \frac{-x_3^*}{\beta(\beta-1)}$ as previously shown, if

$$\log\left(\frac{x}{x_3^*}\right) < \frac{-1}{\beta-1},$$

is satisfied, an increase in σ increases the manager's value π^m. □

A.2. Proof of Simplified Statement

In this subsection, we have simplified the four constraints by eliminating constraints which are not binding in our setting, and have removed a constant from the objective function as well. This subsection provides the derivation in Section 3.

Lemma A.1. *Constraint* (11) *is not binding*, (*i.e.*, $k_1 > 0$).

(proof)

$$k_1 \geq \left(\frac{x_1}{x_2}\right)^{\beta}(k_2 + \Delta\theta) \geq \left(\frac{x_1}{x_2}\right)^{\beta}\Delta\theta > 0.$$

The first and second equalities follow from (9) and (12), respectively. □

Lemma A.2. (12) *is binding* (*i.e.*, $k_2 = 0$).

(proof) In order to optimally choose these pairs to solve the owner's maximization problem, we obtain the Lagrangian as follows:

$$\begin{aligned}\max_{x_1,x_2,k_1,k_2,\lambda_1,\lambda_2,\lambda_3} \mathcal{L} = \frac{x}{r-\mu} - \frac{w}{r} &+ p\left\{\gamma + \theta_1 - \frac{x_1}{r-\mu} + \frac{w}{r} - k_1\right\}\left(\frac{x}{x_1}\right)^{\beta} \\
&+(1-p)\left\{\gamma + \theta_2 - \frac{x_2}{r-\mu} + \frac{w}{r} - k_2\right\}\left(\frac{x}{x_2}\right)^{\beta} \\
&+\lambda_1\left[\left(\frac{x}{x_2}\right)^{\beta}(k_2 - \Delta\theta) - \left(\frac{x}{x_1}\right)^{\beta}k_1\right] \\
&+\lambda_2\left[\left(\frac{x}{x_1}\right)^{\beta}(k_1 + \Delta\theta) - \left(\frac{x}{x_2}\right)^{\beta}k_2\right] + \lambda_3 \cdot k_2,\end{aligned}$$

where λ_i denote the multiplier on these constraints ($i = \{1,2,3\}$). The solution must satisfy the following first-order conditions:

$$(k_1): p - \lambda_1 + \lambda_2 = 0,$$

$$(k_2): \{(1-p) + \lambda_1 - \lambda_2\}\left(\frac{x}{x_2}\right)^{\beta} - \lambda_3 = 0.$$

Rearranging these equations, we obtain $\lambda_3 > 0$. Thus, we can obtain $k_2 = 0$. □

Lemma A.3. *Constraint* (10) *is not binding* (*i.e., holds with strict inequality*).

(proof) Substitution of $k_2 = 0$ into (10) gives

$$\left(\frac{x}{x_1}\right)^{\beta}(k_1 - \Delta\theta) \le 0.$$

Then, $k_1 \le \Delta\theta$ is satisfied from the above equation. There we obtain $k_1 < \Delta\theta$ readily by the owner's payoff maximization problem. □

Proposition A.1. *Two constraints* (10) *and* (12) *can be neglected. Moreover, constraint* (9) *is binding.*

(proof) The former statement is obtained by Lemmas A.1–A.3. As for the latter statement, constraint (9) is not binding at the solution to this problem. Otherwise, the owner could lower the manager's wages while still getting him to accept the contract. □

A.3. Optimal Contracts

In the principal-agent optimization problem, we form the Lagrangian as follows:

$$\max_{x_1,x_2,k_1,\lambda} \mathcal{L} = \frac{x}{r-\mu} - \frac{w}{r} + p\left\{\gamma + \theta_1 - \frac{x_1}{r-\mu} + \frac{w}{r} - k_1\right\}\left(\frac{x}{x_1}\right)^{\beta}$$
$$+(1-p)\left\{\gamma + \theta_2 - \frac{x_2}{r-\mu} + \frac{w}{r}\right\}\left(\frac{x}{x_2}\right)^{\beta}$$
$$+\lambda\left(\left(\frac{x}{x_1}\right)^{\beta} k_1 - \left(\frac{x}{x_2}\right)^{\beta}\Delta\theta\right),$$

where λ denotes the multiplier on the constraint. The first-order conditions with respect to x_1, x_2, k_1, and λ yield:

$$(x_1): \left(\gamma + \theta_1 - \frac{x_1}{r-\mu} + \frac{w}{r} - k_1\right)(-\beta)x_1^{-1} + \left(-\frac{1}{r-\mu}\right) + \frac{\lambda}{p}k_1(-\beta)x_1^{-1} = 0,$$

$$(x_2): \left(\gamma + \theta_2 - \frac{x_2}{r-\mu} + \frac{w}{r}\right)(-\beta)x_2^{-1} + \left(-\frac{1}{r-\mu}\right) - \frac{\lambda}{1-p}(-\beta)x_2^{-1}\Delta\theta = 0,$$

$$(k_1): -p + \lambda = 0,$$

$$(\lambda): k_1 = \left(\frac{x_1}{x_2}\right)^{\beta}\Delta\theta.$$

Rearranging these equations gives solutions $x_1 = x_1^*$, $x_2 = x_3^*$ and $k_1 = (x_1/x_2)^{\beta}\Delta\theta$ where $\theta_3 := \theta_2 - \frac{p}{1-p}\Delta\theta$.

Acknowledgement

I would like to thank Carl Chiarella (University of technology, Sydney), Masaaki Kijima (Kyoto University), Motonari Kurasawa (Yokohama National University), Sofiane Rinaz (Kyoto University) and seminar participants at Kyoto University and Yokohama National University for valuable comments. I would also like to thank an anonymous referee for helpful comments. This research is supported in part by the president's research promotion foundation of Yokohama National University.

References

Bielecki, T. R. and M. Rutkowski (2002) *Credit risk: modeling, valuation and hedging*. Springer.

Brennan, M. J. and E. S. Schwartz (1984) "Valuation of corporate claims", *Journal of Finance* 39: 593–607.

Chiarella, C. (2002) An introduction to derivative security pricing: techniques, methods and applications (Lecture Notes).

Dixit, A. K. and R. S. Pindyck (1994) *Investment under uncertainty*, Princeton Unversity Press.

Fudenberg, D. and J. Tirole (1991) *Game theory*. MIT Press.

Grenadier, S. R. and N. Wang (2005) "Investment timing, agency and information", *Journal of Financial Economics* 75: 493–533.

Kijima, M. (2002) *Stochastic processes with applications to finance*. Chapman & Hall. London.

Harrison, M. J. and D. M. Kreps (1979) "Martingales and arbitrage in multiperiod securities markets", *Journal of Economic Theory* 20: 381–408.

Lando, D. (2004) *Credit risk modeling*. Princeton University Press.

Leland, H. (1994) "Corporate debt value, bond covenants, and optimal capital structure", *Journal of Finance* 49: 1213–1252.

Leland, H. and K. B. Toft (1996) "Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads", *Journal of Finance* 51: 987–1019.

Mæland, J. (2001) "Valuation of an irreversible investment involving agents with private information about stochastic costs", Proceedings in 5th Annual Real Options Conference, Los Angels, 2001.

Mas-Collel, A., M. Whinston and J. R. Green (1995) *Microeconomic theory*. Oxford Student Press.

Mella-Barral, P. and W. Perraudin (1997) "Strategic debt service", *Journal of Finance* 52: 531–556.

Merton, R. C. (1974) "On the pricing of corporate debt: the risk structure of interest rates", *Journal of Finance* 29: 449–470.

Salanié, B. (1997) *The economics of contracts*. MIT Press.