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EQUIVALENCES BETWEEN BLOCKS OF ALTERNATING GROUPS

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ABSTRACT. This article contains the talk given at the meeting on "Cohomology Theory of Finite Groups", held at RIMS, Kyoto University, September 1–5, 2003. We present the results of [10] establishing Broué's abelian defect group conjecture for the alternating groups, using the Chuang-Rouquier theorem proving this for the symmetric groups and a descent result coming from Clifford theory. We also discuss some connections with the conjectures of Dade, and of Donovan-Puig.

1. INTRODUCTION

Let G be the alternating group A_n, $\mathcal{O}$ a complete discrete valuation ring with algebraically closed residue field k of characteristic $p>0$, let b be a block of $\mathcal{O}G$ with defect group D, and let c be the Brauer corresponding of $\mathcal{O}N_G(D)$.

We have shown in [10] that D if is abelian, then the algebras $A=b\mathcal{O}G$ and $B=c\mathcal{O}N_G(D)$ are splendidly derived equivalent, that is, there is a bounded complex X of (A,B)-bimodules such that its components are p-permutation modules whose indecomposable summands have vertices contained in $\delta(D)=\{(u,u)\mid u\in D\}$, and such that $X\otimes_B X^\vee\simeq A$ in the homotopy category of complexes of (A,A)-bimodules, and $X^\vee\otimes_A X\simeq B$ in the homotopy category of (B,B)-bimodules, where $X^\vee$ denotes the $\mathcal{O}$-dual of X. Moreover, there is such an equivalence which is compatible with p'-outer automorphism groups, which means in our case the existence of a tilting complex having an $\mathrm{Aut}(G)/G$-grading. This additional condition is especially important in the case of principal blocks, where it is used to reduce the conjecture to the case of simple groups.

We have used that the conjecture is known to hold for the symmetric group S_n by the work of J. Rickard, J. Chuang, R. Kessar and R. Rouquier, and we show how to "go down" to A_n, by using techniques of graded equivalences, as in [8]. Inspiration also came from the paper [5] of P. Fong and M. Harris, who verified the weaker "isotypy form" of the conjecture for A_n, by using Rouquier's paper [14] on S_n. A similar procedure was developed by E. Dade in [4] leading to the verification of his Invariant Projective Conjecture for A_n.

Recall that Donovan's conjecture states that for a fixed p-group P, *there are only finitely many Morita equivalence classes of blocks of group algebra having P as a defect group.* Similar methods have been used to verify these two conjectures in several particular cases, probably most notable being the case of blocks of symmetric groups, and also other blocks with similar combinatorial structure, by Scopes, Kessar, Hiss, Chuang and Rouquier. It is conjectured that even a refinement of this conjecture would hold. Two blocks with defect group P are called Puig equivalent if their source algebras are isomorphic as $\mathcal{O}P$-interior algebras, or equivalently,

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they are splendidly Morita equivalent. Puig's refinement of Donovan's conjecture states that *there are only finitely many Puig equivalence classes of blocks of group algebras having P as a defect group.*

For symmetric groups, Donovan's conjecture holds by the work of J. Scopes [17], while the refined conjecture was verified by a different method by L. Puig [12]. For alternating groups, G. Hiss [6] deduced the validity of Donovan's conjecture from [17] by an easy general argument, and Puig's conjecture is deduced in a similar manner by R. Kessar [7]. But these arguments do not provide explicit Morita equivalences as in [17] or [12]. In [7, Theorem 1.7] it is shown that Scopes' Morita equivalence between certain blocks of symmetric groups induce Morita equivalence between the blocks of alternating groups covered by them. Our method give a very easy proof of [7, Theorem 1.7] when p is odd, see 3.7 below, and we are also able to deal with Rickard's tilting complex that generalizes Scopes' bimodule. Note that in [7, Theorem 1.9], explicit bounds are given for the number of possible Morita or Puig equivalence classes that can occur in blocks of alternating groups with fixed defect groups.

2. Algebras graded by a cyclic group

The main technical ingredient is that a bimodule over two $\mathcal{O}$-algebras graded by the cyclic group C_n of order n not divisible by p is C_n-graded if and only if the group $\hat{C}_n$ of linear characters of C_n acts on it. If a complex X induces a Rickard equivalence between two strongly C_n-graded algebras R and S, then we obtain a Rickard equivalence between the 1-components R_1 and S_1 provided that X is a complex of C_n-graded bimodules.

2.1. Let $C_n = \langle\sigma\rangle$ be the cyclic group of order n, and let $(\mathcal{K}, \mathcal{O}, k)$ be a p-modular system, where p does not divide n, such that $\mathcal{K}$ contains a primitive n-th root ϵ of unity. The group $\hat{C}_n := \mathrm{Hom}(C_n, \mathcal{K}^\times)$ of characters of C_n is isomorphic to C_n, and we have that $\hat{C}_n = \langle\hat{\sigma}\rangle$, where $\hat{\sigma}(\sigma) = \epsilon$.

2.2. Let $R = \bigoplus_{g\in C_n} R_g$ be a C_n-graded $\mathcal{O}$-algebra, not necessarily strongly graded. Then $\hat{C}_n$ acts on R as automorphisms of C_n-graded algebras by ${}^{\hat{\rho}}r_g = \hat{\rho}(g)r_g$, for all $g \in C_n$, $\hat{\rho} \in \hat{C}_n$, and $R_{\sigma^j} = \{r \in R \mid {}^{\hat{\sigma}}r = \epsilon^j r\}$, for $j = 0, \ldots, n-1$. We may form the skew group algebra $R * \hat{C}_n = \{r\hat{\rho} \mid r \in R,\ \hat{\rho} \in \hat{C}_n\}$.

Proposition 2.3. *The category R-*Gr *of C_n-graded (left) R-modules is isomorphic to the category $R * \hat{C}_n$-*Mod.

Indeed, if $M = \bigoplus_{g\in C_n} M_g$ is a C_n-graded R-module, then M becomes an $R * \hat{C}_n$-module with multiplication defined by $(r\hat{\rho})m_g = \hat{\rho}(g)rm_g$, for all $r \in R$, $g \in C_n$, $m_g \in M_g$ and $\hat{\rho} \in \hat{C}_n$. Conversely, if M is an $R * \hat{C}_n$-module, then the components of the corresponding graded module M are $M_{\sigma^j} = \{m \in M \mid \hat{\sigma}m = \epsilon^j m\}$.

2.4. Let R and S be two C_n-graded $\mathcal{O}$-algebras. Then $\hat{C}_n$ acts on $R \otimes_{\mathcal{O}} S^{\mathrm{op}}$ diagonally, by ${}^{\hat{\rho}}(r \otimes s) = {}^{\hat{\rho}}r \otimes {}^{\hat{\rho}^{-1}}s$, for all $\hat{\rho} \in \hat{C}_n$, $r \in R$ and $s \in S$, so we may consider the skew group algebra $(R \otimes_{\mathcal{O}} S^{\mathrm{op}}) * \hat{C}_n$. As above, *the category R-*Gr*-S of C_n-graded (R, S)-bimodules is isomorphic to the category $(R \otimes_{\mathcal{O}} S^{op}) * \hat{C}_n$-*Mod.

If M is an (R, S)-bimodule and $\hat{\rho} \in \hat{C}_n$, then the $\hat{\rho}$-th conjugate ${}^{\hat{\rho}}M$ of M is defined by

$$ {}^{\hat{\rho}}M = (R \otimes_{\mathcal{O}} S^{\mathrm{op}})\hat{\rho} \otimes_{R\otimes_{\mathcal{O}} S^{\mathrm{op}}} M. $$

Observe that we obtain an isomorphic (R, S)-bimodule if we set ${}^{\hat{\rho}}M = M$ as $\mathcal{O}$-modules, and multiplication $(r \otimes s) \cdot_{\hat{\rho}} m = {}^{\hat{\rho}^{-1}}(r \otimes s) \cdot m$, for all $m \in M$, $r \in R$, $s \in S$ and $\hat{\rho} \in \hat{C}_n$.

2.5. The above constructions are used to obtain a descent theorem for Rickard equivalences, which can also be regarded as an analogue of [4, Theorem 12.2].

Let G^+ be a normal subgroup of the finite group G, with $G/G^+ \simeq C_n$. Let b be a block of $\mathcal{O}G$ with defect group $D \leq G^+$, let $H = N_G(D)$, $H^+ = N_{G^+}(D)$, and let $c \in \mathcal{O}H$ be the Brauer correspondent of b. If e is a block of $\mathcal{O}G^+$ covered by b, then the Brauer correspondent $f \in \mathcal{O}H^+$ of e is covered by c, by the Harris-Knörr correspondence.

The group $\hat{C}_n$ acts on on the blocks of $\mathcal{O}G$ and $\mathcal{O}H$, and for each $\hat{\rho} \in \hat{C}_n$, the Brauer correspondent of ${}^{\hat{\rho}}b$ is ${}^{\hat{\rho}}c$. We denote by $\hat{C}_{n,b}$ the stabilizer of b under this action. The group C_n acts by conjugation of the blocks of $\mathcal{O}G^+$ and $\mathcal{O}H^+$, and for each $g \in C_n$, the Brauer correspondent of ${}^{g}e$ is ${}^{g}f$. Let $C_{n,e}$ denote the stabilizer of e in C_n. Consider the central idempotent

$$b^+ = \sum_{\hat{\rho} \in [\hat{C}_n/\hat{C}_{n,b}]} {}^{\hat{\rho}}b = \sum_{g \in [C_n/C_{n,e}]} {}^{g}e$$

of $\mathcal{O}G^+$, where $[C_n/C_{n,e}]$ denotes a full set of representatives for the left cosets of $C_{n,e}$ in C_n. The second equality follows by [4, Lemma 9.9]. Let c^+ be the similarly defined central idempotent of $\mathcal{O}H^+$, and consider the strongly C_n-graded algebras $R = b^+\mathcal{O}G = \mathcal{O}Ge\mathcal{O}G$ and $S = c^+\mathcal{O}H = \mathcal{O}He\mathcal{O}H$. Note that R is Morita equivalent to $e\mathcal{O}Ge$ and S is Morita equivalent to $f\mathcal{O}Hf$.

The following result is more general than we need in the case of alternating groups.

Theorem 2.6. *Let X be a complex of $(b\mathcal{O}G, c\mathcal{O}H)$-bimodules inducing a Rickard equivalence between $b\mathcal{O}G$ and $c\mathcal{O}H$, and consider the complex*

$$Y = \bigoplus_{\hat{\rho} \in [\hat{C}_n/\hat{C}_{n,b}]} {}^{\hat{\rho}}X$$

of (R, S)-bimodules. If ${}^{\hat{\rho}}Y \simeq Y$ as complexes of (R, S)-bimodules for all $\hat{\rho} \in \hat{C}_n$, then the block algebras $e\mathcal{O}G^+$ and $f\mathcal{O}H^+$ are Rickard equivalent.

3. Blocks of symmetric and alternating groups

For Broué's conjecture, we only need to consider the case $p > 2$. Indeed, if $p = 2$, then by [5, Lemma (7.A)], $D \simeq C_2 \times C_2$. In this case Broué's conjecture holds (even in the extended form) by [16, Section 6.3].

Theorem 3.1. *Let $p > 2$, $G = S_n$, $G^+ = A_n$, $\tilde{G} = \mathrm{Aut}(G^+)$, b^+ a block of $\mathcal{O}G^+$ with nontrivial abelian defect group D, $H^+ = N_{G^+}(D)$, and $c^+ \in \mathcal{O}H^+$ the Brauer correspondent of b^+. Then there exists a splendid tilting complex of $\tilde{G}/G^+$-graded $(b^+\mathcal{O}\tilde{G}, c^+\mathcal{O}\tilde{H})$-bimodules.*

We briefly present the steps in the proof of the theorem.

3.2. The block b^+ is C_2-invariant. Let b be a block of $\mathcal{O}G$ covering b^+ and let $c \in \mathcal{O}H$ be the Brauer correspondent of b. We denote ${}^{\hat{\sigma}}b = b^*$, where $C_2 = \langle\hat{\sigma}\rangle$. If $b \neq b^*$, then $b\mathcal{O}G \simeq b^+\mathcal{O}G^+$ and $c\mathcal{O}H \simeq c^+\mathcal{O}H^+$. Consequently, if X is a splendid tilting complex of $(b\mathcal{O}G, c\mathcal{O}H)$-bimodules, then X is also a splendid tilting complex of $(b^+\mathcal{O}G^+, c^+\mathcal{O}H^+)$-bimodules.

3.3. Assume that $b = b^*$, that is, b is *self associated.* Then $b = b^+$, $c = c^* = c^+$, and $b\mathcal{O}G$ and $c\mathcal{O}H$ are strongly C_2-graded algebras. We can aply Theorem 2.6 if we show that the splendid equivalence constructed in [2] and [3] is induced by a complex of C_2-graded bimodules. As this equivalence is a composition of several equivalences, we shall examine the steps one by one.

The bloc b corresponds uniquely to a p-core κ and a p-weight $w < p$, and $D \simeq C_p \times \cdots \times C_p$ (w times). Write $n = pw + t$. Then, by [2, Section 3], $c\mathcal{O}H \simeq \mathcal{O}N_{S_{pw}}(D) \otimes_{\mathcal{O}} \mathcal{O}S_t c_0$, where c_0 is the block of defect zero of $\mathcal{O}S_t$ corresponding to the p-core κ. Recall also that since b is self associated, κ is also self associated, that is, its diagram is symmetric with respect to the main diagonal. Moreover, $\mathcal{O}N_{S_{pw}}(D) \simeq \mathcal{O}((C_p \rtimes C_{p-1}) \wr S_w)$.

3.4. It was conjectured by R. Rouquier that *there are blocks of weight w of symmetric groups which are Morita equivalent to the principal block $B_0(S_p \wr S_w)$ of $\mathcal{O}(S_p \wr S_w)$*. This conjecture was proved in [2, Section 4], where one of these blocks was defined as follows.

Consider an abacus having $w + i(w-1)$ beads on the i-th runner, $i = 0, 1, \ldots, p-1$, and let ρ be the p-core having this abacus representation. Note that the core ρ is self-associated.

Let V be a set containing the disjoint union $U = U_1 \cup \cdots \cup U_w$ of sets of cardinality p, and let e be a block of $\mathcal{O}S(V)$ with defect group D corresponding to the p-core ρ. Let $\tilde{N}$ be the subgroup of $S(U)$ consisting of permutations sending each U_i to some U_j, let $N = \tilde{N} \times S(V \backslash U)$, and let $f \in \mathcal{O}N$ be the Brauer correspondent of e.

Then $\tilde{N} \simeq S_p \wr S_w$, and $f\mathcal{O}N \simeq B_0(S_p \wr S_w) \otimes_{\mathcal{O}} \mathcal{O}S_r f_0$, where f_0 is the block of defect zero corresponding to the core ρ, and $r = |V \setminus U|$.

By [2, Theorem 2], the Green correspondent M of $e\mathcal{O}S(V)$ with respect to $(S(V) \times S(V), S(V) \times N, \delta(D))$ induces a Morita equivalence

$$e\mathcal{O}S(V)\text{-Mod} \sim f\mathcal{O}N\text{-Mod},$$

and we have shown in [10] that M is a C_2-graded $(e\mathcal{O}S(V), f\mathcal{O}N)$-bimodule.

3.5. To see that there is a C_2-graded Rickard equivalence

$$\mathcal{H}^b(\mathcal{O}((C_p \rtimes C_{p-1}) \wr S_w) \otimes_{\mathcal{O}} \mathcal{O}S_t c_0) \sim \mathcal{H}^b(B_0(S_p \wr S_w) \otimes_{\mathcal{O}} \mathcal{O}S_t c_0),$$

note first that if $R = R_1 \oplus R_{-1}$ and $S = S_1 \oplus S_{-1}$ are C_2-graded algebras, then $R \otimes_{\mathcal{O}} S$ is C_2-graded in a natural way. Moreover, the wreath product $R \wr S_w = R^{\otimes w} * S_w$ is C_2-graded by

$$\deg(r_1 \otimes \cdots \otimes r_w)\sigma = \operatorname{sgn}(\sigma) \deg r_1 \ldots \deg r_w,$$

where $r_1, \ldots, r_w \in R$ are homogeneous elements and $\sigma \in S_w$.

By [15] there is a Rickard equivalence between $\mathcal{O}(C_p \rtimes C_{\frac{p-1}{2}})$ and $B_0(A_p)$, which, by [8, Example 5.5], extends to a C_2-graded equivalence between $\mathcal{O}(C_p \rtimes C_{p-1})$ and $B_0(S_p)$, induced by a complex X. Then by [8, Theorem 4.3], the complex $X \wr S_w$ induces a Rickard equivalence between $\mathcal{O}((C_p \rtimes C_{p-1}) \wr S_w)$ and $B_0(S_p \wr S_w)$.

Moreover, by [10, 3.5], $X \wr S_w$ is a complex of C_2-graded $(\mathcal{O}((C_p \rtimes C_{p-1}) \wr S_w), B_0(S_p \wr S_w))$-bimodules.

3.6. A C_2-graded Morita equivalence between the block $c_0\mathcal{O}S_t$ and $f_0\mathcal{O}S_r$ of defect zero is obtained as follows.

We have that $c_0 \in \mathcal{O}A_t$ and $f_0 \in \mathcal{O}A_r$ since the p-cores κ and ρ are self-associated, but these idempotents decompose as $c_0 = c' + c''$ and $f_0 = f' + f''$ in $\mathcal{O}A_t$ and $\mathcal{O}A_r$ respectively, where c', c'', respectively f', f'' are C_2-conjugated.

Let V' be a $(c'\mathcal{O}A_t, f'\mathcal{O}A_r)$-bimodule inducing a Morita equivalence. We may take $V' = U' \otimes_{\mathcal{O}} W'$, where U' is the unique simple left $c'\mathcal{O}A_t$-module, and W' is the unique simple right $f'\mathcal{O}A_r$-module. Let $V'' = U'' \otimes_{\mathcal{O}} W''$, where U'' and W'' are the C_2-conjugates of U' and W' respectively. Then $V := V' \oplus V''$ is a $(c_0\mathcal{O}A_t \otimes_{\mathcal{O}} (f_0\mathcal{O}A_r)^{\mathrm{op}})$-module, which extends to the diagonal subalgebra $\Delta = \Delta(c_0\mathcal{O}S_t \otimes_{\mathcal{O}} (f_0\mathcal{O}S_r)^{\mathrm{op}})$, hence by [8, Theorem 3.4], $\operatorname{Ind}_{\Delta}^{c_0\mathcal{O}S_t \otimes_{\mathcal{O}} (f_0\mathcal{O}S_r)^{\mathrm{op}}} V$ induces the desired C_2-graded Morita equivalence.

3.7. There is a C_2-graded derived equivalence

$$\mathcal{H}^b(b\mathcal{O}S_n) \sim \mathcal{H}^b(e\mathcal{O}S(V)).$$

In fact, Rickard [13] has conjectured that *any two blocks of the same weight w of symmetric groups are derived equivalent.* He proposed a candidate for a tilting complex which is a generalization of Scopes' Morita equivalence [17]. The conjecture has been recently verified by Chuang and Rouquier [3]. Actually, the derived equivalence between $b\mathcal{O}S_n$ and $e\mathcal{O}S(V)$ is obtained as a composition of equivalences between blocks forming a so called $[w:k]$ pair, defined as follows.

Assume that $a\mathcal{O}S_n$ is a block of weight w of $\mathcal{O}S_n$ corresponding to an abacus whose j-th runner has k more beads than the $(j-1)$-th runner. Switching the number of beads on these two runners, we obtain a block $b\mathcal{O}S_{n-k}$ of weight w of $\mathcal{O}S_{n-k}$. If $k \geq w$, Scopes [17] proved that $a\mathcal{O}S_n$ and $b\mathcal{O}S_{n-k}$ are Morita equivalent. Observe that $M := a\mathcal{O}S_n b$ is an $(a\mathcal{O}S_n, b\mathcal{O}S_{n-k} \otimes_{\mathcal{O}} \mathcal{O}S_k)$-bimodule. Then the Morita equivalence is induced by $M \otimes_{\mathcal{O}S_k} \mathcal{O}$, and we show in [10, 3.7.1] that $M \otimes_{\mathcal{O}S_k} \mathcal{O}$ is an $(A \otimes_{\mathcal{O}} B^{\mathrm{op}}) * \hat{C}_2$-module, hence a C_2-graded (A, B)-bimodule by 2.4.

For arbitrary k, Rickard's complex is a generalization of Scopes' bimodule. We recall its construction following [13] and [3]. Let $r = \max\{i \in \mathbb{N} \mid i(k+i) \leq w\}$, and for $0 \leq i \leq r$ let b_i be the block of $\mathcal{O}S_{n-k-i}$ having $w - i(k+i)$ and represented by an abacus obtained from the abacus of b by moving i of the beads on the j-th runner onto the $(j-1)$-th runner. Consider the $(a\mathcal{O}S_n, b\mathcal{O}S_{n-k})$-bimodule

$$Y_i = a\mathcal{O}S_n b_i \otimes_{b_i \mathcal{O}S_{n-k-i}} b_i \mathcal{O}S_{n-k} b.$$

Using the map

$$b_{i-1}\mathcal{O}S_{n-k-i+1} b_i \otimes_{b_i \mathcal{O}S_{n-k-i}} b_i \mathcal{O}S_{n-k-i+1} b_{i-1} \to b_i \mathcal{O}S_{n-k-i+1}$$

induced by multiplication, and the bimodule isomorphisms

$$a\mathcal{O}S_n b_{i-1} \otimes_{b_{i-1}\mathcal{O}S_{n-k-i+1}} b_{i-1}\mathcal{O}S_{n-k-i+1} b_i \simeq a\mathcal{O}S_n b_i,$$

$$b_i\mathcal{O}S_{n-k-i+1} b_{i-1} \otimes_{b_{i-1}\mathcal{O}S_{n-k-i+1}} b_{i-1}\mathcal{O}S_{n-k} b \simeq b_i \mathcal{O}S_{n-k} b,$$

one obtains a map $Y_i \to Y_{i-1}$ of $(a\mathcal{O}S_n, b\mathcal{O}_{n-k})$-bimodules. In order to obtain a complex, the additional structure of these bimodules is needed. Let

$$X_i = (a\mathcal{O}S_n b_i \otimes_{\mathcal{O}S_{k+i}} \mathcal{O}) \otimes_{b_i\mathcal{O}S_{n-k-i}} (\mathcal{O}^- \otimes_{\mathcal{O}S_i} b_i \mathcal{O}S_{n-k} b).$$

The map $Y_i \to Y_{i-1}$ induces a map $X_i \to X_{i-1}$. By [2],

$$X := (\cdots \to 0 \to X_r \to \cdots \to X_1 \to X_0 \to 0 \to \dots)$$

is a splendid tilting complex of $(a\mathcal{O}S_n, b\mathcal{O}S_{n-k})$-bimodules, and we show in [10] that the map $X_i \to X_{i-1}$ is $(A \otimes_{\mathcal{O}} B^{\mathrm{op}}) * \hat{C}_2$-linear.

3.8. Finally, the compatibility with p'-outer automorphism groups also holds, and in fact there are very few cases to look at. With the notations of 3.1, assume that b^+ is the principal block of $\mathcal{O}G^+$ and b the principal block of $\mathcal{O}G$.

Denoting $\tilde{G} = \mathrm{Aut}(G^+)$ and $\tilde{H} = N_{\tilde{G}}(D)$, we have that $G \leq \tilde{G}$, and $G = \tilde{G}$ if $n \neq 6$ and $|\tilde{G}/G| = 2$ if $n = 6$.

Let $n \neq 6$. If $b \neq b^*$, then the algebras $b\mathcal{O}G$ and $b^+\mathcal{O}G^+$ are isomorphic, and in this case, the compatibility holds by [8, (5.4)]. If $b = b^* = b^+$, then the required compatibility just means that there is a C_2-graded Rickard equivalence between $b\mathcal{O}G$ and $c\mathcal{O}H$, and this is what we have proved above.

Let $n = 6$, so $|G^+| = 2^3 \cdot 3^2 \cdot 5$. If $p = 5$, then there is a $\tilde{G}/G^+$-graded Rickard equivalence between $b^+\mathcal{O}\tilde{G}$ and $c^+\mathcal{O}\tilde{H}$ by [15] and [8, Example 5.5]. If $p = 3$, then $D \simeq C_3 \times C_3$. In this case Okuyama constructed in [11] (by using a different method) a Rickard equivalence between $b^+\mathcal{O}G^+$ and $c^+\mathcal{O}G^+$, and this is compatible with p'-extensions by [9, Example 3.11].

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