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A Model of Skill Acquisition: An Inquiry into the Effect of Skill Variety on Educational Choice

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While numerous studies have specifically examined the role of education as an investment, a signal, or a consumption, this paper addresses another important aspect of education: it provides people with prospects for their future. Through education, people become able to obtain information about their own abilities and aptitudes, which facilitates their choice of a field in which they can excel. A skill-acquisition mechanism based on this aspect of education is presented, in which people who are uncertain about their educational outcomes are sorted into distinct skill-learning paths. An educational choice depends not only on what one can expect from acquiring a particular skill, but also on the variety of skills and the proximity among skills.

Keywords: skill acquisition mechanism, education under uncertainty, skill variety
JEL Classification Numbers: I21, J24

1. Introduction

Numerous economists have examined the close relationship linking education and economic development. The broad consensus is that an accumulation of human capital is essential for economic development. In fact, many studies have been carried out from exactly that viewpoint, to examine the effects of education on economic development from both theoretical and empirical aspects. For example, Lucas (1988) theoretically described the role of human capital in economic growth in his well-known endogenous growth model. Mankiw, Romer, and Weil (1992) examined cross-country growth differences using a neoclassical growth model in which they have included human capital. Becker, Murphy, and Tamura (1990) show that an increase in the initial stock of human capital tends to raise physical investment, thereby making it easier for developing countries to catch up with the leading economies.

On the other hand, Bils and Klenow (2000), based on a calibration of economic growth model, point out the importance of the reverse causality channel from growth to schooling. They demonstrate that the more that growth is foreseen, the greater its effect on schooling, and the stronger its effect of reverse causality. However, studies in this direction have just begun, and they remain insufficient due, we believe, to the lack of theory for education to address this problem.

In order to address the problem, this paper introduces a fairly new view of education. Whereas the role of education as an investment,[1)], a signal[2)], or a consumption[3)] is well known, this paper demonstrates education has another important aspect: through education, people are informed of their expected outcomes of future education. In the traditional theory for education, it is assumed that the educational outcomes are known in advance. However, in reality, they are unknown because people do not know of their innate abilities or aptitudes in advance, and also because educational outcomes might change by accident. Instead, people obtain information about their own abilities or aptitudes through education, which allows them to predict the outcomes of their future education. Consequently, educated people become able to work in fields where they can perform best. In fact, some elementary school students learn a wide range of subjects such as mathematics, baseball, and piano, not necessarily in the hope of being a mathematician, a professional baseball player, or a pianist, but rather in the hope of finding a field in which they excel; they cannot know such abilities in advance.

Indeed, education under uncertainty is recently attracting more attention. Heckman (2005) stresses the importance of uncertainty in education, which has often been neglected in the precedent literature. Brunello, Giannini and Ariga (2005) try to account for the optimal timing of tracking, given noise in educational outcomes. Our idea of education is more closely related to that of Heckman, Lochner and Todd (2003), who study on the option value of education under uncertainty.

This paper presents a model of a skill-acquisition mechanism that manages this property of education theoretically. The model describes the process wherein people acquire skills, evaluate their innate abilities, and decide their career paths. We allow several types of skill to coexist and the level of each skill to vary among individuals. Given the price of each skill, workers respectively determine the supply of each skill. The demand side of skills is also examined to derive the general equilibrium of the model. We see that the conditional probability obtained through education is necessary for personal determination of a career path.

This paper, then, examines the effect of skill variety on educational demand by numerically analyzing our model. The main implication is that an educational choice depends not only on what one can expect from acquiring a particular skill but also on the variety of skills in a region. We see that economic diversity alters people's preferences for general skills. The value of learning general skills

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[1)] Becker (1964) and Schultz (1963) are well known studies that discuss the role of education as an investment.

[2)] Spence (1973) and Arrow (1973) point out the signaling aspect of education. Stiglitz (1976) discusses the screening aspect of education, which resembles the role of education as a signal.

[3)] See for example, discussion in Gullason (1989).

is demonstrably higher in diversified economies where more jobs are available: to be informed of individual traits has greater value in those countries.

This paper is organized as follows. The mechanism of skill acquisition is presented in Section 2. The interaction of skill choice and skill variety using a numerical analysis on the specific case of the model is examined in Section 3. The conclusion is presented in Section 4.

2. Mechanism of Skill Acquisition

2.1. Supply Side of the Model

We begin by constructing a three-period economic model of skill acquisition. In this economy, L (a constant) individuals are born in every period; each individual lives for three periods. In each period, individuals allocate their time to work or skill acquisition (education). Individuals can acquire a skill only by spending a period of time to learn it. Therefore, to work at a particular job, an individual must first acquire the skill needed for the job. Assume that individuals are risk-neutral, and that the intertemporal discount rate is 0. Under such conditions, individuals are interested only in their expected lifetime income[4)].

Let there be three types of skills: A, B, and C. Presumably, individuals face uncertainty of educational outcomes, in which the productivity of each skill is expressed as a random variable α $(0 \leq \alpha \leq M_A)$, β $(0 \leq \beta \leq M_B)$, and γ $(0 \leq \gamma \leq M_C)$. The prices of respective skills per unit productivity in the labor market are denoted as p_A, p_B, and p_C. At the outset, individuals know the joint density functions of α, β, and γ, i.e., $f(\alpha, \beta, \gamma)$, but not their own values. They will know their own level of skill with certainty only after they have spent time acquiring each skill.

An important point to note is that, as long as α, β, and γ are correlated, knowing the value of α, for example, will allow an individual to infer his productivity in skill B and C. In fact, after learning skill A, that individual will revise his prior distribution over β to a conditional density function $f(\beta|\alpha)$. Therefore, through an acquisition of a certain skill, one obtains prospects for abilities of other skills, which decreases the uncertainty for the future, thereby making the individual more productive.

Let us limit our analysis to a case in which all individuals start their careers by learning skill A[5)]. In the second period, then, each individual has three options. One is to begin a job career in the occupation where skill A is used for inputs. (We call it occupation A. The same applies for occupations B and C.) In this case, the individual receives $p_A\alpha$ (her productivity times the price of skill A) as a wage for the succeeding two periods; thus, earnings are $2p_A\alpha$ for a lifetime. Otherwise, the individual can spend time to acquire either skill B or C during the second period.

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4) It is further assumed later that individuals are not allowed to save, so that capital does not exist in the economy: the interest rate becomes inconsequential.

5) We take a strategy verifying whether this behavior of individuals is justified as the equilibrium state afterward.

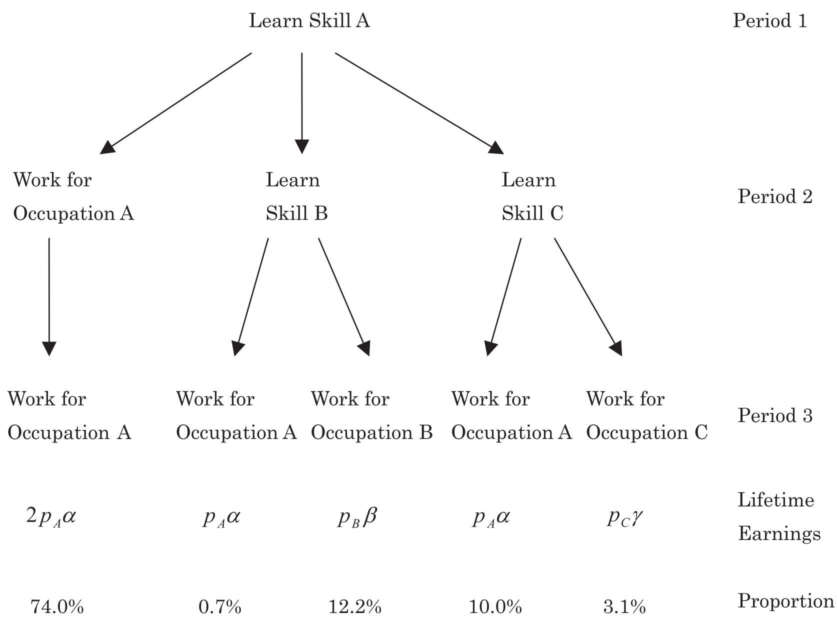

Figure 1 Skill acquisition paths. (The proportional rate in each skill acquisition path is based on the numerical analysis in Section 3.)

Individuals who spend time to acquire skill B (C) in the second period can work at either occupation A or B (C) in the third period. Lifetime earnings are $p_A\alpha$ and $p_B\beta$ ($p_C\gamma$) in each case. Since they only have three periods, individuals never waste time learning a skill in the third period.

Fig. 1 shows a branching diagram for five possible skill acquisition paths. Individuals choose a skill path to take, depending upon their realized value of skills, population distribution of the level of three skills (α, β, γ), and the price vector (p_A, p_B, p_C).

Note that the model can be interpreted in a different way. In reality, firms often determine the task that a worker will perform. The firm often trains workers to acquire a basic skill, evaluates their ability, and assigns them to a department where they seem to perform best. Consequently, this sorting process is also consistent with the allocation choices of a profit-maximizing firm.

We can describe a sorting process by solving the decision-making problem backward. Two nodes require decisions in the third period after learning skill B or C in the second period. After the values of α and β (γ) are realized, the choice that the individual faces in the third period can be described as

$$\max\{p_A\alpha, p_B\beta\} \;\; (\max\{p_A\alpha, p_C\gamma\}\,). \tag{1}$$

If $p_A\alpha$ is greater than $p_B\beta$ ($p_C\gamma$), the individual decides to work at occupation A; otherwise, one works for occupation B (C).

Based on this solution in the third period, each in the second period compares $2p_A\alpha$ to the expected value of learning skill B and skill C, given the realized value of α. Notation V_B and V_C respectively denote the values of learning skill B and skill C. Thus, the choice in the second period can be described as

$$\max\{2p_A\alpha, V_B, V_C\}, \tag{2}$$

where

$$V_B = E(\max\{p_A\alpha, p_B\beta\}|\alpha), \tag{3}$$

and

$$V_C = E(\max\{p_A\alpha, p_C\gamma\}|\alpha). \tag{4}$$

The critical value of α can be found where the decision to work at occupation A is indifferent from the decision to learn skill B by equalizing $2p_A\alpha$ and V_B to get

$$2p_A\alpha = E(\max\{p_A\alpha, p_B\beta\}|\alpha) \tag{5}$$

or

$$2p_A\alpha = \int_0^{\frac{p_A}{p_B}\alpha} p_A\alpha f(\beta|\alpha)d\beta + \int_{\frac{p_A}{p_B}\alpha}^{M_B} p_B\beta f(\beta|\alpha)d\beta. \tag{6}$$

Rearranging eq. (6) using $2p_A\alpha = 2p_A\alpha\Big(\int_0^{\frac{p_A}{p_B}\alpha} f(\beta|\alpha)d\beta + \int_{\frac{p_A}{p_B}\alpha}^{M_B} f(\beta|\alpha)d\beta\Big)$, we obtain

$$\alpha = \int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} (\hat{p}_B\beta - \alpha) f(\beta|\alpha)d\beta, \tag{7}$$

where we used the relative price $\hat{p}_B \equiv p_B/p_A$.

The left side of eq. (7) expresses the cost of learning skill B: the foregone wage that could otherwise have been earned in occupation A. The right side represents the expected benefit of learning skill B. We can calculate eq. (7) further using the integration-by-parts formula to get

$$\alpha = \frac{1}{2}\left(\hat{p}_B M_B - \int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} \hat{p}_B F(\beta|\alpha)d\beta\right), \tag{8}$$

where we used $F(\beta|\alpha)$, which stands for the distribution function, and the fact that $F(M_B|\alpha) = 1$. Similarly, we can obtain

$$\alpha = \frac{1}{2}\left(\hat{p}_C M_C - \int_{\frac{1}{\hat{p}_C}\alpha}^{M_C} \hat{p}_C F(\gamma|\alpha)d\gamma\right) \tag{9}$$

by equalizing $2p_A\alpha$ and V_C, where we again used the relative price $\hat{p}_C \equiv p_C/p_A$. Let us respectively define the functions on the right sides of eq. (8) and (9) as $\hat{V}_B(\alpha)$ and $\hat{V}_C(\alpha)$.

While the shape of $\hat{V}_B(\alpha)$ depends on how skills A and B are correlated, we can ensure the existence of a solution in eq. (8), which we summarize in the following proposition.

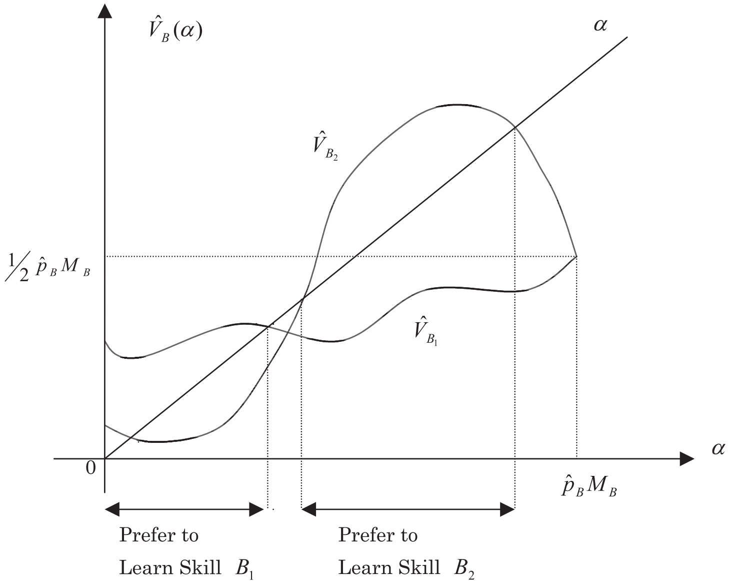

Figure 2 $\hat{V}_B(\alpha)$ and the 45° line.

Proposition 2.1. *If $F(\beta|\alpha)$ is continuous, there exists at least one solution in eq.* (8) *in the interval* $[0, \hat{p}_B M_B]$.

Proof. Since

$$\hat{V}_B(\hat{p}_B M_B) = \frac{1}{2}\hat{p}_B M_B, \quad \textit{and}$$

$$\begin{aligned} \hat{V}_B(0) &= \frac{1}{2}\left(\hat{p}_B M_B - \int_0^{M_B} \hat{p}_B F(\beta|0)d\beta\right) \\ &\geq \frac{1}{2}\left(\hat{p}_B M_B - \int_0^{M_B} \hat{p}_B d\beta\right) = 0, \end{aligned}$$

the continuity of $\hat{V}_B(\alpha)$ guarantees at least one solution of eq. (8). □

The same is true of the existence of a solution in eq. (9).

The 45° line and two types of $\hat{V}_B(\alpha)$ are drawn in Fig. 2 ($\hat{V}_{B_1}(\alpha)$ and $\hat{V}_{B_2}(\alpha)$). Because the 45° line and $\hat{V}_B(\alpha)$ respectively represent the cost and the benefit of learning skill B, individuals will decide to learn skill B when $\hat{V}_B(\alpha)$ is above the 45° line. Otherwise, they will decide to work at occupation A. Note that the form of $\hat{V}_B(\alpha)$ is an important factor to characterize the skill. In Fig. 2, skill B_1 is relatively elementary, which is preferred by people with a low value of skill A. On the other hand, people who have acquired a high value of skill A tend to prefer skill B_2, which can be considered to be rather sophisticated. Consequently, we can make sure that to learn skill A not only enables individuals to use the skill; it also provides them with information about their prospects for other skills.

Using the following definitions of notations,

$$\begin{aligned}
\Delta_A &\equiv \{\alpha | \alpha \geq \hat{V}_B(\alpha), \alpha \geq \hat{V}_C(\alpha), \alpha \in [0, M_A]\}, \\
\Delta_B &\equiv \{\alpha | \hat{V}_B(\alpha) \geq \alpha, \hat{V}_B(\alpha) \geq \hat{V}_C(\alpha), \alpha \in [0, M_A]\}, \\
\Delta_C &\equiv \{\alpha | \hat{V}_C(\alpha) \geq \alpha, \hat{V}_C(\alpha) \geq \hat{V}_B(\alpha), \alpha \in [0, M_A]\},
\end{aligned}$$

the decisions of individuals in the second period and in the third period are summarized as the following lemmas.

Lemma 2.1. *The solution of the decision-making problem that individuals face in the second period, given the realized value of α, is as follows*

(i). *If $\alpha \in \Delta_A$, individuals decide to work at occupation A.*

(ii). *If $\alpha \in \Delta_B$, individuals decide to learn skill B.*

(iii). *If $\alpha \in \Delta_C$, individuals decide to learn skill C.*

Lemma 2.2. *The sorting process in the third period, after learning skill B or C, is described as follows.*

(i). *Individuals who learn skill B in the second period will decide to work at occupation B if $\hat{p}_B\beta \geq \alpha$; otherwise, they decide to work at occupation A.*

(ii). *Individuals who learn skill C in the second period will decide to work at occupation C if $\hat{p}_C\gamma \geq \alpha$; otherwise, they decide to work at occupation A.*

The notations L_A, L_B, and L_C are used to denote the people working respectively at occupations A, B, and C, whereas T_B and T_C refer respectively to the people trained to acquire skills B and C. Using Lemmas 2.1 and 2.2, and the marginal density function $f(\alpha)$, they can be described as follows.

$$\begin{aligned}
L_A &= 2L \int_{\Delta_A} f(\alpha) d\alpha + L \int_{\Delta_B} f(\alpha) \int_0^{\frac{1}{\hat{p}_B}\alpha} f(\beta|\alpha) d\beta d\alpha \\
&\quad + L \int_{\Delta_C} f(\alpha) \int_0^{\frac{1}{\hat{p}_C}\alpha} f(\gamma|\alpha) d\gamma d\alpha \qquad (10) \\
L_B &= L \int_{\Delta_B} f(\alpha) \int_{\frac{1}{\hat{p}_B}\alpha}^{M_B} f(\beta|\alpha) d\beta d\alpha \qquad (11) \\
L_C &= L \int_{\Delta_C} f(\alpha) \int_{\frac{1}{\hat{p}_C}\alpha}^{M_C} f(\gamma|\alpha) d\gamma d\alpha \qquad (12) \\
T_B &= L \int_{\Delta_B} f(\alpha) d\alpha \qquad (13) \\
T_C &= L \int_{\Delta_C} f(\alpha) d\alpha. \qquad (14)
\end{aligned}$$

Likewise, the aggregate supply of each skill can be determined as

$$\begin{aligned} S_A &= 2L\int_{\Delta_A}\alpha f(\alpha)d\alpha + L\int_{\Delta_B}\alpha f(\alpha)\int_0^{\frac{1}{\hat{p}_B}\alpha} f(\beta|\alpha)d\beta d\alpha \\ &\quad + L\int_{\Delta_C}\alpha f(\alpha)\int_0^{\frac{1}{\hat{p}_C}\alpha} f(\gamma|\alpha)d\gamma d\alpha, \end{aligned} \quad (15)$$

$$S_B = L\int_{\Delta_B} f(\alpha)\int_{\frac{1}{\hat{p}_B}\alpha}^{M_B}\beta f(\beta|\alpha)d\beta d\alpha, \quad (16)$$

$$S_C = L\int_{\Delta_C} f(\alpha)\int_{\frac{1}{\hat{p}_C}\alpha}^{M_C}\gamma f(\gamma|\alpha)d\gamma d\alpha. \quad (17)$$

2.2. Demand Side of the Model

The aggregate demand for skills is now determined by solving firms' cost-minimization problem. It is assumed for simplicity that, in each occupation, skills are transformed into middle-sector goods using one-to-one technology. The production in each occupation is denoted as m_A, m_B, and m_C. Each occupation is assumed to be completely competitive so that the price of goods produced by that occupation is equal to the price of a skill used for their inputs. In this economy, it is also assumed that only one final good exists, whose inputs are the three middle-sector goods. Final goods are consumed immediately by individuals whose utility functions are linear to them and who are not allowed to save their earnings or products. Firms that have identical production functions produce final goods:

$$Y = m_A^{\mu} m_B^{\nu} m_C^{1-\mu-\nu}. \quad (18)$$

The optimal behavior of each firm is to minimize costs for any value of Y. We can determine the demand of firm i for each good in each period, which we denote as D_A^i, D_B^i, and D_C^i by solving

$$\begin{aligned} &\min_{m_A, m_B, m_C} \; p_A m_A(i) + p_B m_B(i) + p_C m_C(i) \\ &s.t.\;\; Y(i) = m_A(i)^{\mu} m_B(i)^{\nu} m_C(i)^{1-\mu-\nu}. \end{aligned} \quad (19)$$

Consequently, the demand for each good is given simply as

$$D_A^i = \lambda^i \frac{\mu}{p_A} Y(i) \quad (20)$$

$$D_B^i = \lambda^i \frac{\nu}{p_B} Y(i) \quad (21)$$

$$D_C^i = \lambda^i \frac{1-\mu-\nu}{p_C} Y(i), \quad (22)$$

where each firm assigns the same proportion of its sale (shadow price λ^i times output $Y(i)$) to each middle-sector good.

2.3. General Equilibrium

Finally, the general equilibrium of the model can be derived based on a supply and demand analysis. Using eqs. (15) ~ (17) and eqs. (20) ~ (22), the general equilibrium of the economy is determined via the following proposition.

Proposition 2.2. $(\hat{p}_B^*, \hat{p}_C^*)$ *is the relative price vector of the general equilibrium in the economy if and only if it satisfies the following simultaneous equations:*

$$\hat{p}_B^* = \frac{\nu}{\mu}\frac{S_A(\hat{p}_B^*, \hat{p}_C^*)}{S_B(\hat{p}_B^*, \hat{p}_C^*)} \equiv \frac{\nu}{\mu}\frac{S_A^*}{S_B^*} \tag{23}$$

and

$$\hat{p}_C^* = \frac{1-\mu-\nu}{\mu}\frac{S_A(\hat{p}_B^*, \hat{p}_C^*)}{S_C(\hat{p}_B^*, \hat{p}_C^*)} \equiv \frac{1-\mu-\nu}{\mu}\frac{S_A^*}{S_C^*}. \tag{24}$$

Proof. (necessity condition) In equilibrium, the aggregate demand of each skill must be equal to the aggregate skill used for inputs in each occupation in each period. Therefore, we get

$$D_A = \sum_i D_A^i = \sum_i \lambda^i \frac{\mu}{p_A} Y(i) = S_A \tag{25}$$

$$D_B = \sum_i D_B^i = \sum_i \lambda^i \frac{\nu}{p_B} Y(i) = S_B \tag{26}$$

$$D_C = \sum_i D_C^i = \sum_i \lambda^i \frac{1-\mu-\nu}{p_C} Y(i) = S_C. \tag{27}$$

Now, we can obtain eq. (23) using eqs. (25) and (26), while eq. (24) can be derived using eqs. (25) and (27).
(sufficient condition) Because $\sum_i \lambda^i Y(i) = p_A D_A + p_B D_B + p_C D_C$, it can easily be shown that $D_A(\hat{p}_B^*, \hat{p}_C^*) = S_A^*$, $D_B(\hat{p}_B^*, \hat{p}_C^*) = S_B^*$, and $D_C(\hat{p}_B^*, \hat{p}_C^*) = S_C^*$ hold under the equilibrium price using eqs. (23) and (24). □

Note that no unemployment exists in this economy because individuals who want to work can always earn marginal value products, i.e., the price of a skill times the amount of skill offered. For that reason, the labor market-clearing condition is also satisfied in equilibrium.

3. Skill Variety and Education

As an empirical application, we now examine the effect of skill variety on educational choice by numerically analyzing a somewhat simple version of the model presented in the previous section. The main implication is that a choice of skills is relevant to their variety and the proximity among them, essentially because of the property that education presents a learner with prospects for the future.

Let us simplify the model such that β and γ each take only two values, meaning that individuals have only two results of learning skill B and C: Individuals either succeed in acquiring skill B (C) to gain productivity M_B (M_C) with probability $1 - u(\alpha)$ ($1 - v(\alpha)$) or fail to acquire skill B (C) and end up with productivity 0 with probability $u(\alpha)$ ($v(\alpha)$).

Two reasons exist for setting these as functions. One is to be able to analyze the general equilibrium numerically; the other is to clarify that skill B and C are rather specialized (or risky) skills compared to skill A, which we suppose to be a rather general skill. The joint distribution of skill productivities is, then, specified as follows.

$$f(\alpha,\beta,\gamma) = \begin{cases} u(\alpha)v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,0) \\ (1-u(\alpha))\,v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,0) \\ u(\alpha)\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,M_C) \\ (1-u(\alpha))\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,M_C) \\ 0 & \text{elsewhere} \end{cases}, \tag{28}$$

where we suppose $u(\alpha)$ to be linear, $v(\alpha)$ to be concave, and $\hat{f}(\alpha)$ to be a truncated normal distribution (so that α is guaranteed to take values in $[0, M_A]$). All functional forms and values of parameters used for numerical analyses are summarized in the Appendix.

It is easy to confirm that

$$f(\alpha) = \sum_{\gamma}\sum_{\beta} f(\alpha,\beta,\gamma) = \hat{f}(\alpha)$$

and

$$f(\beta|\alpha) = \begin{cases} u(\alpha) & (\beta = 0) \\ 1-u(\alpha) & (\beta = M_B) \end{cases} \tag{29}$$

$$f(\gamma|\alpha) = \begin{cases} v(\alpha) & (\gamma = 0) \\ 1-v(\alpha) & (\gamma = M_C) \end{cases} \tag{30}$$

It is also noteworthy that the concave property of function $v(\alpha)$ characterizes skill C as a more sophisticated skill, which requires a certain degree of basic ability to acquire. On the contrary, linearity of function $u(\alpha)$ characterizes skill B as simpler and easier to acquire, even for individuals with less α[6].

Table 1 presents the results that describe the equilibrium state of this economy. The equilibrium skill price of the model (solution of simultaneous eqs. (23) and (24)) along with Δ_A, Δ_B and Δ_C completely describe the process of how individuals are sorted into distinct skill-learning paths. In addition, a proportion of L_A, L_B, L_C, T_B, and T_C within the labor force population (i.e. 2L) is calculable using eqs. (10)

[6] It also characterizes two other minor properties. (1) Both $u(\alpha)$ and $v(\alpha)$ are decreasing functions of α: Skill A and B (C) are positively correlated such that an individual with a high ability of skill A is more likely to succeed in acquiring skill B (C). (2) $u(0) = v(0) = 1$, $\exists b, c,\ u(b) = v(c) = 0$: An individual whose value of skill A is 0 always fails to acquire skill B (C), while an individual who has a value of skill A larger than b (c) will surely succeed in acquiring skill B (C).

Table 1 Equilibrium state of the economy

Interval for the choice in the first period		
Δ_A	[2.08, 5.25]	
Δ_B	[0, 2.08]	
Δ_C	[5.25, 8]	
Equilibrium skill price		
Skill B ($\hat{p}_B^*$)	1.07	
Skill C ($\hat{p}_C^*$)	2.67	
Proportion within the labor force population		
Occupation A	79.3%	
Occupation B	6.1%	
Occupation C	1.5%	
Learning B	6.5%	
Learning C	6.5%	
Average wage		
	Market-observed	Latent
Occupation A	4.1	4.0
Occupation B	4.3	4.2
Occupation C	53	2.1

$\sim$ (14). The proportion of the population sorted into each skill acquisition path is as shown in the bottom of Fig. 1.

The market-observed average wage and the latent average wage for a worker in each occupation are also collected in Table 1. The market-observed average wage refers to the wage that can be seen in the equilibrium market (i.e., E_A, $\hat{p}_B^* M_B$ and $\hat{p}_C^* M_C$, respectively), while the latent average wage refers to the expected wage of the individual if he or she were to work for each occupation after learning a corresponding skill. One noteworthy point is the existence of inter-job wage differences in this economy. Average wages in occupation A and B are observed to be far below the wage that workers in occupation C earn. This inequality essentially stems from both the differences in ability among individuals and the sorting mechanism in the economy. Workers in occupation C earn much more than workers in occupations A or B, not because they happened to choose to work at that particular job, but because they were qualified to learn skill C and succeeded in acquiring the skill because they had sufficiently high aptitude[7)].

Finally, we can ensure that the behaviors of individuals in their respective first

[7)] Roy (1951) is a classical study that presents insight into how some jobs are superior and attract superior laborers, whereas other jobs are inferior and expand inter-job wage differences when abilities of skills are positively correlated.

periods are also equilibrium strategies by showing that no individual has an incentive to deviate from this state, that is, to start their careers by learning either skill B or C instead. This is done by comparing the expected lifetime income in our model where all individuals start learning from skill A and that where an individual deviates from the situation and starts learning from either skill B or C. We refer to those respectively as $E(I_A)$,$E(I_B^{DEV})$, and $E(I_C^{DEV})$.

It is clear that

$$E(I_A) = \frac{1}{L}\left(S_A^* + \hat{p}_B^* S_B^* + \hat{p}_C^* S_C^*\right). \tag{31}$$

By following the same procedure in the previous section, $E(I_B^{DEV})$ and $E(I_C^{DEV})$ can be verified and expressed as

$$E(I_B^{DEV}) = \max\{I_{BA}, I_{BC}\} \tag{32}$$
$$E(I_C^{DEV}) = \max\{I_{CA}, I_{CB}\} \tag{33}$$

where

$$I_{BA} = \int_0^{M_A} \alpha u(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_A} \hat{p}_B^* M_B\left(1 - u(\alpha)\right)\hat{f}(\alpha)d\alpha, \tag{34}$$

$$\begin{aligned} I_{BC} &= \int_0^{M_A} \hat{p}_C^* M_C\left(1 - v(\alpha)\right)u(\alpha)\hat{f}(\alpha)d\alpha \\ &\quad + 2\int_0^{M_A} \hat{p}_B^* M_B\left(1 - u(\alpha)\right)\hat{f}(\alpha)d\alpha, \end{aligned} \tag{35}$$

$$I_{CA} = \int_0^{M_A} \alpha v(\alpha)\hat{f}(\alpha)d\alpha + 2\int_0^{M_A} \hat{p}_C^* M_C\left(1 - v(\alpha)\right)\hat{f}(\alpha)d\alpha, \tag{36}$$

$$\begin{aligned} I_{CB} &= \int_0^{M_A} \hat{p}_B^* M_B\left(1 - u(\alpha)\right)v(\alpha)\hat{f}(\alpha)d\alpha \\ &\quad + 2\int_0^{M_A} \hat{p}_C^* M_C\left(1 - v(\alpha)\right)\hat{f}(\alpha)d\alpha. \end{aligned} \tag{37}$$

Table 2 (third column) shows the results, where $E(I_A)$ has the largest value. Those results justify our inference that the behavior for all individuals to start from learning skill A is an equilibrium strategy in this case.

Table 2 Expected lifetime income in two-skill and in three-skill economies

	2-skill economy	3-skill economy
$E(I_A)$	8.16	8.72
$E(I_B^{DEV})$	8.87	8.50
$E(I_C^{DEV})$	–	8.61

We now turn to demonstrate that the skill availability, i.e., the number of skills that can be acquired in a country, is an important factor for the preference that individuals hold with regard to skills. It can be shown that the existence of skill C can change people's preferences in their first period for the acquisition between skill A and B by comparing our three-skill model to an economy with only two skills: A and B. The economic model with two skills can be analyzed just as same as the economy with three skills[8)]. Those results are shown in Table 2 (second column): people in a two-skill economy have an incentive to not start learning from skill A, but instead, to start their careers by learning skill B.

The preference relation of individuals between skills A and B in the first period has changed because information on the prospects for their future that skill A provides is richer than that provided by skill B in a three-skill economy rather than in a two-skill economy because skill C is more correlated to skill A than skill B. This adds more value as an option in learning skill A in a three-skill economy: learning skill A is more valuable. Generally, the more skills there are, the greater the value of learning general skills, as far as those skills have a correlation with many other skills, providing information about which skill to learn next. This result depends greatly on parameter values. However, an important point to notice is that the value of education depends not only on the expected outcome of that education, but also on the variety of other available educations and on the correlations of their expected outcomes. People might prefer to learn rather specific skills if only two skills exist, whereas they might prefer to learn more general skills if there were more than three skills. We summarize this notion in the following proposition.

Proposition 3.1. *Skill A might be preferred to skill B in a three-skill region, whereas skill B might be preferred to skill A in a two-skill region.*

It is noteworthy that this proposition suggests the possibility of differences in cross-country economic development. Some countries might develop more rapidly because people in such countries are more likely to acquire skills that are effective for economic development; the opposite applies for other countries. As far as we can believe that primary education (which is, by definition, broadly correlated to other levels of education) is very important for economic development[9)], our framework provides a conceptualization and a mechanism showing why some countries

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8) To be exact, some devices in eq. (18) are necessary for the analysis to be feasible. We presume another production technology by which skill A and B are the only inputs and where firms can choose whichever technology they want to use. Alternatively, we might presume that firms can obtain m_C with fixed cost $\bar{p}_C$ outside the country, in both cases, maintaining the relation that skills A and B are demanded in the same ratio ($\frac{\nu}{\mu}$).

9) Although few studies that support this view exist, Haveman and Wolfe (1984) examine some educational externalities that might have resulted from primary education. Some empirical studies have reported that secondary education matters more for growth than primary education (e.g. Krueger and Lindahl, 2001), but because primary education provides people with knowledge for skills or jobs that one would be most suited for, the estimated effects of primary and secondary education on economic growth would be biased.

remain undeveloped. As in our model, in less developed countries, where less variety of skills pertains, people might well tend to choose risky skills, preventing the country from sound development.

4. Conclusion

A model is presented that describes a mechanism of skill acquisition. The role of education, as a provider of future prospects to individuals, is expressed in the model theoretically. Individuals who are uncertain of their educational outcomes are sorted into distinct skill acquisition paths because of this property. In the latter part of this paper, we demonstrated that our theoretical framework makes it possible for us to understand the effect of skill variety on educational choice. It has been implied that people in a region where few skills are available have a tendency not to learn general skills since they have less incentive to obtain information about their innate abilities. However, the measurement of quantitative effects of diversity on education would be difficult, and it would be an appropriate goal for further study.

5. Appendix

This Appendix summarizes the functional forms and values of parameters used for numerical analyses in Section 3.

Functional forms used for numerical analyses are

$$f(\alpha,\beta,\gamma) = \begin{cases} u(\alpha)v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,0) \\ (1-u(\alpha))\,v(\alpha)\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,0) \\ u(\alpha)\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (0,M_C) \\ (1-u(\alpha))\,(1-v(\alpha))\,\hat{f}(\alpha) & \text{if} \quad (\beta,\gamma) = (M_B,M_C) \\ 0 & \text{elsewhere} \end{cases}, \tag{38}$$

$$\hat{f}(\alpha) = \frac{\phi(\alpha) - \phi(0)}{\int_0^{M_A} \phi(\xi)d\xi - M_A\phi(0)} \tag{39}$$

where $\phi(\alpha)$ is the density function of the normal distribution with a mean m and standard deviation σ, and

$$u(\alpha) = \begin{cases} -\frac{1}{b}\alpha + 1 & (0 \le \alpha \le b) \\ 0 & (\alpha \ge b) \end{cases} \tag{40}$$

$$v(\alpha) = \begin{cases} -\frac{1}{c^2}\alpha^2 + 1 & (0 \le \alpha \le c) \\ 0 & (\alpha \ge c) \end{cases}. \tag{41}$$

Values of parameters used for numerical analyses are as follows.

$M_A = 8$	$b = 1$	$m = 4$	$\nu/\mu = 0.08$
$M_B = 4$	$c = 10$	$\sigma = 2$	$(1 - \mu - \nu)/\mu = 0.25$
$M_C = 20$	$L = 1$		

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