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ON THE HOLOMORPHIC SOLUTION OF NONLINEAR FIRST ORDER EQUATIONS WITH SEVERAL SPACE VARIABLES

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ABSTRACT. We will establish the existence and uniqueness of the holomorphic solution of the nonlinear first order partial differential equation

$$t\frac{\partial u}{\partial t} = F\Big(t, x_1, \ldots, x_n, u, \frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_n}\Big).$$

Chen and Tahara [4, 5] asserted this fact in the case when the space variable x is one-dimensional. Chen and Luo [2], and Shirai [12] offered nontrivial generalizations to several space variables. This paper offers yet another nontrivial generalization using a different tool to prove the convergence of the formal solution.

1. INTRODUCTION AND MAIN RESULT

Consider the nonlinear nonlinear singular partial differential equation

$$\text{(E)} \qquad t\frac{\partial u}{\partial t} = F\Big(t, x_1, \ldots, x_n, u, \frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_n}\Big)$$

with independent variables $(t, x) = (t, x_1, ..., x_n) \in \mathbb{C}_t \times \mathbb{C}_x^n$. The function $F(t, x, u, v)$ is assumed to be holomoprhic in a neighborhood of the origin $(0, 0, 0, 0) \in \mathbb{C}_t \times \mathbb{C}_x^n \times \mathbb{C}_u \times \mathbb{C}_v^n$ and satisfies

$$F(0, x, 0, 0) \equiv 0 \qquad \text{near } x = 0.$$

Hence, near the origin, we have the expansion

$$F(t, x, u, v) = a(x)\, t + b(x)\, u + \sum_{j=0}^{n} c_j(x)\, v_j + \sum_{p+q+|\mu|\geq 2} a_{p,q,\mu}(x)\, t^p u^q v_1^{\mu_1} \cdots v_n^{\mu_n}.$$

We may then focus our attention on the coefficients $c_j(x)$ and consider several cases. If each $c_j(x)$ vanishes identically near the origin, then (E) is called a *nonlinear Fuchs type equation* (since its linear part is a PDE of Fuchs type) or a *Briot-Bouquet type equation* (since it is one possible generalization into PDEs of the ODE studied by Briot and Bouquet). This case was studied quite thoroughly by Gérard and Tahara (see for example [6, 7, 8]) in the early 1990s. However if for some $1 \leq j \leq n$ we have a nonzero $c_j(0)$, then we can solve the equation (E) for $\partial u/\partial x_j$ and invoke the Cauchy-Kowalevsky Theorem to assert the existence of a unique holomorphic solution $u(t, x)$ satisfying $u(0, x) \equiv 0$ and $u(t, 0) \equiv 0$. Hence this second possibility is not so interesting.

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It now remains to consider the third case, namely, when each $c_j(0)$ is equal to zero but $c_j(x)$ is not identically equal to zero. In this case, Chen and Tahara [4, 5] called equation (E) a *nonlinear equation of totally characteristic type*. They established the unique existence of the solution for the case of a one-dimensional space variable x and an indicial operator of regular singularity, and under a non-resonance condition. Here is their result.

Theorem 1.1 (Chen-Tahara). *Suppose the space variable x is of one dimension and $c(x) := c_1(x) = x\gamma(x)$ with $\gamma(0) \neq 0$. If there exists a $\sigma > 0$ such that for all $(k, l) \in \mathbb{N}^* \times \mathbb{N}$, we have*

$$|k - b(0) - l\gamma(0)| \geq \sigma(k + l + 1),$$

then (E) *has a unique holomorphic solution $u(t, x)$ satisfying $u(0, x) \equiv 0$.*

It must be noted that there is a big gap between the case when $c(x) = x\gamma(x)$ and when $c(x) = x^p\gamma(x)$, where $p \geq 2$. In the latter case, the indicial operator has irregular singularity and the formal series solution is in general not convergent. (The interested reader is referred to the paper of Chen, Luo and Tahara [3].)

Chen and Luo later gave the following nontrivial extension of the above theorem to the case when the space variable x is multi-dimensional.

Theorem 1.2 (Chen-Luo). *Suppose that for each j, $c_j(x) = x_j\gamma_j(x)$ with $\gamma_j(0) \neq 0$. If there exists a $\sigma > 0$ such that for all $(k, \mu) \in \mathbb{N}^* \times \mathbb{N}^n$, we have*

$$\left|k - b(0) - \sum_{j=0}^{n} \mu_j\gamma_j(0)\right| \geq \sigma(k + |\mu| + 1),$$

then (E) *has a unique holomorphic solution $u(t, x)$ satisfying $u(0, x) \equiv 0$.*

Shirai further extended this result to several time-space variables. Applied to the equation being considered, but keeping time one-dimensional, his result gives the following.

Theorem 1.3 (Shirai). *Suppose that $c_j(0) = 0$ for each j, and let $\gamma_1, \ldots, \gamma_n$ be the eigenvalues of the matrix $[(\partial c_j/\partial x_i)(0)]$. If there exists a $\sigma > 0$ such that for all $(k, \mu) \in \mathbb{N}^* \times \mathbb{N}^n$, we have*

$$\left|k - b(0) - \sum_{j=0}^{n} \mu_j\gamma_j(0)\right| \geq \sigma(k + |\mu| + 1),$$

then (E) *has a unique holomorphic solution $u(t, x)$ satisfying $u(0, x) \equiv 0$.*

Note that in the generalizations of Chen-Luo and Shirai, the Poincaré condition forces all $\gamma_j(0)$'s to be nonzero.

This paper presents another nontrivial extension of Theorem 1.1 to the case of several space variables. We will employ another method of proof and thus come up with an alternative proof of Theorem 1.1. The following is our main result.

Theorem 1.4. *Suppose that for each j, $c_j(x) = x_1\gamma_j(x)$ with $\gamma_1(0) \neq 0$. If there exists a $\sigma > 0$ such that for all $(k, \mu) \in \mathbb{N}^* \times \mathbb{N}^n$, we have*

$$|k - b(0) - \mu_1\gamma_1(0)| \geq \sigma(k + \mu_1 + 1),$$

then (E) *has a unique holomorphic solution $u(t, x)$ satisfying $u(0, x) \equiv 0$.*

Note that the current setup allows the possibility for some $\gamma_j(0)$'s to be zero; in fact, the $c_j(x)$'s may be, with the exception of course of $c_1(x)$, identically zero.

2. Proof of Main Result

We will make use of a family of majorant functions to establish the convergence of the formal power series solution of (E). This family is a modified version of the one used by Lax [10].

For each nonnegative integer i, we define the function

$$\varphi_i(z) = \frac{1}{4S}\sum_{k=0}^{\infty}\frac{z^k}{(k+1)^{2+i}}.$$

Here, the constant S is equal to $\pi^2/6\,(= 1 + 1/4 + 1/9 + \cdots)$, and was introduced by Tahara to greatly facilitate computations. (Kobayashi [9] also used this type of majorant function but he did not make use of the constant S. The interested reader can compare how computations are greatly simplified by the mere addition of this constant in the definition of the majorant function.) It is easy to check that the series converges and thus defines a holomorphic function in the domain $\{z \in \mathbb{C};\ |z| < 1\}$.

These majorant functions satisfy interesting majorant relations[1] that are rather easy to verify. We state them without proof.

Proposition 2.1. *The following hold for any nonnegative integer i:*

(a) $\varphi_i(z)\varphi_i(z) \ll 2^i\varphi_i(z)$,

(b) $\varphi_{i+1}(z) \ll \varphi_i(z)$,

(c) $z\varphi_i(z) \ll 2^{2+i}\varphi_i(z)$,

(d) $\left(\frac{1}{2}\right)^{2+i}\varphi_i(z) \ll \varphi_{i+1}'(z) \ll \varphi_i(z)$.

The following proposition provides a majorant for the product of a holomorphic function and one of the functions in this family. We also omit the easy proof. (See Lope-Tahara [11].)

Proposition 2.2. *Let $0 < \varepsilon < 1$ and let i be a nonnegative integer. Then there exists a constant $C_{i,\varepsilon} > 0$ such that*

$$\frac{1}{1-\varepsilon z}\,\varphi_i(z) \ll C_{i,\varepsilon}\varphi_i(z).$$

Let us now prove Theorem 1.4. From our assumptions, equation (E) can we rewritten as

$$(\mathrm{E}') \qquad t\frac{\partial u}{\partial t} = a(x)t + b(x)u + \sum_{j=1}^{n} x_1\gamma_j(x)\frac{\partial u}{\partial x_j} + \sum_{p+q+|\mu|\ge 2} a_{p,q,\mu}(x)\, t^p\, u^q \left(\frac{\partial u}{\partial x_1}\right)^{\mu_1}\cdots\left(\frac{\partial u}{\partial x_n}\right)^{\mu_n}.$$

[1] We will follow the usual notation to express majorant relations, that is, we will write $\sum a_\alpha x^\alpha \ll \sum A_\alpha x^\alpha$ to mean $|a_\alpha| \le A_\alpha$ for all $\alpha \in \mathbb{N}^n$.

Since we are interested in solutions that satisfy $u(0,x) \equiv 0$, we now assume a formal solution of the form $\sum_{k=1}^{\infty} u_k(x)t^k$. Substituting this formal series into (E′) and comparing the coefficients of equal powers of t, we see that the coefficients $u_k(x)$ must satisfy

$$\gamma_1(x)x_1\frac{\partial u_1}{\partial x_1} - [1-b(x)]u_1 + x_1\sum_{j=2}^{n}\gamma_j(x)\frac{\partial u_1}{\partial x_j} = -a(x)$$

and for $k \geq 2$,

$$\gamma_1(x)x_1\frac{\partial u_k}{\partial x_1} - [1-b(x)]u_k + x_1\sum_{j=2}^{n}\gamma_j(x)\frac{\partial u_k}{\partial x_j} = H_k(x),$$

where $H_k(x)$ is a function of the previous coefficients $u_1(x), \ldots, u_{k-1}(x)$ and of their first derivatives.

The above equations are first order linear Fuchsian partial differential equations and have formal solutions $u_k(x)$ provided

$$k - b(0) - l\gamma_1(0) \neq 0 \quad \text{for all } (k,l) \in \mathbb{N}^* \times \mathbb{N}.$$

The Poincaré condition assumed in our theorem guarantees that the above holds. We are therefore left to show that the formal sum $\sum_{k=1}^{\infty} u_k(x)t^k$ is indeed convergent. To do so, we will prove the existence of a holomorphic function that majorizes the formal solution obtained above.

Suppose that the function $F(t,x,u,v)$ is holomorphic in a neighborhood of $\{(t,x,u,v) \in \mathbb{C}_t \times \mathbb{C}_x^n \times \mathbb{C}_u \times \mathbb{C}_v^n;\ |t| \leq r_0,\ |x_j| \leq R,\ |u| \leq \rho \text{ and } |v| \leq \rho\}$, and is bounded there by some constant M. Then in this neighborhood, we have the following bounds for the coefficients of the partial Taylor expansion of F:

$$|a(x)| \leq \frac{M}{r_0}, \quad |b(x)| \leq \frac{M}{\rho}, \quad |\gamma_j(x)| \leq \frac{M}{\rho R_0} \quad \text{and} \quad |a_{p,q,\mu}(x)| \leq \frac{M}{r_0^p\, \rho^{q+|\mu|}}.$$

Hence, setting $\psi(x) = \left(1 - (x_1 + \cdots + x_n)/R_0\right)^{-1}$, we obviously have

$$a(x) \ll \frac{M}{r_0}\psi(x), \qquad b(x) \ll \frac{M}{\rho}\psi(x),$$

$$\gamma_j(x) \ll \frac{M}{\rho R_0}\psi(x) \qquad \text{and} \qquad a_{p,q,\mu}(x) \ll \frac{M}{r_0^p\, \rho^{q+|\mu|}}\psi(x).$$

For any function $g(x)$, let us denote by $\widetilde{g}(x)$ the function $g(x) - g(0)$. Using this notation, we can now rewrite (E′) as

$$\text{(E}''\text{)} \quad t\frac{\partial u}{\partial t} - b(0)u - \gamma_1(0)x_1\frac{\partial u}{\partial x_1}$$
$$= a(x)t + \widetilde{b}(x)u + \widetilde{\gamma}_1(x)x_1\frac{\partial u}{\partial x_1} + \sum_{j=2}^{n} x_1\gamma_j(x)\frac{\partial u}{\partial x_j}$$
$$+ \sum_{p+q+|\mu|\geq 2} a_{p,q,\mu}(x)\, t^p\, u^q \left(\frac{\partial u}{\partial x_1}\right)^{\mu_1} \cdots \left(\frac{\partial u}{\partial x_n}\right)^{\mu_n}.$$

In view of this and of the Poincaré condition of Theorem 1.4, we see that any $w(t,x)$ satisfying the following set of relations is a majorant of the formal solution:

$$
\text{(M)}\quad
\begin{cases}
\sigma\Big(t\dfrac{\partial w}{\partial t}+x_1\dfrac{\partial w}{\partial x_1}+w\Big) \gg \dfrac{Mt}{r_0}\psi(x)+\dfrac{Mw}{\rho}\widetilde{\psi}(x)+\dfrac{M\widetilde{\psi}(x)}{\rho R_0}\Big(x_1\dfrac{\partial w}{\partial x_1}\Big) \\
\qquad\qquad + \displaystyle\sum_{j=2}^{n}\frac{Mx_1\psi(x)}{\rho R_0}\frac{\partial w}{\partial x_j} \\
\qquad\qquad + \displaystyle\sum_{p+q+|\mu|\geq 2}\frac{M\psi(x)}{r_0^p\rho^{q+|\mu|}}t^p w^q\Big(\frac{\partial w}{\partial x}\Big)^{\mu}, \\
w(0,x)\equiv 0.
\end{cases}
$$

We claim that for a suitably chosen set of constants L, c, η, r and R, the holomorphic function

$$
(2.1)\qquad w(t,x)=Lt\varphi_1\Big(\frac{t}{cr}+\frac{x_1}{\eta R}+\frac{x_2+\cdots+x_n}{R}\Big)
$$

satisfies (M), and hence is one majorant of the formal solution $u(t,x)$.

The rest of the section is devoted to this task.

Let us consider first the left-hand side of (M). For convenience, we set $X=t/cr+x_1/\eta R+(x_2+\cdots+x_n)/R$. Simple applications of the properties of the functions $\varphi_i(z)$ yield

$$
t\frac{\partial w}{\partial t}=Lt\Big(t\frac{d\varphi_1(X)}{dX}\frac{1}{cr}+\varphi_1(X)\Big)\gg\frac{Lt^2}{8cr}\varphi_0(X)+Lt\varphi_1(X)
$$

and

$$
x_1\frac{\partial w}{\partial x_1}=Ltx_1\frac{d\varphi_1(X)}{dX}\frac{1}{\eta R}\gg\frac{Ltx_1}{8\eta R}\varphi_0(X).
$$

Hence, we have

$$
(2.2)\quad \sigma\Big(t\frac{\partial w}{\partial t}+x_1\frac{\partial w}{\partial x_1}+w\Big)\gg 2\sigma Lt\varphi_1(X)+\frac{\sigma Lt^2}{8cr}\varphi_0(X)+\frac{\sigma Ltx_1}{8\eta R}\varphi_0(X).
$$

Let us now turn to the right-hand side. We will separately majorize each of the appearing terms. Since $\widetilde{\psi}(x)=(x_1+\cdots+x_n)\psi(x)/R_0$, we have

$$
\begin{aligned}
\frac{M\widetilde{\psi}(x)w}{\rho}&=\frac{MLt}{\rho}\frac{x_1+\cdots+x_n}{R_0}\psi(x)\varphi_1(X)\\
&\ll\frac{MRLt}{\rho R_0}\frac{x_1+\cdots+x_n}{R}C_1\varphi_1(X),
\end{aligned}
$$

where the constant C_1 is the one that results after an application of Proposition 2.2. It actually depends also on R but for simplicity in notation, we will only indicate its dependence on i. (We will also do the same for the other constants that result after an application of this proposition.) We then apply Proposition 2.1 (c) to obtain

$$\text{(2.3)} \qquad \frac{M\widetilde{\psi}(x)w}{\rho} \ll \frac{8MLRt}{\rho R_0} C_1\varphi_1(X).$$

As for expressions involving derivatives, we use Proposition 2.1 (d) to get

$$\text{(2.4)} \qquad \begin{aligned} \frac{M\widetilde{\psi}(x)}{\rho R_0}\left(x_1 \frac{\partial w}{\partial x_1}\right) &\ll \frac{M}{\rho R_0} \frac{x_1 + \cdots + x_n}{R_0} \psi(x) \frac{Ltx_1}{\eta R} \frac{d\varphi_1(X)}{dX} \\ &\ll \frac{MLtx_1}{\rho R_0^2} \frac{x_1 + \cdots + x_n}{\eta R} C_0\varphi_0(X) \\ &\ll \frac{4MLtx_1}{\rho\eta R_0^2} C_0\varphi_0(X) \end{aligned}$$

and

$$\text{(2.5)} \qquad \begin{aligned} \sum_{j=2}^{n} \frac{Mx_1\psi(x)}{\rho R_0} \frac{\partial w}{\partial x_j} &\ll \sum_{j=2}^{n} \frac{MLtx_1}{\rho R_0 R} \psi(x)\varphi_0(X) \\ &\ll (n-1)\frac{MLtx_1}{\rho R_0 R} C_0\varphi_0(X). \end{aligned}$$

Before we majorize the remaining two terms in the right-hand side of (M), we rewrite them as

$$\begin{aligned} &\frac{Mt\psi(x)}{r_0} + \sum_{p+q+|\mu|\geq 2} \frac{M\psi(x)}{r_0^p \rho^{q+|\mu|}} t^p w^q \left(\frac{\partial w}{\partial x}\right)^\mu \\ &\quad = \sum_{p=1}^{\infty} M\psi(x)\left(\frac{t}{r_0}\right)^p + \sum_{\substack{p+q+|\mu|\geq 2 \\ q+|\mu|\geq 1}} \frac{M\psi(x)}{r_0^p \rho^{q+|\mu|}} t^p w^q \left(\frac{\partial w}{\partial x}\right)^\mu. \end{aligned}$$

The first summation on the right is easily seen to satisfy

$$\text{(2.6)} \qquad \begin{aligned} \sum_{p=1}^{\infty} M\psi(x)\left(\frac{t}{r_0}\right)^p &\ll \frac{Mt}{r_0}\psi(x)\frac{1}{1-t/r_0} 4S\varphi_1(X) \\ &\ll \frac{4SMt}{r_0} \frac{1}{1-t/r_0 - x/R_0}\varphi_1(X) \\ &\ll \frac{4SMt}{r_0} C_1\varphi_1(X). \end{aligned}$$

The last step is an application of Proposition 2.2. Note that the constant C_1 can be chosen to be the same as in (2.3). As for the second summation, we have

(2.7)

$$\sum_{\substack{p+q+|\mu|\geq 2\\ q+|\mu|\geq 1}} \frac{M\psi(x)}{r_0^p\,\rho^{q+|\mu|}}\, t^p w^q \Big(\frac{\partial w}{\partial x}\Big)^{\mu}$$

$$\ll \sum_{\substack{p+q+|\mu|\geq 2\\ q+|\mu|\geq 1}} \frac{M\psi(x)}{r_0^p\,\rho^{q+|\mu|}}\, t^p (Lt\varphi_0(X))^q \Big(\frac{Lt\varphi_0(X)}{\eta R}\Big)^{\mu_1}\Big(\frac{Lt\varphi_0(X)}{R}\Big)^{|\mu|-\mu_1}$$

$$\ll M\psi(x)\varphi_0(X) \sum_{\substack{p+q+|\mu|\geq 2\\ q+|\mu|\geq 1}} \Big(\frac{t}{r_0}\Big)^p\Big(\frac{Lt}{\rho}\Big)^q\Big(\frac{Lt}{\rho\eta R}\Big)^{\mu_1}\Big(\frac{Lt}{\rho R}\Big)^{|\mu|-\mu_1}$$

$$\ll \frac{Mt^2\varphi_0(X)\Big(\dfrac{1}{r_0}+\dfrac{L}{\rho}+\dfrac{L}{\rho\eta R}+\dfrac{(n-1)L}{\rho R}\Big)^2}{1-\dfrac{t}{r_0}-\dfrac{Lt}{\rho}-\dfrac{Lt}{\rho\eta R}-\dfrac{(n-1)Lt}{\rho R}-\dfrac{x_1+\cdots+x_n}{R_0}}$$

$$\ll \Big(\frac{1}{r_0}+\frac{L}{\rho}+\frac{L}{\rho\eta R}+\frac{(n-1)L}{\rho R}\Big)^2 Mt^2C_0\varphi_0(X),$$

where the last simplification is possible if we assume that

$$\frac{1}{r_0}+\frac{L}{\rho}+\frac{L}{\rho\eta R}+\frac{(n-1)L}{\rho R}<\frac{1}{cr_0}. \tag{2.8}$$

Note further that the constant C_0 above can be chosen to be the same as the constant C_0 that appeared in the earlier computations.

Having majorized or minorized all the terms appearing in (M), let us now compare the majorant relation obtained in (2.2) to the relations obtained in equations (2.3)–(2.7). We can then see that in order for the holomorphic function $w(t,x)$ in (2.1) to satisfy (M), the following inequalities must hold, in addition to (2.8):

$$2\sigma L \geq \frac{8MLRC_1}{\rho R_0}+\frac{4SMC_1}{r_0} \tag{2.9a}$$

$$\frac{\sigma}{8\eta R} \geq \frac{4MC_0}{\rho\eta R_0^2}+\frac{(n-1)MC_0}{\rho R_0 R} \tag{2.9b}$$

$$\frac{\sigma L}{8cr} \geq MC_0\Big(\frac{1}{r_0}+\frac{L}{\rho}+\frac{L}{\rho\eta R}+\frac{(n-1)L}{\rho R}\Big)^2 \tag{2.9c}$$

Recall that we are free to choose the constants R, L, η and c. To satisfy the above, we first choose and fix an $R<\min(\sigma\rho R_1/4MC_1, \sigma\rho R_0^2/32MC_0)$, after which we choose a large L and a small η so that (2.9a) and (2.9b) will both hold. Finally, we choose a small constant c so that (2.9c) and (2.8) are satisfied.

To summarize, we have shown that for suitably chosen constants, the function $w(t,x) = Lt\varphi_1(X)$ is indeed a majorant of the formal power series solution $u(t,x)$. Since $w(t,x)$ is a holomorphic function in

$$\Omega = \left\{(t,x) \in \mathbb{C} \times \mathbb{C}^n;\ \left|\frac{t}{cr} + \frac{x_1}{\eta R} + \frac{x_2 + \cdots + x_n}{R}\right| < 1\right\},$$

we are assured that $u(t,x)$ converges to a holomorphic function at least in the domain Ω.

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