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Capital investment under output demand and investment cost ambiguity*

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1 Introduction

Today, the business environment is more complex and uncertain than ever before, and managers face numerous uncertainties including demand uncertainty and cost uncertainty. In addition to these uncertainties, expenditure on capital investment is generally a sunk cost. This creates the problem of when and how much firms should invest in capital in an effort to increase their value. In this study, we investigate a firm's capital investment problem under output demand and investment cost uncertainty and derive the optimal investment strategy.

Many researchers have investigated the impact of uncertainty and irreversibility on capital investment problems. Hartman (1972), Able (1983), and Abel and Eberly (1994) investigated how a firm's capital investment is influenced by output price uncertainty. They showed that output price uncertainty promotes capital investment. Meanwhile, real options analysis shows that uncertainty postpones investment when the investment expenditure is a sunk cost, that is, capital investment is irreversible. See Dixit and Pindyck (1994) for more details on real options analysis.

In this study, we consider the case in which the firm's manager does not have sufficient confidence to predict either future output demand or the price of capital. That is, the firm's manager faces output demand and investment cost ambiguity (Knightian uncertainty). See, for example, Camerer and Weber (1992), Etner et. al. (2012) and Guidolin and Rinaldi (2013) for further details on decision-making under ambiguity. Nishimura and Ozaki (2007), Trojanowska and Kort (2010), Wang (2010), and Thijssen (2011) examine the irreversible capital investment problem under ambiguity using the framework of κ-ignorance developed by Chen and Epstein (2002), in which the ambiguity only affects the drift terms of the associated processes. On the other handConversely, we adopt the framework developed by Kast and Lapied (2010) and Kast et al. (2014), in which the ambiguity affects both the drift terms and diffusion terms of the associated processes. The ambiguities of output demand and the price of capital are expressed by Choquet–Brownian motions. Tsujimura (2017) investigated a firm's capital expansion and reduction problem when output demand is ambiguous. However, he assumed that the price of capital was constant. We only consider capital expansion in an ambiguous business environment, but we extend Tsujimura's model by incorporating investment cost ambiguity.

In general, if two random variables are incorporated into models, they are usually solved by either reducing the dimension or employing numerical analysis. McDonald and Siegel (1986)

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investigated an irreversible project investment problem in which the project value and investment cost were both random variables by reducing the dimension by changing one variable and assuming homogeneity of degree one in the boundary condition. Adkins and Paxson (2011) relaxed the homogeneity and derived an implicit analytical solution, while Stokey (2009, Section 11.5) examined an irreversible investment problem in which the demand and price of capital were both random variables and derived an explicit solution without assuming homogeneity.

This study explores a firm's capital investment problem when future output demand is ambiguous. To this end, we formulate the firm's problem as a singular stochastic control problem. In this study, output demand ambiguity is expressed by a Choquet–Brownian motion developed by Kast and Lapied (2010) and Kast et al. (2014), whereas the previous studies mentioned above adopted the framework of κ-ignorance developed by Chen and Epstein (2002) to incorporate ambiguity. We solve the firm's problem by using variational inequalities to derive the optimal investment strategy, as in Stokey (2009, Section 11.5). This is described by a threshold that determines the amount of capital expansion. Furthermore, we conduct a comparative static analysis of some of the parameters. As a result of this analysis, we find that if the firm's manager is more ambiguity-averse, investment is delayed. Furthermore, an increase in the volatility of the output price and the price of capital also delays investment.

The rest of the paper is organized as follows. Section 2 describes the setup of the firm's problem. Section 3 presents a solution to the firm's problem. The numerical analysis is presented in Section 4, and Section 5 concludes.

2 The Model

We consider a firm's capital investment problem. The firm produces a single output using capital K and sells it in a competitive market. If the output demand increases sufficiently, the firm must expand its capital to meet the increased demand. In this study, we assume that output demand and the price of capital are uncertain. In this context, the firm's manager is not confident of predicting either future output demand or the price of capital. In other words, the firm's manager faces ambiguity. We adopt the framework developed by Kast and Lapied (2010) and Kast et al. (2014) to cope with this ambiguity.

Let X_t be the output demand at time t. The dynamics of the output demand is governed by the following stochastic differential equation:

$$\mathrm{d}X_t = \mu_X X_t \mathrm{d}t + \sigma_X X_t \mathrm{d}W_t^{X,c}, \quad X_{0^-} = x > 0, \tag{2.1}$$

where $\mu_X > 0$ and $\sigma_X > 0$ are constants. $W_t^{X,c}$ is a generalized Wiener process with mean $m = 2c - 1$ and variance $s^2 = 4c(1-c)$:

$$\mathrm{d}W_t^{X,c} = m\mathrm{d}t + s\mathrm{d}W_t^X, \tag{2.2}$$

where W_t^X is a standard Brownian motion. c $(0 < c < 1)$ is the constant conditional Choquet capacity, which indicates the firm manager's attitude toward ambiguity. Then, output demand follows a Choquet–Brownian motion as follows:

$$\mathrm{d}X_t = (\mu_X + m\sigma_X) X_t \mathrm{d}t + s\sigma_X X_t \mathrm{d}W_t^X, \quad X_{0^-} = x > 0. \tag{2.3}$$

If $c < \frac{1}{2}$ (resp. $c > \frac{1}{2}$), the firm manager is ambiguity-averse (resp. ambiguity-lovingembracing). If $c = \frac{1}{2}$, then $m = 0$ and $s = 1$. This means that $\mathrm{d}W_t^{X,c} = \mathrm{d}W_t^X$, ambiguity disappears, and the firm manager has perfect confidence in the dynamics of the output demand.

The firm expands its business by accumulating capital at price P. The dynamics of the price of capital is governed by:

$$\mathrm{d}P_t = \mu_P P_t \mathrm{d}t + \sigma_P P_t \mathrm{d}W_t^{P,c}, \quad P_{0^-} = p > 0, \tag{2.4}$$

where $\mu_P > 0$ and $\sigma_P > 0$ are constants. $W_t^{P,c}$ is a generalized Wiener process with mean $m = 2c - 1$ and variance $s^2 = 4c(1-c)$:

$$\mathrm{d}W_t^{P,c} = m\mathrm{d}t + s\mathrm{d}W_t^P, \tag{2.5}$$

where $\mathbb{E}[\mathrm{d}W_t^X \mathrm{d}W_t^P] = \rho \mathrm{d}t$ with $\rho \in [-1, 1]$. Then, the dynamics of the price of capital is rewritten as:

$$\mathrm{d}P_t = (\mu_P + m\sigma_P)P_t \mathrm{d}t + s\sigma_P X_t \mathrm{d}W_t^P, \quad P_{0^-} = p > 0. \tag{2.6}$$

Let I_t be the cumulative expansion of capital until time t. This is right-continuous with left-hand limited adapted processes, nonnegative, and nondecreasing, with $I_{0-} = 0$. Then, the dynamics of the price of capital is given by:

$$\mathrm{d}K_t = -\delta K_t \mathrm{d}t + \mathrm{d}I_t, \quad K_{0^-} = k > 0, \tag{2.7}$$

where $\delta \in (0, 1)$ is a constant depreciation rate.

The firm's operating profit $\hat{\pi}$ at time t is given by:

$$\hat{\pi}(K_t, X_t) = K_t^\alpha X_t^\beta, \tag{2.8}$$

where $\alpha \in (0, 1)$ and $\beta > 0$. The firm's expected discounted net profit $J(k, x, p; I)$ is given by:

$$\hat{J}(k, x, p; I) = \mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}\hat{\pi}(K_t, X_t)\mathrm{d}t - \int_0^\infty \mathrm{e}^{-rt} P_t \mathrm{d}I_t\right], \tag{2.9}$$

where $r > 0$ is the discount rate and $I := \{I_t\}_{t \geq 0}$ is the investment strategy. The investment strategy is admissible when $I \in \mathcal{A}$, where $\mathcal{A}$ is the set of all admissible investment strategies. In this context, it is assumed that:

$$\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}\hat{\pi}(K_t, X_t)\mathrm{d}t\right] < \infty, \tag{2.10}$$

and

$$\mathbb{E}\left[\int_0^\infty \mathrm{e}^{-rt}\mathrm{d}I_t\right] < \infty. \tag{2.11}$$

Therefore, the firm's problem is to maximize the expected discounted net profit over $\mathcal{A}$:

$$\hat{V}(k, x, p) = \sup_{I \in \mathcal{A}} \hat{J}(k, x, p; I) = \hat{J}(k, x, p; I^*), \tag{2.12}$$

where $\hat{V}$ is the value function and I^* is the optimal investment strategy.

3 Variational Inequalities

The firm's investment problem (2.12) is formulated as a singular stochastic control problem. Then, we conjecture that the firm maintains its capital stock level within a given region, so that whenever the capital stock is below a certain level, the firm invests in additional capital. Note that the boundary of the capital stock region depends on the level of demand and the price of capital. To verify this conjecture regarding the investment strategy, we solve the firm's problem (2.12) using variational inequalities.

The variational inequalities of the firm's problem (2.12) are given as follows.

Definition 3.1 (Variational Inequalities) *The following relationships are called the variational inequalities in the firm's problem (2.12):*

$$\hat{\mathcal{L}}\hat{V}(k,x,p) + \hat{\pi}(k,x,p) \leq 0, \tag{3.1}$$

$$\hat{V}_K(k,x,p) \leq p, \tag{3.2}$$

$$[\hat{\mathcal{L}}\hat{V}(k,x,p) + \hat{\pi}(k,x,p)][\hat{V}_K(k,x,p) - p] = 0, \tag{3.3}$$

where $\hat{\mathcal{L}}$ is the operator, defined by:

$$\begin{aligned}\hat{\mathcal{L}} :=& -\delta k\frac{\partial}{\partial K} + (\mu_X + m\sigma_X)x\frac{\partial}{\partial X} + (\mu_P + m\sigma_P)p\frac{\partial}{\partial P} \\ &+ \frac{1}{2}s^2\sigma_X^2 x^2\frac{\partial^2}{\partial X^2} + s^2\sigma_X\sigma_P\rho xp\frac{\partial^2}{\partial X\partial P} + \frac{1}{2}s^2\sigma_P^2 p^2\frac{\partial^2}{\partial P^2} - r.\end{aligned}$$

The variational inequalities are summarized as:

$$\max\left\{\hat{\mathcal{L}}\hat{V}(k,x,p) + \hat{\pi}(k,x,p), \hat{V}_K(k,x,p) - p\right\} = 0. \tag{3.4}$$

Let $\hat{\mathcal{H}}$ be the continuation region, given by:

$$\hat{\mathcal{H}} := \{(k,x,p); \hat{V}_K(k,x,p) < p\}. \tag{3.5}$$

For analytical tractability, we assume that $\beta = 1-\alpha$ and the change variables are $Y_t := K_t/X_t$, as in Tsujimura (2017). Then, the profit function and the value function, respectively, can be rewritten as follows:

$$\hat{\pi}(K_t, X_t) = K_t^{\alpha}X_t^{1-\alpha} = Y_t^{\alpha}X_t = \pi(Y_t)X_t. \tag{3.6}$$

$$\hat{V}(k,x,p) = x\hat{V}\left(\frac{k}{x}, 1, p\right) = xV(y,p). \tag{3.7}$$

It follows from (3.7) that we have $\hat{V}_K(k,x,p) = V_Y(y,p)$, $\hat{V}_X(k,x,p) = V(y,p) - yV_Y(y,p)$, $\hat{V}_{XX}(k,x,p) = (y^2/x)V_{YY}(y,p)$, $\hat{V}_P(k,x,p) = xV_P(y,p)$, $\hat{V}_{PP}(k,x,p) = xV_{PP}(y,p)$, and $\hat{V}_{XP}(k,x,p) = V_P(y,p) - yV_{YP}(y,p)$. The variational inequalities (3.1)–(3.3) can also be rewritten as:

$$\mathcal{L}V(y,p) + \pi(y) \leq 0, \tag{3.8}$$

$$V_Y(y,p) \leq p, \tag{3.9}$$

$$[\mathcal{L}V(y,p)+\pi(y)][V_Y(y,p)-p]=0, \tag{3.10}$$

where $\mathcal{L}$ is the operator defined by:

$$\begin{aligned}\mathcal{L}:=&-(\delta+\mu_X+m\sigma_X)y\frac{\partial}{\partial Y}+(\mu_P+m\sigma_P+s^2\sigma_X\sigma_P\rho)p\frac{\partial}{\partial P}\\&+\frac{1}{2}s^2\sigma_X^2y^2\frac{\partial^2}{\partial Y^2}-s^2\sigma_X\sigma_P\rho py\frac{\partial}{\partial YP}+\frac{1}{2}s^2\sigma_P^2p^2\frac{\partial^2}{\partial P^2}-(r-\mu_X-m\sigma_X).\end{aligned} \tag{3.11}$$

The continuation region (3.5) can be rewritten as:

$$\mathcal{H}:=\{y;V_Y(y,p)<p\}. \tag{3.12}$$

For $y\in\mathcal{H}$, the variational inequalities (3.8)–(3.10) lead to the following ordinary differential equation:

$$\mathcal{L}V(y,p)+\pi(y)=0. \tag{3.13}$$

Let $\phi(y,p)=Ay^\gamma p^\eta$ be a candidate function of the homogeneous part of (3.13). If $\beta+\gamma=1$, then ϕ is homogeneous of degree one. In general, the homogeneity does not necessarily satisfy (Adkins and Paxson, 2011). Then, the general solution of the homogeneous part of (3.13) is given by:

$$\phi^H(y,p)=A_1y^{\gamma_1}p^{\eta_1}+A_2y^{\gamma_2}p^{\eta_2}+A_3y^{\gamma_3}p^{\eta_3}+A_4y^{\gamma_4}p^{\eta_4},\qquad y\in\mathcal{H}, \tag{3.14}$$

where A_1–A_4 are constants to be determined. γ_1–γ_4 and η_1–η_4 are the solutions to the following characteristic equation:

$$\begin{aligned}&\frac{1}{2}s^2\sigma_X^2\gamma(\gamma-1)-s^2\sigma_X\sigma_P\rho\gamma\eta+\frac{1}{2}s^2\sigma_P^2\eta(\eta-1)-(\mu_X+m\sigma_X+\delta)\gamma\\&\qquad+(\mu_P+m\sigma_P+s^2\sigma_X\sigma_P\rho)\eta-(r-\mu_X-m\sigma_X)=0.\end{aligned} \tag{3.15}$$

The discriminant of the characteristic equation (3.15) is:

$$\frac{1}{2}s^2\sigma_X^2\frac{1}{2}s^2\sigma_P^2-\frac{-s^2\sigma_X\sigma_P\rho}{4}>0.$$

The characteristic equation (3.15) is the elliptic equation (see Figure 1). $\{\gamma_1,\eta_1\}$ with $\gamma_1\geq 0$, $\eta_1\geq 0$ are the solutions of the first quadrant, $\{\gamma_2,\eta_2\}$ with $\gamma_2\geq 0$, $\eta_2\leq 0$ are the solutions of the second quadrant, $\{\gamma_3,\eta_3\}$ with $\gamma_3\leq 0$, $\eta_3\leq 0$ are the solutions of the third quadrant, and $\{\gamma_4,\eta_4\}$ with $\gamma_4\leq 0$, $\eta_4\geq 0$ are the solutions of the fourth quadrant.

The particular solution of (3.13) is calculated as:

$$\phi^P(y,p)=By^\alpha, \tag{3.16}$$

where $B:=(r-\mu_X-m\sigma_X)+(\delta+\mu_X+m\sigma_X)\alpha-\frac{1}{2}s^2\sigma_X^2\alpha(\alpha-1)$. It follows from assumption (2.10) that $B>0$. The general solution of (3.13) is:

$$\phi(y,p)=\phi^H(p,y)+\phi^P(y,p),\qquad y\in\mathcal{H}. \tag{3.17}$$

It follows from the definition of the firm's problem that the function ϕ satisfies the following inequality:

$$\phi(y)>By^\alpha. \tag{3.18}$$

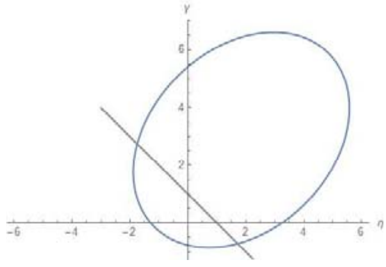


Figure 1: Graph of the characteristic equation (3.15).

The general solution of the homogeneous part of (3.13) ϕ^H represents the option value to invest in capital. This implies that the constant to determine A_i must be nonnegative.

Suppose that the output demand X decreases to 0, or the level of capital K goes to ∞, i.e., y goes to ∞. Then, there is no economic justification for expanding capital at any price of capital p:

$$\lim_{y\to\infty} \phi^H(y,p) \to 0, \quad \forall p.$$

This implies that $\gamma < 0$. Next, suppose that the price of capital P goes to ∞. Then, there is no economic justification for expanding capital for any y:

$$\lim_{p\to\infty} \phi^H(y,p) \to 0, \quad \forall y.$$

This means that $\eta < 0$. Then, the homogeneous solution (3.14) is:

$$\phi^H(y,p) = A_3 y^{\gamma_3} p^{\eta_3}. \tag{3.19}$$

Thus, we obtain the following form for ϕ:

$$\phi(y,p) = \begin{cases} \psi(\underline{y}(p),p) - p(\underline{y}(p) - y), & y \le \underline{y}(p), \\ \psi(y,p) := A_3 y^{\gamma_3} p^{\eta_3} + B y^{\alpha}, & y > \underline{y}(p). \end{cases} \tag{3.20}$$

The four unknowns A_3, $\underline{y}(p)$, γ_3, and η_3 are determined by the following simultaneous equations:

$$\psi_Y(\underline{y}(p),p) = p, \qquad \forall p, \tag{3.21}$$

$$\psi_{YY}(\underline{y}(p),p) = 0, \qquad \forall p. \tag{3.22}$$

Equation (3.21) is the smooth-pasting condition and equation (3.22) is the super-contact condition (see Dumas (1991) for details).

It follows from (3.21) and (3.22) that we have:

$$\underline{y}(p) = \left[\frac{\gamma_3 - 1}{(\gamma_3 - \alpha)\alpha}\frac{1}{B}\right]^{\frac{1}{\alpha-1}} p^{\frac{1}{\alpha-1}}, \tag{3.23}$$

$$A_3 = \frac{1}{\gamma_3} p^{-\eta_3}\left[p\underline{y}(p)^{1-\gamma_3} - \alpha B \underline{y}(p)^{\alpha-\gamma_3}\right]. \tag{3.24}$$

Substituting (3.23) into (3.21) yields:

$$\gamma_3 A_3 \left[\frac{\gamma_3 - 1}{(\gamma_3 - \alpha)\alpha}\frac{1}{B}\right]^{\frac{\gamma_3-1}{\alpha-1}} p^{\eta_3+\frac{\gamma_3-1}{\alpha-1}} + \frac{\alpha-1}{\gamma_3-\alpha}p = 0. \tag{3.25}$$

To Satisfying the equality in hold (3.25) gives the following equations:

$$\eta_3 + \frac{\gamma_3 - 1}{\alpha - 1} = 1, \tag{3.26}$$

$$\gamma_3 A_3 \left[\frac{\gamma_3 - 1}{(\gamma_3 - \alpha)\alpha}\frac{1}{B}\right]^{\frac{\gamma_3-1}{\alpha-1}} = \frac{\alpha-1}{\gamma_3-\alpha}. \tag{3.27}$$

It follows from (3.26) that we obtain:

$$\eta_3 = \frac{\alpha - \gamma_3}{\alpha - 1}. \tag{3.28}$$

4 Numerical Analysis

In this section, we calculate the four unknowns, A_3, $\underline{y}(p)$, γ_3, and η_3, and investigate the effects of changes in the parameters on the threshold, $\underline{y}(p)$. The basic parameter values are set out as follows: $r = 0.05$, $\delta = 0.1$, $\mu_X = 0.01$, $\sigma_X = 0.2$, $\mu_P = 0.01$, $\sigma_P = 0.2$, $\rho = 0.3$, $\alpha = 0.4$, $p = 10$, and $c = 0.4$. Then, we obtain $A_3 = 14.0086$, $\underline{y}(p) = 0.058602$, $\gamma_3 = -0.232449$, and $\eta_3 = -1.05408$.

The results of the comparative static analysis of the threshold, $\underline{y}(p)$, are shown in Figures 3–7. Note that the blue circle represents the result of comparative static analysis when the firm's manager is ambiguity-averse ($c = 0.4$), while the red cross represents the results of comparative static analysis when the firm's manager is ambiguity-neutral ($c = 0.5$) in Figures 3–7.

Figure 2 shows how investment decision-making is influenced by the manager's attitude toward ambiguity, c. Figure 2 shows that the capital investment threshold, $\underline{y}(p)$, is increasing with c. This means that if the firm's manager is more ambiguity-averse, capital investment is delayed. An ambiguity-averse manager tends to hesitate to invest in capital in an ambiguous business environment.

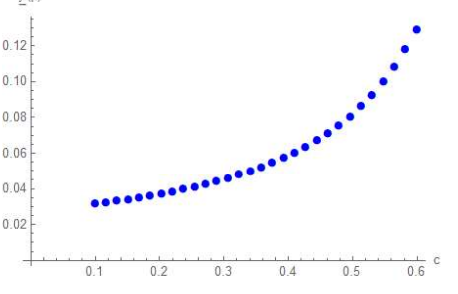
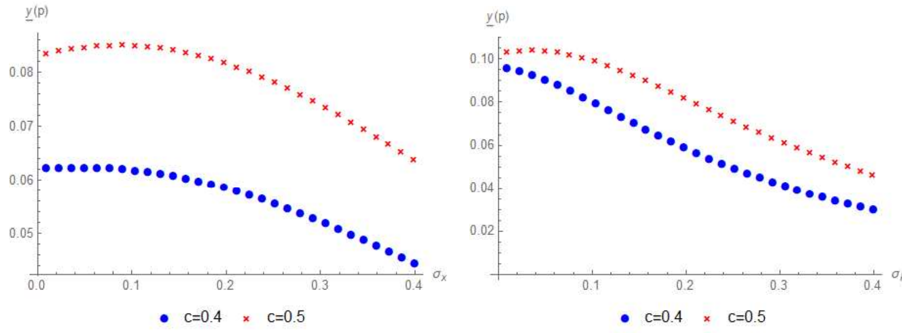


Figure 2: The effect of changing the ambiguity attitude parameter, c, on the threshold $\underline{y}(p)$.

Figure 3 shows the impact of volatility of both output demand, σ_X, and the price of capital, σ_P. If the firm's manager is ambiguity-averse ($c = 0.4$), the threshold $\underline{y}(p)$ is decreasing in both volatilities. This means that investment is delayed when either volatility increases. This is consistent with the standard result of real options analysis. Conversely, if the firm's manager is ambiguity-neutral ($c = 0.5$), the threshold $\underline{y}(p)$ is initially increasing in both volatilities. However, beyond a certain level in each volatility (about $\sigma_X = 0.091$ and $\sigma_P = 0.037$), the thresholds start to decrease.



(a) σ_X (b) σ_P

Figure 3: The effect of changing the volatility of output demand and capital price, σ_X and σ_P, on the threshold $\underline{y}(p)$.

Figure 4 shows the impact of the correlation between the two Brownian motions, W_t^X and W_t^P, on investment decision-making. The investment threshold $\underline{y}(p)$ is increasing with the correlation coefficient ρ.

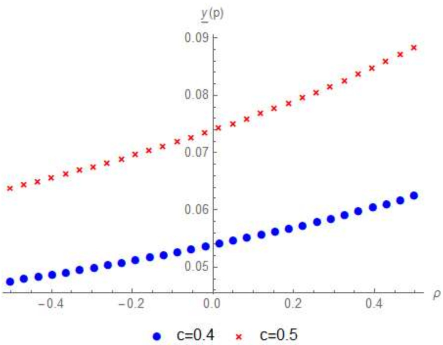
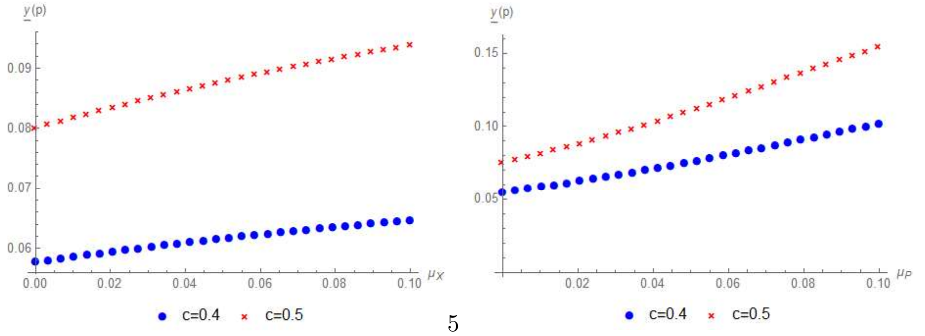


Figure 4: The effect of changing the correlation coefficient between W_t^X and W_t^P, ρ, on the threshold $\underline{y}(p)$.

Figure 5 shows how investment decision-making is influenced by the expected rate of change of both the output demand and the price of capital, μ_X and μ_P. The investment threshold $\underline{y}(p)$ is increasing in both the expected rates of change. This implies that investment is hastened when the expected rate of change of the output demand and the price of capital is increasing.



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(a) μ_X (b) μ_P

Figure 5: The effect of changing the drift coefficient of output demand and the price of capital, μ_X and μ_P, on the threshold $\underline{y}(p)$.

Figure 6 shows the impact of the output elasticity of capital, α, on investment decision-making. Recall that output per unit of capital increases in α. Figure 6 shows that the threshold $\underline{y}(p)$ is initially increasing with the output elasticity of capital. However, beyond a certain level (about $\alpha = 0.3586$ if $c = 0.4$, and $\alpha = 0.3759$ if $c = 0.5$), the threshold starts to decrease.

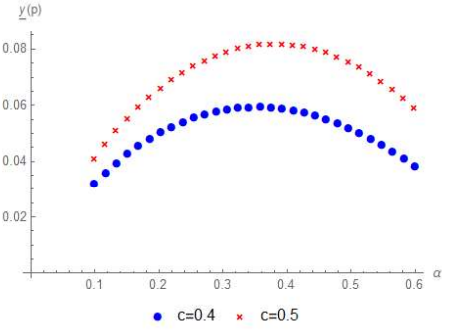

Figure 6: The effect of changing the output elasticity of capital, α, on the threshold $\underline{y}(p)$.

Figure 7 shows how the price of capital affects investment decision-making. Figure 7 shows that the threshold $\underline{y}(p)$ decreases as the price of capital decreases. This implies that a higher price of capital delays investment.

These results provide useful insights into investment decision-making when output demand and the price of capital are ambiguous.

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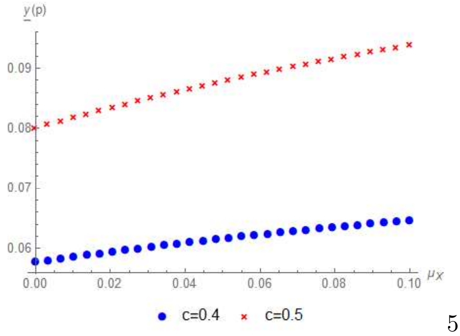
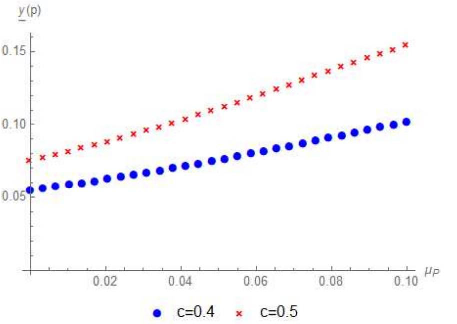


(a) μ_X



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(b) μ_P

Figure 5: The effect of changing the drift coefficient of output demand and the price of capital, μ_X and μ_P, on the threshold $\underline{y}(p)$.

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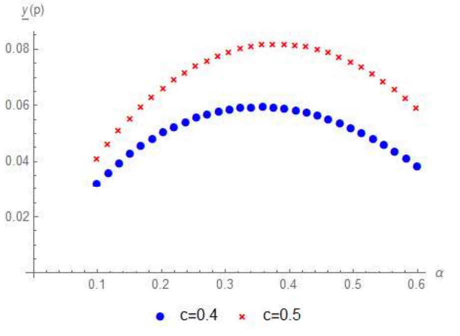

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