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Exploitation of uncertain weather forecast data in power network management ⋆

Y. Tomisawa * K. Ohki *,** H. Sugihara ***,** K. Kashima *,** Y. Ohta *,**

* *Graduate School of Informatics, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto, Japan (e-mail: ohki@i.kyoto-u.ac.jp).*
** *JST CREST*
*** *Graduate School of Engineering, Osaka University, 2-1 Yamadaoka, Suita, Osaka, Japan*

Abstract: In power network management, the heat capacity of transmission lines originally arises from the line temperature constraint. The line temperatures are affected not only by the Joule heat but also by the thermal environment. This motivates us to exploit weather forecast data to improve the power management performance. The goal of this paper is to propose a stochastic model predictive control scheme for this purpose. In particular, we compensate for the probabilistic uncertainty by means of chance constraint optimization. The effectiveness of the proposed control scheme is examined through numerical simulation of power grids under the area dependent uncertainty and transmission line failure.

Keywords: Stochastic model predictive control, Applications, Electric power systems, Electrical network, Parallel circuit transmission lines

1. INTRODUCTION

Recent power grids are going green and smart (Amin and Wollenberg (2005); Farhangi (2010); Chakrabortty and Ilic (2012)). Several renewable energies are now available in practice and a variety of current sensor and network technologies serve the real time monitoring, automatic operating technology of the power flow. Furthermore, the prediction techniques of climates, demands of the power have been developed and the accuracy of the prediction has been improved.

However, it is still difficult to predict the amount of the renewables, the demands, and the climates with high accuracy. These large prediction errors can make the grid fragile; for example, too much power transmittance can damage the transmission lines and electric facilities, which causes blackout, breaking of wire, and other serious accidents. Therefore, it is necessary to handle the power generation with taking into account the prediction errors.

To avoid such a situation, the power generators conventionally had to transmit the power under the allowable current. This approach only considers the steady state of the heat dynamics of the transmission lines, and therefore is conservative. As this conservative method tends to cost the fuel, other approaches to protect the transmission lines have recently been proposed (Banakar et al. (2005); Olsen et al. (2013); Sugihara et al. (2015); Alguacil et al. (2005); Almassalkhi and Hiskens (2015a,b); Adrees and Milanovic (2016)). In particular, since the damage of the transmission lines is mainly caused by their Joule heat, the heat of the transmission lines has directly been considered (Banakar et al. (2005); Sugihara et al. (2015); Almassalkhi and Hiskens (2015a,b)).

Almassalkhi and Hiskens (2015a) proposed a model predictive control (MPC) scheme to manipulate a given power grid with storages and renewables in a deterministic manner. However, as mentioned above, it is preferable to consider the randomness due to the prediction errors directly and we should clarify how the randomness affects the temperature of the transmission lines.

In this paper, we consider an optimization problem to minimize the generation cost by the stochastic MPC for a given power grid with transmission line temperature fluctuation. Transmission lines are usually outside and exchange the heat energy with the environment whose temperature is predicted with errors. Since the temperature around each transmission line depends on the area and the prediction accuracy and the power generation cost of each generator is usually different, we examine how the difference of prediction errors affects the total generation cost numerically. Furthermore, we investigate a power system that consists of parallel circuit transmission lines and one of its transmission lines is broken. For simplicity, we only consider parallel single-circuit lines in this paper. Since transmission lines are necessary to be a carrier of power supply even if some of them are accidentally broken, parallel transmission lines are used to assure reliability in general. If one of the parallel lines is broken, the electric current of the other line doubles and increases the line temperature drastically. We examine single circuit fault on the parallel-single circuit lines while the proposed stochastic MPC scheme is being applied and check how it works well.

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The remainder of this paper is organized as follows. The model of the power grid and the dynamics of the transmission lines are introduced in Section 2. We also set an optimization problem of power generation scheduling in the section and give a relaxation to a quadratic programming problem in Section 3. The numerical results is shown in Section 4 and we summarizes the paper in Section 5.

Throughout this paper, we use following notations: $\mathbb{R}$ and $\mathbb{C}$ are real numbers and complex numbers, respectively, and $j := \sqrt{-1}$. E_n denotes n-dimensional identity matrix. $\otimes$ denotes the Kronecker product and I^* implies complex conjugate of $I \in \mathbb{C}$.

2. MODEL

2.1 Power grid and line temperatures

In this paper, we consider the network model that consists of middle transmission lines and the electrical property of each line is described by the Π model (Kundur (1994)). Let $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ be an undirected and connected graph representing the network of a given power grid, where $\mathcal{V} := \{1, \ldots, n\}$ is a set of all nodes and $\mathcal{E}$ is a set of edges. We assume that all edges have same admittance $g(j\omega)$. Each node represents a consumer or a generator in the grid, and each edge represents the corresponding transmission line. We sort the nodes so that $\{1, \ldots, n_g\}$ are the generators.

Assume that all transmission lines have same mechanical and electrical properties in this paper. Let $\dot{Y}' \in \mathbb{C}^{n\times n}$ [S] be the node-admittance matrix of $\mathcal{G}$ defined by

$$\dot{Y}' := g(j\omega)L + H(j\omega),$$

where $L \in \mathbb{R}^{n\times n}$ is the graph Laplacian of $\mathcal{G}$, $g(j\omega) \in \mathbb{C}$ [S] is a admittance of transmission line, $H(j\omega) \in \mathbb{C}^{n\times n}$ [S] is a diagonal matrix which reflects small load of the corresponding node, and $\omega \in \mathbb{R}$ is a fixed angular velocity. For given node-admittance matrices $\dot{Y}_1, \dot{Y}_2 \in \mathbb{C}^{n\times n}$ of $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively, the admittance matrix $\dot{Y}_{\mathrm{par}} \in \mathbb{C}^{2n\times 2n}$ of parallel single-circuit lines are modeled as

$$\dot{Y}_{\mathrm{par}} := \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \dot{Y}_1 + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \otimes \dot{Y}_2, \tag{1}$$

Then the reduced node-admittance matrix $\dot{Y} \in \mathbb{C}^{n\times n}$ is

$$\dot{Y} := ([1\ \ 0] \otimes E_n)\, \dot{Y}_{\mathrm{par}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes E_n \right). \tag{2}$$

Then, describing the voltage in phasor representation of node k by $\dot{V}_k$ [V], the current $\dot{I}_k$ [A] at the node k is calculated by the Kirchhoff's law; $\dot{I}_k := \sum_{l=1}^{n} \dot{Y}_{kl}\dot{V}_l$. The active and reactive powers of node k are described as P_k [W] and Q_k [var], respectively, and they should satisfy the following power flow equations:

$$P_k := \mathrm{Re}\Big(\dot{V}_k \dot{I}_k^*\Big) = V_k \sum_{l=1}^{n} V_l Y_{kl} \cos(\theta_l - \theta_k - \phi_{lk}), \tag{3}$$

$$Q_k := \mathrm{Im}\Big(\dot{V}_k \dot{I}_k^*\Big) = V_k \sum_{l=1}^{n} V_l Y_{kl} \sin(\theta_l - \theta_k - \phi_{lk}), \tag{4}$$

where $\dot{V}_k = V_k e^{j\theta_k}$ and $\dot{Y}_{kl} = Y_{kl} e^{j\phi_{kl}}$, and since the voltage V_{kl} of each node usually fluctuates small, assume that every V_{kl}, $(k,l) \in \mathcal{E}$ is a given constant. Furthermore, since we only use the difference of the phases $\{\theta_k\}$, we assume that $\theta_1 = 0$, i.e., node 1 is the reference node. We use the active powers of generators as control inputs,

$$u := \Big(P_1,\ P_2,\ \cdots,\ P_{n_g}\Big). \tag{5}$$

The rest of active powers are consumers' demands, which are assumed to be given *a priori*.

The dynamics of the line temperatures is driven by the power flow fluctuations and meteorological conditions. Each line temperature $\tilde{T}_{kl}$ [°C] at the line $(k,l) \in \mathcal{E}$ is described by the line heat balance equation (Banakar et al. (2005)). Since the absorption of heat solar radiation and the dissipation heat are small enough in our setting, we consider the following simplified dynamics; for each $(k,l) \in \mathcal{E}$,

$$\frac{d\tilde{T}_{kl}(t)}{dt} = \frac{R_{ac}(\tilde{T}_{kl}(t)) I_{kl}(t)^2}{C} + \lambda(T_{kl}^{out}(t) - \tilde{T}_{kl}(t)), \tag{6}$$

where C is the thermal line capacitance, $I_{kl}(t)$ is the line current at time t, $R_{ac}(\tilde{T}_{kl})$ is the line resistance, λ is the thermal diffusivity of the transmission line, and $T_{kl}^{out}(t)$ denotes the outside temperature of the environment surrounding the transmission line at time t. In what follows, we assume that the outside temperature T_{kl}^{out} can be decomposed to predicted one and its error;

$$T_{kl}^{out}(t) = T_{kl}^{pre}(t) + \sigma_{kl} w_{kl}(t), \tag{7}$$

where σ_{kl} is the amplitude of the prediction error and $w_{kl}(t)$ is a stationary standard white Gaussian noise (formally, derivative of the standard Wiener process).

2.2 Problem formulation

Let us consider a power generation scheduling problem associated with the above power grid. A naive scheduling problem is minimization of the cost of power generation satisfying consumers' demand and sustainable manipulation. In order to avoid desynchronization and overheating of the transmission lines, we impose the following mathematical constraints, respectively;

$$|\theta_k - \theta_l| < \frac{\pi}{6}, \quad \hat{T}_{kl}(t) \le T_{\max}, \tag{8}$$

where $T_{\max} > 0$ is the upper bound of the admissible temperature of the transmission lines. Then, we consider an optimization problem with chance constraints. We set the objective function as

$$J(u_{[t_0, t_0+\tau]}) := \int_{t_0}^{t_0+\tau} u^\top(t) S u(t) dt, \tag{9}$$

where $S = \mathrm{diag}(s_1, \ldots, s_{n_g})$ and $s_i > 0$ is the cost for the generator i. The line temperatures should be lower than $T_{\max}$ with high probability, which is prespecified by a given number $p \in [0, 1)$. Finally, we end up with the following chance constrained optimization problem.

Problem A

$$\min_{u_{[t_0,t_0+\tau]}} \quad J(u_{[t_0,t_0+\tau]}), \tag{10}$$

$$\begin{aligned} \text{s.t. } & (3),\ (6),\ |\theta_k(t)-\theta_l(t)| < \frac{\pi}{6}, \\ & \text{and } \Pr(\tilde{T}_{kl}(t) \le T_{\max}) > p, \\ & \forall t \in [t_0, t_0+\tau],\ \forall (k,l) \in \mathcal{E}. \end{aligned} \tag{11}$$

We use the first several input sequences $u(t)$, $t \in [t_0, t_0+\delta t]$ and reset the initial time of Problem **A** to $t_0+\delta t$, and then solve it recurrently.

Problem **A** is usually hard to solve directly due to the nonlinearity of (3) and (6), though, it is well known that there are several approximation techniques which work well in practice. We use the so-called direct method for solving power flow equations (3) and (4) , and linearization for line temperature equation (6). We describe the details below.

3. REDUCTION TO QUADRATIC PROGRAMMING

3.1 Approximations

Assume that the differences between θ_k and θ_l, $\forall (k,l) \in \mathcal{E}$, are small enough and the resistance of every line is much less than its reactance. Then, we apply the directed current method (DC method) to the calculation of the power flow equation (3), i.e., linearize Eq. (3) and ignore some small constant parameters (Momoh (2008), pp. 100–102). As a result, we obtain the following equation

$$P = M\theta, \tag{12}$$

where $\theta = (\theta_k)$, $P = (P_k)$, $k \in \mathcal{V}$, and $M \in \mathbb{R}^{n\times n}$ is the matrix whose component is $M_{kl} := V_k V_l Y_{kl} \sin(\phi_{lk})$ for $k \ne l$ and $M_{kk} = -\sum_{l=1, l\ne k}^{n} M_{kl}$. Note that the DC method allows us to neglect the reactive powers. Since we assume $\theta_1 = 0$, Eq. (12) is

$$P = \bar{M}\bar{\theta}, \tag{13}$$

where

$$\bar{M} := \begin{bmatrix} M_{12} & \dots & M_{1n} \\ M_{22} & \dots & M_{2n} \\ \vdots & & \vdots \\ M_{n2} & \dots & M_{nn} \end{bmatrix} \text{ and } \bar{\theta} := \begin{bmatrix} \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}.$$

If the rank of $\bar{M}$ is $n-1$, then the solution of Eq. (13) is

$$\bar{\theta} = \left(\bar{M}^\top \bar{M}\right)^{-1} \bar{M}^\top P. \tag{14}$$

This rank condition holds unless at least one of the transmission lines is broken.

Next, let us approximate the line temperature equation (6). Due to small differences among the phases and the temperature independent approximation of the impedance $R_{ac}(T) \simeq R := \mathrm{Re}(g(j\omega)^{-1})$, the Joule heat of each line can be approximated as

$$\begin{aligned} & R_{ac}(T) I_{kl}(t)^2 \\ \simeq & R C_e Y_{kl}^2 \left((V_l^2 - V_k^2)^2 - V_l V_k (\theta_l - \theta_k)^2\right), \end{aligned} \tag{15}$$

where C_e is

$$\begin{aligned} C_e := & \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin^2 \Big(\omega t \\ & + \angle \left(Y_{kl} \left(V_k e^{j(\phi_{kl}+\theta_k)} - V_l e^{j(\phi_{kl}+\theta_l)}\right)\right)\Big) dt. \end{aligned}$$

From (12), the Joule heat is approximated by the quadratic function B_{kl} of $u(t)$. Since $P = \left[u^\top,\ d^\top\right]^\top$, where d represents the demand of consumers, Eq. (14) is represented by

$$\bar{\theta} = A_u u + A_d d,$$

where $A_u \in \mathbb{R}^{(n-1)\times n_g}$ and $A_d \in \mathbb{R}^{(n-1)\times(n-n_g)}$ are defined by $[A_u\ A_d] = \left(\bar{M}^\top \bar{M}\right)^{-1} \bar{M}^\top$. Using $E_k \in \mathbb{R}^{n-1}$ where the lth component of E_k is 1 and the others are 0, θ_k, $k \ge 2$, can be represented by $E_k^\top \bar{\theta}$. Then

$$B_{kl}(u) = u^\top N_{kl} u + \beta_{kl}^\top u + \alpha_{kl},$$

where $N_{kl} = N_{lk} \in \mathbb{R}^{n_g \times n_g}$, $\beta_{kl} = \beta_{lk} \in \mathbb{R}^{n_g}$, and $\alpha_{kl} = \alpha_{lk} \in \mathbb{R}$ are defined as follows;

$$\begin{aligned} N_{kl} := & -\frac{R C_e Y_{kl}^2 V_k V_l}{C} A_u^\top (E_l - E_k)(E_l - E_k)^\top A_u, \\ \beta_{kl} := & -2\frac{R C_e Y_{kl}^2 V_k V_l}{C} d^\top A_d^\top (E_l - E_k)(E_l - E_k)^\top A_u, \\ \alpha_{kl} := & \frac{R C_e Y_{kl}^2}{C} \Big((V_k - V_l)^2 \\ & - V_k V_l d^\top A_d^\top (E_l - E_k)(E_l - E_k)^\top A_d d\Big) \end{aligned}$$

for $k, l \ge 2$, and

$$\begin{aligned} N_{k1} := & -\frac{R C_e Y_{k1}^2 V_k V_1}{C} A_u^\top E_k E_k^\top A_u, \\ \beta_{k1} := & -2\frac{R C_e Y_{k1}^2 V_k V_1}{C} d^\top A_d^\top E_k E_k^\top A_u, \\ \alpha_{k1} := & \frac{R C_e Y_{k1}^2}{C} \Big((V_k - V_1)^2 - V_k V_1 d^\top A_d^\top E_k E_k^\top A_d d\Big). \end{aligned}$$

Then, we consider the following equation as the line temperature dynamics:

$$\begin{aligned} \frac{dT_{kl}(t)}{dt} = & -\lambda T_{kl}(t) + B_{kl}(u(t)) \\ & + \lambda (T_{kl}^{pre}(t) + \sigma_{kl} w_{kl}(t)). \end{aligned} \tag{16}$$

To deal with the optimization problem numerically, the time interval $[t_0, t_0+\tau]$ is divided into N segments. Let $t_i := t_{i-1} + \frac{\tau}{N}$, $i = 1, \cdots, N$ be the i–th time with $t_N = t_0\tau$ [minutes]. The objective function $J(u)$ is also discretized as

$$\tilde{J}(u_{\{t_0,\dots,t_N\}}) := \sum_{i=0}^{N} u(t_i)^\top S u(t_i) \Delta t. \tag{17}$$

where $\Delta t = \frac{\tau}{N}$. From above, the reduced problem is as follows.

Problem B

$$\min_{u} \quad \tilde{J}(u_{\{t_0,\dots,t_N\}}), \tag{18}$$

$$\begin{aligned} \text{s.t. } & P(t) = M\theta(t),\ (16), \\ & |\theta_k(t) - \theta_l(t)| < \frac{\pi}{6}, \\ & \text{and } \Pr(T_{kl}(t) \le T_{\max}) > p, \\ & \forall t \in \{t_0, \dots, t_N\},\ \forall (k,l) \in \mathcal{E}. \end{aligned}$$

3.2 Quadratic Programming

Let us describe Problem **B** as the corresponding quadratic programming problem. The explicit solution of Eq. (16) is

$$\begin{aligned} T_{kl}(t) =& e^{-\lambda t} T_{kl}(t_0) + \int_{t_0}^{t} e^{-\lambda(t-\tau)} B_{kl}(u(\tau)) d\tau \\ &+ \lambda \int_{t_0}^{t} e^{-\lambda(t-\tau)} T_{kl}^{pre}(\tau) d\tau \\ &+ \lambda \sigma_{kl} \int_{t_0}^{t} e^{-\lambda(t-\tau)} w_{kl}(\tau) d\tau. \end{aligned} \tag{19}$$

This implies that $T_{kl}(t)$ obeys

$$T_{kl}(t) \sim \mathcal{N}\left(\mu_{kl}(t),\ \bar{\sigma}_{kl}(t)^2\right), \tag{20}$$

where $\mathcal{N}(\mu_{kl}, \bar{\sigma}_{kl}^2)$ denotes the normal distribution with mean μ_{kl} and the variance σ_{kl}^2, and

$$\begin{aligned} \mu_{kl}(t) :=& e^{-\lambda(t-t_0)} T_{kl}(t_0) + \int_{t_0}^{t} e^{-\lambda(t-\tau)} B_{kl}(u(\tau)) d\tau \\ &+ \lambda \int_{t_0}^{t} e^{-\lambda(t-\tau)} T_{kl}^{pre}(\tau) d\tau, \end{aligned} \tag{21}$$

$$\bar{\sigma}_{kl}(t) := \lambda \sigma_{kl} \sqrt{\frac{1}{2\lambda}\left(1 - e^{-2\lambda(t-t_0)}\right)}. \tag{22}$$

The second term in the right hand side of (21) at $t = t_n$ can be approximated as

$$\sum_{i=0}^{n} e^{-\lambda(t_n - t_i)} B_{kl}(u(t_i)) \Delta t,$$

which is again a quadratic function of u. Other terms can be calculated numerically.

By virtue of (21), the chance constraint (11) is equivalent to

$$\mu_{kl}(t) + F(p)\bar{\sigma}_{kl}(t) < T_{\max} \tag{23}$$

with the function $F : [0, 1] \to \mathbb{R}$ defined as

$$F(p) := F_1^{-1}(p),$$

where

$$F_1(x) = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{x} e^{-\frac{y^2}{2}} dy.$$

Since B_{kl} is the quadratic function of the active power generation u, Eq. (23) implies quadratic constraints with respect to θ; for every $(k, l) \in \mathcal{E}$,

$$\begin{pmatrix} \theta(t_0)^\top A_{kl} \theta(t_0) \\ \vdots \\ \theta(t_{N-1})^\top A_{kl} \theta(t_{N-1}) \end{pmatrix} > \frac{\Gamma_b^{-1}(T_{kl}(t_0)\Gamma_a + F(p)\boldsymbol{\sigma}_{kl} - T_{\max}\mathbf{1}) - C_1^{kl}\mathbf{1} - \lambda \boldsymbol{T}_{kl}^{pre}}{C_2^{kl}},$$

where $\mathbf{1}$ is a N-dimensional vector whose element is 1, $A_{kl} = (A_{kl}(i, m)) \in \mathbb{R}^{n \times n}$ is

$$A_{kl}(i, m) := \begin{cases} 1, & \text{if } i = m, \\ -1, & \text{if } (i, m) \in \mathcal{E}, \\ 0, & \text{otherwise}, \end{cases}$$

and

$$\Gamma_a := \begin{pmatrix} \gamma \\ \gamma^2 \\ \vdots \\ \gamma^N \end{pmatrix}, \ \Gamma_b := \Delta t \begin{pmatrix} 1 & & & \\ \gamma^1 & 1 & & \\ \vdots & & \ddots & \\ \gamma^{N-1} & \cdots & \gamma & 1 \end{pmatrix},$$

$$\boldsymbol{T}_{kl}^{pre} := (T_{kl}(t_1),\ T_{kl}(t_2),\ \ldots,\ T_{kl}(t_N))^\top,$$

$$\boldsymbol{\sigma}_{kl} := (\bar{\sigma}_{kl}(t_1),\ \bar{\sigma}_{kl}(t_2),\ \ldots,\ \bar{\sigma}_{kl}(t_N))^\top,$$

$$C_1^{kl} := RC_e Y_{kl}^2 (V_k - V_l)^2,\ C_2^{kl} := RC_e Y_{kl}^2 V_k V_l,$$

where $\gamma = e^{-\lambda \Delta t}$. Then, the problem we should solve is formulated as follows.

Problem C

$$\min_{\theta} \sum_{l=0}^{N-1} u(t_l)^\top S u(t_l), \tag{24}$$

$$\begin{aligned} \text{s.t. } & P(t) = M\theta(t),\ (21),\ (22), \\ & |\theta_k(t) - \theta_l(t)| < \frac{\pi}{6}, \\ & \mu_{kl}(t) + F(p)\bar{\sigma}_{kl}(t) < T_{\max}, \\ & \forall t \in \{t_0, \ldots, t_N\},\ \forall (k, l) \in \mathcal{E}. \end{aligned}$$

We examine our stochastic MPC scheme numerically in the following section.

4. NUMERICAL EXPERIMENTS

In this section, we apply our control scheme to two power systems. Hereafter, we set the parameters as follows: $\lambda = 0.9$ [sec^{-1}], $\alpha = 35$ °C, $T_{\max} = 75$ °C, $\Delta t = 3$ minutes, $g(j\omega) = 1.05 + j31.5$ S, and $H(j\omega) = 10^{-6}$ S. The network of each grid consists of parallel single-circuit lines and both single-circuit lines are identical unless the line failure occurs. The time interval for the stochastic MPC is $\tau = 18$ minutes and we only use the input at the first step of the obtained optimal input sequences. Assume that the demands are required from $t = 0$ and the generators run after $t = 0$. The initial line temperature $T_{kl}(0)$ of each transmission line follows the normal distribution $\mathcal{N}(\alpha, \bar{\sigma}_{kl}(\infty))$. We put $p = 0.997$ for the parameter of the chance constraint in Problem **C**.

With above parameters and conditions, we evaluate two issues that should be considered in practice. First, we evaluate how the prediction accuracy around each line affects the total cost of the power generation and temperature constraints. We examine the case that the prediction accuracies depend on the area, and how it affects the power generation and temperature constraints. Secondly, we also consider the case that the line failure is occurred during the control operation. Mathematically, it is represented by the change of topology, i.e., one of the graph Laplacians of the parallel single-circuit lines is changed at the time the failure occurs. In each simulation, we solve Problem **C** and obtain $u(t)$ each time, and the phases θ is calculated again by solving the power flow equation (3), and then the dynamics (16) is calculated by newly obtained θ^*. Each simulation is run 100 times to evaluate the each issue statistically.

To this end, we consider a simple power grid model consists of two generators and one consumer, which are connected

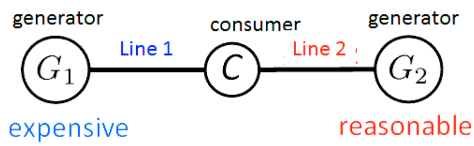

Fig. 1. A power system model

as in Fig. 1. Node 1, 2, and 3 are G_1, G_2, and C, respectively, and Line 1 and Line 2 denote the edges $(1, 3)$ and $(2, 3)$, respectively. Assume that the generator G_1 is more expensive than G_2 such that $S = \mathrm{diag}(10, 1)$ and consumer requires 500 MW. Under the above conditions, we examine how prediction accuracies of the outside temperatures affect the system for the 3 types of the prediction accuracies:

Case 1: $(\sigma_{13}, \sigma_{23}) = (1, 1)$,
Case 2: $(\sigma_{13}, \sigma_{23}) = (1, 5)$,
Case 3: $(\sigma_{13}, \sigma_{23}) = (5, 1)$.

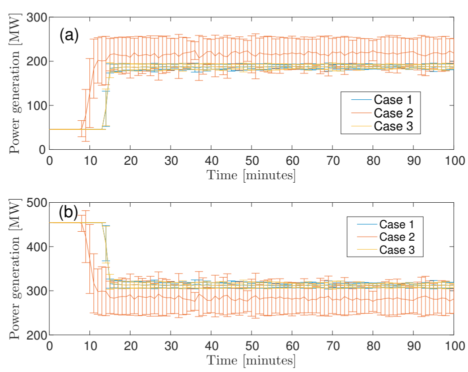

Fig. 2. Ensemble average of the power generation with error bars of (a) Generator 1 and (b) Generator 2.

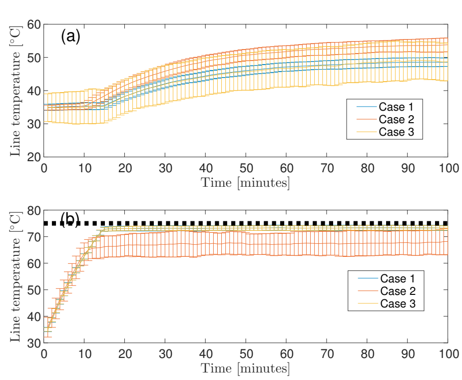

Fig. 3. Ensemble average of the line temperature with error bars of (a) Line 1 and (b) Line 2. The black dot-line represents the constraint.

Figs. 2 and 3 show the ensemble averages of power generation and line temperatures of the above cases, respectively. As shown in Fig. 2, the ensemble average of the power generation between Case 1 and Case 3 are almost same, and it is reflected in the total costs shown in Table 1, where the total costs are calculated by $\sum_{l=0}^{N_f} u(t_l)^\top S u(t_l) \Delta t$, with $t_0 = 0$ and $t_{N_f} = 100$ minutes. In Case 1 and Case 3, the temperature around Line 1 is not predicted with high accuracy. However, since our control scheme tends to avoid using the generator G_1, the accuracy of the prediction of the temperature around Line 1 is not important for supplying power to the consumer.

On the other hand, the less prediction accuracy of the environment temperature around Line 2 makes difference. Since Line 2 has large uncertainty, the generator G_2 supplies power to the consumer in order to keep the line temperatures lower than 75°C. Fig. 3 shows that the ensemble average of line temperatures of each line. From Fig. 3 (b), the average of the temperatures of Line 2 in Case 2 is much lower than the others, this implies that the generation cost is more expensive if the prediction errors around the reasonable generators are worse.

The chance constraints admit only about 0.3% of the samples are over 75 °C, though, since Problem **C** is an approximation of Problem **A**, the percentages of constraint-breaking samples is higher than 0.3%; at the time 100, 2 samples in Case 1, 9 samples of Case 2, and 1 sample in Case 3 are in higher temperature than 75°C. The main reason of the inconsistency comes from the approximations of the power equation. The DC method is a well-known approximation to solve power flow equations, though, the approximation errors in our setting are utmost about 2%.

Table 1. Total cost in each case

Case	Generation cost
Case 1	$(2.117 \pm 0.016) \times 10^6$
Case 2	$(2.691 \pm 0.073) \times 10^6$
Case 3	$(2.117 \pm 0.017) \times 10^6$

Next, we examine the case of the line failure and its effect. Assume that one of the parallel single–circuit lines between G_2 and the consumer in Fig. 1 is broken during the control operation and the effective impedance of the line is changed at time 100 minutes. The control scheme is changed in 10 minutes after the failure. Since we focus on the line failure effect, we consider Case 1 for simplicity.

Figs. 4 and 5 show the ensemble average of the currents and the line temperatures with error bars, respectively. When the failure is occurred at the time 100, the current in Line 2 drastically increases and keep large current until the control scheme based on the new node admittance matrix is applied. The line temperature of Line 2 also increases and breaks the temperature constraint after the failure occurs, though, it returns to the same values after the new control runs and keeps almost same mean and variance as those of the values before the failure happens. Consequently, our proposed control scheme works well in this simple power grid model even if one line failure occurs.

5. CONCLUSION

We proposed a stochastic model predictive control scheme for power generation scheduling problem and examined its efficiency via numerical examples. The results show that the required prediction accuracy depends on the network topology and the generation cost.

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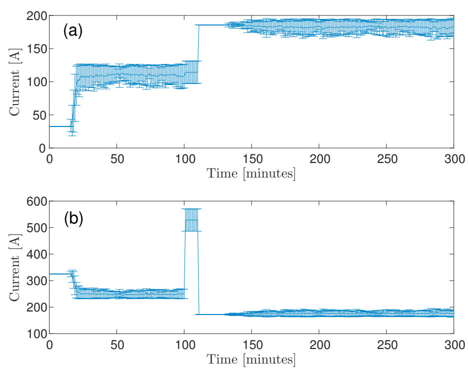

Fig. 4. Ensemble average of the line current with error bars: (a) Line 1 and (b) Line 2.

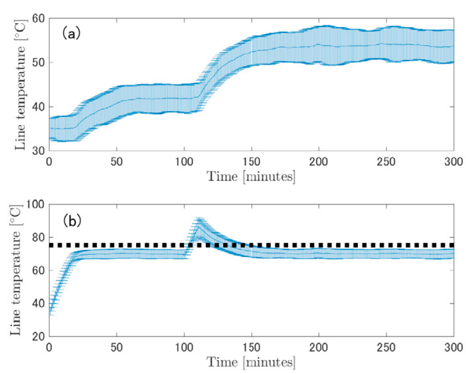

Fig. 5. Ensemble average of the line temperatures with error bars: (a) Line 1 and (b) Line 2. The black dot-line represents the constraint.

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