数理解析研究所講究録 第2118巻 2019年 80-86

HERMITIAN OPERATORS ON SPACES OF VECTOR-VALUED LIPSCHITZ MAPS

SHIHO OI
NIIGATA PREFECTURAL NAGAOKA HIGH SCHOOL

ABSTRACT. We characterize unital surjective linear isometries on algebra of Lipschitz maps with the values in matrix algebras with the ℓ_1 norm $\|\cdot\|_\infty + L(\cdot)$. The fact that Hermitian operators completely characterizes unital surjective linear isometries lets us prove this. The method is sometime referred to as Lumer's method (see [5]). To apply Lumer's method, one needs to characterize Hermitian operators.

1. INTRODUCTION

We study Hermitian operators in Section 2 and surjective linear isometries in Section 3 . The relations between these operators emerge most clearly, when we see the fact that if T is a Hermitian operator and U is a surjective linear isometry then UTU^{-1} is a Hermitian operator. Lumer in [10, 11] initiated a research to characterize surjective linear isometries by applying this fact, and this method is called 'Lumer's method' now.

If a semi-inner product $[\cdot,\cdot]$ satisfies $[v,v]=\|v\|_v^2$ for every v in V, then $[\cdot,\cdot]$ is said to be a semi-inner product compatible with the norm of V. In this paper we abbreviate a semi-inner product compatible with the norm as a semi-inner product.

Definition 1.1. Let $[\cdot,\cdot]$ be a semi-inner product on a complex Banach space V. Then a bounded linear operator T on V is said to be a Hermitian operator if $[Tv,v]\in\mathbb{R}$ for all $v\in V$.

We first introduce the definition of the space of all $E-$valued Lipschitz maps.

Definition 1.2. Let X be a compact metric space. For any continuous map $F:X\to E$, we call a Lipschitz map if

$$L(F)=\sup\{\frac{\|F(x)-F(y)\|_E}{d(x,y)}:x,y\in X\ x\neq y\}<\infty.$$

The space of all $E-$valued Lipschitz maps is denoted by $\mathrm{Lip}(X,E)$. We define some complete norms on $\mathrm{Lip}(X,E)$. We introduce two norms among them, which are known as simple but important norms on $\mathrm{Lip}(X,E)$. First is

$$\|\cdot\|_{\max}=\max\{L(\cdot),\|\cdot\|_\infty\},$$

and second is

$$\|\cdot\|_L=L(\cdot)+\|\cdot\|_\infty.$$

Both norms make $\mathrm{Lip}(X,E)$ a complete, so $\mathrm{Lip}(X,E)$ is a Banach space. However there are points of agreement and difference between them. In this paper, we shall compare two norms. For any $f\in\mathrm{Lip}(X)$ and $e\in E$ we define the tensor product $f\otimes e$ by

$$(f\otimes e)(x)=f(x)e\in E,\quad x\in X.$$

We have $f\otimes e\in\mathrm{Lip}(X,E)$ by definition.

2. Hermitian operator

Fleming and Jamison in [4] proved the characterization of Hermitian operators on $C(X, E)$.

Theorem 2.1. [4, Theorem 4] *Let X be a compact Hausdorff space and E a complex Banach space. Suppose that $C(X, E)$ is the Banach space of all continuous functions on X with values in E with the supremum norm. A bounded linear operator T on $C(X, E)$ is a Hermitian operator if and only if for each $x \in X$ there is a Hermitian operator $\phi(x)$ on E such that for any $F \in C(X, E)$ we have*

$$TF(x) = \phi(x)F(x), \quad x \in X.$$

In the case of Banach spaces of vector-valued Lipschitz maps, the study of Hermitian operator has been researched.

2.1. Hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_{\max}$.

Defining $\|\cdot\|_{\max}$ on $\mathrm{Lip}(X, E)$, some researchers study Hermitian operators. The following theorem is proved by Botelho, Jamison, Jiménez-Vargas and Villegas-Vallecillos.

Theorem 2.2. [2, Theorem 2.4] *Let X be a compact and 2- connected metric space and E a Banach space. Then T is a Hermitian operator on $(\mathrm{Lip}(X, E), \|\cdot\|_{\max})$ if and only if there exists a Hermitian operator $\phi : E \to E$ such that*

$$TF(x) = \phi(F(x)), \quad F \in \mathrm{Lip}(X, E), \quad x \in X.$$

2.2. Hermitian operators on $\mathrm{Lip}(X, E)$ with $\|\cdot\|_L$.

We conjecture that when we consider Hermitian operators on $\mathrm{Lip}(X, E)$ with respect to $\|\cdot\|_L$, a similar argument will essentially still work to give similar results. But no proof of the characterization of Hermitian operators in full generality has been published. General case is still open. However Botelho, Jamison, Jiménez-Vargas and Villegas-Vallecillos showed that similar statement with [2, Theorem 2.4] holds in the case when $E = \mathbb{C}$ in [3].

Theorem 2.3. [3, Theorem 3.1] *Let X be a compact metric space. A bounded linear operator T on $\mathrm{Lip}(X)$ is Hermitian operator if and only if there exists a real number λ such that*

$$T = \lambda \cdot I.$$

Moreover Hatori and the author have characterized in the case where E is a uniform algebra.

Theorem 2.4. [6, Theorem 8] *Let X be a compact metric space and A be a uniform algebra. Then a bounded linear operator $T : \mathrm{Lip}(X, A) \to \mathrm{Lip}(X, A)$ is a Hermitian operator if and only if there exists a real-valued function $f \in A$ such that*

$$T = 1 \otimes f \cdot I.$$

In the case of non-commutative, do we obtain a similar statement? When E is a Banach space of a finite dimension, we obtain the following the theorem in [12].

Theorem 2.5. *Let X be a compact metric space and E a Banach space of a finite dimension. Then T is a Hermitian operator on $\mathrm{Lip}(X, E)$ if and only if there exists a Hermitian operator $\phi : E \to E$ such that*

$$TF(x) = \phi(F(x)), \quad F \in \mathrm{Lip}(X, E), \quad x \in X.$$

We shall briefly outline the necessary condition of Theorem 2.5. First we need a key lemma. The next lemma is fundamental in this article.

Lemma 2.6. *Let E be a Banach space of a finite dimension. Then we have*

$$\mathrm{Lip}(X, E) = \mathrm{Lip}(X) \otimes E.$$

If E is infinite dimension, we have $\mathrm{Lip}(X) \otimes E \subset \mathrm{Lip}(X, E)$.This is the main reason why we assume that E is a Banach space of a finite dimension.

Proof of Theorem 2.5. For any $e \in E$, we obtain $L(1 \otimes e) = 0$. Moreover, T is a Hermitian operator, we have $\mathbb{R} \ni [T(1 \otimes e), 1 \otimes e]_L$. This shows that $L(T(1 \otimes e)) = 0$. This implies that $T(1 \otimes e) \in 1 \otimes E$ for all $e \in E$. Thus we define a Hermitian operator $\phi : E \to E$ by

$$T(1 \otimes e) = 1 \otimes \phi(e), \quad e \in E.$$

Next, we define a Hermitian operator $T_0 : \mathrm{Lip}(X, E) \to \mathrm{Lip}(X, E)$ by

$$(T_0 F)(x) = (TF)(x) - \phi(F(x)).$$

We divide our proof into two steps.
(**Step 1**) For all $e \in E$ with $\|e\| = 1$, we define $S_e : \mathrm{Lip}(X) \to \mathrm{Lip}(X)$ as follows.

$$\begin{array}{ccc} \mathrm{Lip}(X) & \xrightarrow{S_e} & \mathrm{Lip}(X) \\ \Downarrow & & \Downarrow \\ f & \longmapsto & [T_0(f \otimes e)(\cdot), e]_E \end{array}$$

We also get $S_e(1)(x) = [T_0(1 \otimes e)(x), e]_E = 0$. Since we see that S_e is a Hermitian operator on $\mathrm{Lip}(X)$, by Theorem 2.3 we have

$$[T_0(f \otimes e)(x), e]_E = 0, \quad f \in \mathrm{Lip}(X).$$

(**Step 2**) For all $f \in \mathrm{Lip}(X), x \in X$, we define $S_{fx} : E \to E$ as follows.

$$\begin{array}{ccc} E & \xrightarrow{S_{fx}} & E \\ \Downarrow & & \Downarrow \\ e & \longmapsto & T_0(f \otimes e)(x) \end{array}$$

By $[T_0(f \otimes e)(x), e]_E = 0$ for any $e \in E$ with $\|e\| = 1$, applying [10, Theorem 5] we obtain

$$T_0(f \otimes e)(x) = 0.$$

By Lemma 2.6, we have

$$T_0(F) = 0, \quad F \in \mathrm{Lip}(X, E).$$

We conclude that

$$(TF)(x) = \phi(F(x)).$$

□

3. SURJECTIVE LINEAR ISOMETRY

The basic problem of interest is whether every surjective linear isometry U of $\mathrm{Lip}(X, E)$ that carries 1 into 1 is of the form

$$U(F) = F \circ \phi,$$

where $\phi : X \to X$ is a homeomorphism.

3.1. **Isometry on** $\mathrm{Lip}(X,E)$ **with** $\|\cdot\|_{\max}$. If we define the norm $\|\cdot\|_{\max}$ on $\mathrm{Lip}(X,E)$, there are some results for isometries. We now introduce one of them.

Theorem 3.1. [1, Theorem 6] *Let X and Y be compact metric spaces and E and D quasi sub-reflexive Banach spaces*[1] *with trivial centralizers. Let $U : (\mathrm{Lip}(X,E), \|\cdot\|_{\max}) \to (\mathrm{Lip}(Y,D), \|\cdot\|_{\max})$ be a surjective linear isometry so that U and U^{-1} have property Q.*[2] *Then there exist $\varphi : Y \to X$ with $L(\varphi) \le \max\{1, \mathrm{diam}(X)\}$, $L(\varphi^{-1}) \le \max\{1, \mathrm{diam}(Y)\}$ and Lipschitz map $y \to \psi(y)$, where $\psi(y) : E \to D$ is surjective linear isometry for $y \in Y$ such that*

$$UF(y) = \psi(y)F(\varphi(y)), \quad F \in \mathrm{Lip}(X,E), y \in Y.$$

The next theorem is critical to their arguments in [1]. Note that a map δ_x is the evaluation function on $\mathrm{Lip}(X,E)$ for $x \in X$. In addition, a map Γ is an isometric linear embedding from $\mathrm{Lip}(X,E)$ with the norm $\|\cdot\|_{\max}$ into the algebra of continuous map on a compact Hausdorff space (We omit the details, which are notationally complicated. See [1]).

Theorem 3.2. *Let X be compact metric spaces and E quasi sub-reflexive Banach spaces. If $x \in X$ and $e^* \in \mathrm{ext}(E^*)$, we have*

$$e^* \circ \delta_x \in \mathrm{ext}(\Gamma(\mathrm{Lip}(X,E)))^*$$

3.2. **Isometry on** $\mathrm{Lip}(X,E)$ **with** $\|\cdot\|_L$. The case when we consider $\mathrm{Lip}(X,E)$ with the norm $\|\cdot\|_L$ can be proved. Jarosz and Pathak proved that this statement holds in the case when $E = \mathbb{C}$.

Theorem 3.3. [9, Example 8] *Let X_j be a compact metric space for $j = 1,2$. Suppose that $U : \mathrm{Lip}(X_1) \to \mathrm{Lip}(X_2)$ is a map.Then U is a surjective isometry if and only if there exists $\alpha \in \mathbb{C}$ with $|\alpha| = 1$, a surjective isometry $\phi : X_2 \to X_1$ such that*

$$U(f) = \alpha f \circ \phi(x), \quad f \in \mathrm{Lip}(X_1), x \in X_2.$$

Moreover Hatori and the author have characterized in the case where E is a unital commutative C^* algebra.

Theorem 3.4. [7, Corollary 14] *Let X_j be a compact metric space and Y_j a compact Hausdorff space for $j = 1,2$. Suppose that $U : \mathrm{Lip}(X_1, C(Y_1)) \to \mathrm{Lip}(X_2, C(Y_2))$ is a map.Then U is a surjective isometry if and only if there exists $h \in C(Y_2)$ with $|h| = 1$ on Y_2, a continuous map $\phi : X_2 \times Y_2 \to X_1$ such that $\phi(\cdot, y) : X_2 \to X_1$ is a surjective isometry for every $y \in Y_2$, and a homeomorphism $\tau : Y_2 \to Y_1$ which satisfy that*

$$U(F)(x,y) = h(y)F(\phi(x,y), \tau(y)), \quad (x,y) \in X_2 \times Y_2$$

Essential to the Lumer's method is the following theorem.

Theorem 3.5. *Let V_1 and V_2 be Banach spaces. Suppose that T is a Hermitian operator on V_1 and U is an isometry from V_1 onto V_2. Then we have UTU^{-1} is a Hermitian operator.*

It is interesting that this can result in a significant loss of convex or extreme points. Note that $M_n(\mathbb{C})$ is the Banach algebra of complex matrices of degree n by $M_n(\mathbb{C})$. The metric we consider on $M_n(\mathbb{C})$ is operator norm. Theorem 2.5 deduce Theorem 3.6 in [12] by applying above Theorem 3.5.

[1] E is quasi sub-reflexive if $e^* \in \mathrm{ext}(E^*) \Longrightarrow \exists e \in S(E)$ *s.t.* $e^*(e) = 1$.

[2] U have property Q if $\forall y \in Y, \forall u \in D, \exists F \in \mathrm{Const}(X,E)$ *s.t.* $UF(y) = u$

Theorem 3.6. *Let X_j be a compact metric space for $j = 1, 2$. Then $U : \mathrm{Lip}(X_1, M_n(\mathbb{C})) \to \mathrm{Lip}(X_2, M_n(\mathbb{C}))$ is a linear surjective isometry such that $U(1) = 1$ if and only if there exists a unitary matrix $V \in M_n(\mathbb{C})$, and a surjective isometry $\varphi : X_2 \to X_1$, such that*

$$(UF)(x) = VF(\varphi(x))V^{-1}, \quad F \in \mathrm{Lip}(X_1, M_n(\mathbb{C})),\ x \in X_2$$

or

$$(UF)(x) = VF^t(\varphi(x))V^{-1}, \quad F \in \mathrm{Lip}(X_1, M_n(\mathbb{C})),\ x \in X_2,$$

where $F^t(y)$ denote transpose of $F(y)$ for $y \in X_1$.

We now prove Theorem 3.6. We shall omit the proof of sufficient condition for Theorem 3.6. We assume that $U : \mathrm{Lip}(X_1, M_n(\mathbb{C})) \to \mathrm{Lip}(X_2, M_n(\mathbb{C}))$ be a surjective linear isometry with $U(1) = 1$. We denote the set of all Hermitian elements of $M_n(\mathbb{C})$ by $H(M_n(\mathbb{C}))$. We denote a left multiplication operator for $A \in M_n(\mathbb{C})$ by M_A, which is a bounded operator such that

$$M_A(B) = AB$$

for $B \in M_n(\mathbb{C})$.

Lemma 3.7. *If $A \in H(M_n(\mathbb{C}))$, then M_A is a Hermitian operator.*

Theorem 3.8. [13, Corollary] *A operator T be a Hermitian on a unital C^* algebra if and only if there exist a Hermitian element E and a *-derivation D such that*

$$T = M_E + iD.$$

Moreover, by an argument for the Banach algebra of complex matrices, we obtain the next theorem.

Theorem 3.9. *If D on $M_n(\mathbb{C})$ is *-derivation then there exists $B \in M_n(\mathbb{C})$ with $B^* = -B$ such that*

$$D(A) = BA - AB, \quad A \in M_n(\mathbb{C}).$$

Applying above two theorem and Theorem 2.5, we obtain a characterization of Hermitian operators on $\mathrm{Lip}(X, M_n(\mathbb{C}))$.

Theorem 3.10. *A operator $T : \mathrm{Lip}(X, M_n(\mathbb{C})) \to \mathrm{Lip}(X, M_n(\mathbb{C}))$ is a Hermitian if and only if there exist $E \in H(M_n(\mathbb{C}))$ and D: *-derivation on $M_n(\mathbb{C})$ such that*

$$T = M_{1\otimes E} + i\hat{D}.$$

Note that

$$M_{1\otimes E}(F)(x) = EF(x), \quad F \in \mathrm{Lip}(X, M_n(\mathbb{C}))$$

and

$$\begin{aligned}\hat{D}(F)(x) &= D(F(x)), \quad F \in \mathrm{Lip}(X, M_n(\mathbb{C})), x \in X \\ &= BF(x) - F(x)B, \quad B \in M_n(\mathbb{C}) \text{ with } B^* = -B.\end{aligned}$$

Lemma 3.11. *For any $E \in H(M_n(\mathbb{C}))$, there exists $E_0 \in H(M_n(\mathbb{C}))$ such that*

$$U(1 \otimes E) = 1 \otimes E_0$$

Proof. Since $E \in H(M_n(\mathbb{C}))$ we have $M_{1\otimes E}$ is a Hermitian operator on $\mathrm{Lip}(X, M_n(\mathbb{C}))$. By Theorem 3.5 $UM_{1\otimes E}U^{-1}$ is a Hermitian operator. Applying Theorem 3.10 there exists $E_0 \in H(M_n(\mathbb{C}))$ and *-derivation D on $M_n(\mathbb{C})$ such that

$$(UM_{1\otimes E}U^{-1})(F)(x) = E_0F(x) + iD(F(x))$$

for any $F \in \mathrm{Lip}(X_1, M_n(\mathbb{C}))$ and $x \in X_2$. This implies that

$$U(1 \otimes E)(x) = (UM_{1\otimes E}U^{-1})(1)(x) = E_0$$

for all $x \in X_2$. □

We define a map $\psi_0 : H(M_n(\mathbb{C})) \to H(M_n(\mathbb{C}))$ by

$$U(1 \otimes E) = 1 \otimes \psi_0(E). \tag{1}$$

Since $M_n(\mathbb{C}) = H(M_n(\mathbb{C})) + iH(M_n(\mathbb{C}))$, we also define a map $\psi : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ by

$$\psi(A) = \psi(E_1 + iE_2) = \psi_0(E_1) + i\psi_0(E_2).$$

We have

$$U(1 \otimes A) = 1 \otimes \psi(A), \quad A \in M_n(\mathbb{C}). \tag{2}$$

Since U is a surjective linear isometry, we deduce the next lemma.

Lemma 3.12. *The map ψ is a complex linear isometry from $M_n(\mathbb{C})$ onto itself.*

Lemma 3.13. *For any $f \in \mathrm{Lip}(X_1)$, there exists $g \in \mathrm{Lip}(X_2)$ such that*

$$U(f \otimes 1) = g \otimes 1$$

Proof. For any $B \in M_n(\mathbb{C})$ with $B^* = -B$, we define $D(A) := BA - AB$ for $A \in M_n(\mathbb{C})$. We have

$$U^{-1}i\widehat{D}U = M_{1\otimes E} + i\widehat{D'} = i\widehat{D'}$$

because $0 = (U^{-1}i\widehat{D}U)(1) = (M_{1\otimes E} + i\widehat{D'})(1) = 1 \otimes E$. In addition we have

$$\begin{aligned} iU^{-1}(BU(f \otimes 1) - U(f \otimes 1)B)(x) &= (U^{-1}i\widehat{D}U)(f \otimes 1)(x) \\ &= i\widehat{D'}(f \otimes 1)(x) = 0. \end{aligned}$$

This shows $BU(f \otimes 1) = U(f \otimes 1)B$. Thus there exists $g(x) \in \mathbb{C}$ such that

$$U(f \otimes 1)(x) = g(x)1.$$

It is easy to see that $g \in \mathrm{Lip}(X_2)$ since $U(f \otimes 1) \in \mathrm{Lip}(X_2, M_n(\mathbb{C}))$. □

By Theorem Jarosz in [8], we get the next lemma.

Lemma 3.14. *There exists a surjective isometry $\varphi : X_2 \to X_1$ such that*

$$U(f \otimes 1)(x) = f(\varphi(x)) \otimes 1 \tag{3}$$

for all $f \in \mathrm{Lip}(X_1)$ and $x \in X_2$.

Proof of Theorem 3.6. By (1) and (3), for $E \in H(M_n(\mathbb{C}))$, $f \in \mathrm{Lip}(X_1)$ and $x \in X_2$, we have that

$$\begin{aligned} U(f \otimes E)(x) &= U(M_{1\otimes E}(f \otimes 1))(x) = UM_{1\otimes E}U^{-1}U(f \otimes 1)(x) \\ &= (M_{1\otimes\psi_0(E)} + i\widehat{D})(U(f \otimes 1))(x) \\ &= M_{1\otimes\psi_0(E)}(U(f \otimes 1))(x) + i\widehat{D}(U(f \otimes 1))(x) \\ &= \psi_0(E)(U(f \otimes 1)(x)) = f(\varphi(x))\psi_0(E) \end{aligned}$$

For any $A \in M_n(\mathbb{C})$, there exists $E_1, E_2 \in H(M_n(\mathbb{C}))$ such that $A = E_1 + iE_2$. Applying (2), we get

$$\begin{aligned} U(f \otimes A)(x) &= U(f \otimes (E_1 + iE_2))(x) \\ &= U(f \otimes E_1)(x) + iU(f \otimes E_2)(x) \\ &= f(\varphi(x))\psi_0(E_1) + if(\varphi(x))\psi_0(E_2) \\ &= f(\varphi(x))\psi(A) \\ &= \psi((f \otimes A)(\varphi(x))) \end{aligned}$$

for any $f \in \mathrm{Lip}(X_1)$ and $x \in X_2$. By Lemma 2.6 and the theorem of Schur in [14], we conclude Theorem 3.6. □

In Theorem 3.6 we characterize unital surjective linear isometries on the algebra of Lipschitz maps from X into $M_n(\mathbb{C})$ whose centralizers are trivial. Comparison of Theorem 6 in [1] and Theorem 3.6 gives a fact that a similar result holds even though way to prove are different.

References

[1] F. Botelho, R. J. Fleming and J. E. Jamison, *Extreme points and isometries on vector-valued Lipschitz spaces,* J. Math. Anal. Appl. **381** (2011), 821–832.

[2] F. Botelho and J. Jamison, A. Jiménez-Vargas and M. Villegas-Vallecillos, *Hermitian operators on Lipschitz function spaces,* Studia Math., **215** (2013), 127–137.

[3] F. Botelho and J. Jamison, A. Jiménez-Vargas and M. Villegas-Vallecillos, *Hermitian operators on Banach algebras of Lipschitz functions,* Proc. Amer. Math, Soc. **142** (2014), 3469–3481.

[4] R. J. Fleming and J. E. Jamison, *Hermitian Operators on C(X,E) and the Banach-Stone Theorem,* Math. Z., **170** (1980), 77–84.

[5] R. J. Fleming and J. E. Jamison, *Isometries on Banach spaces,* Chapman & Hall/CRC Monographs and Surveys in Pure and Applied Mathematics, 129. Chapman & Hall/CRC, Boca Raton, FL, 2003. x+197 pp. ISBN: 1-58488-040-6.

[6] O. Hatori and S. Oi, *Hermitian operators on Banach algebras of vector-valued Lipschitz maps,* J. Math. Anal. Appl. **452** (2017), 378–387; *Corrigendum to "Hermitian operators on Banach algebras of vector-valued Lipschitz maps" [J. Math. Anal. Appl. 452 (2017) 378–387] [MR3628025].* J. Math. Anal. Appl. **453** (2017), 1151–1152.

[7] O. Hatori and S. Oi, *Isometries on Banach algebras of vector-valued maps,* to appear in Acta Sci. Math.

[8] K. Jarosz, *Isometries in semisimple, commutative Banach algebras,* Proc. Amer. Math, Soc. **94** (1985), 65–71.

[9] K. Jarosz and V. D. Pathak, *Isometries between function spaces,* Trans. Amer. Math. Soc. **305** (1988), 193–206.

[10] G. Lumer, *Semi-inner product of bounded maps into Banach space,* Trans. Amer. Math. Soc. **100** (1961), 26–43.

[11] G. Lumer, *On the isometries of reflexive Orlicz spaces,* Ann. Inst. Fourier, (1963), 99–109.

[12] S. Oi, *Hermitan operators and isometries on algebras of matrix-valued Lipschitz maps,* to submitted.

[13] A. M. Sinclair, *Homomorphisms and Derivations on Semisimple Banach Algebras,* Proc. Amer. Math, Soc. **24** (1970), 209–214.

[14] I. Schur, *Einige Bermerkungen zur determinanten theorie,* S.B. Preuss Akad. Wiss. Berlin **25** (1925), 454—463.

Current address: Niigata prefectural Hakkai High School
E-mail address: shiho.oi.pmfn20@gmail.com