

数理解析研究所講究録 2120

RIMS 共同研究 (公開型)

Profinite monodromy, Galois representations,  
and Complex functions

京都大学数理解析研究所

2019年7月

数理解析研究所講究録は、京都大学数理解析研究所の共同利用研究集会および共同研究の記録として1964年に刊行が開始されました。当研究所が全国共同利用研究所として発足した翌年のことでしたが、以来半世紀、毎年数十巻を刊行し、2016年には第2000巻が刊行されるに至りました。第1巻から第2000巻までに収録された論文数は29,265編、総頁数は342,960頁という膨大なものであり、最先端の数学・数理科学分野の研究状況を伝えるのみならず、我が国の数学・数理科学の発展の歴史を留める文献として、他に類例を見ない論文集となっています。

講究録の内容は当研究所のウェブサイトおよび京都大学の学術情報リポジトリにおいても公開され、年間の総アクセス数は1,380,032回（2017年度）を数えるなど、多数の方にご利用いただいています。

講究録の使用言語は論文著者の判断に任されていますが、結果的に日本語が多用されていることが特徴の一つとなっています。その結果、講究録は、数学・数理科学の広い領域における最先端の専門知識に母国語でアクセスできるものとして、近年の英語化の流れの中で、重要な文献となりつつあります。

当研究所の共同利用事業に参加し講究録の論文を執筆していただいた多数の方々に対し、講究録を大きく成長させていただいたことを深く感謝いたしますとともに、これからも、当研究所の国際共同利用・共同研究拠点(\*)としての活動にご参加いただき、講究録の発展にご協力いただけますよう心よりお願い申し上げます。

\*数理解析研究所は2018年11月13日、共同利用・共同研究拠点の認定が廃止され、新しく国際共同利用・共同研究拠点に認定されました。

*RIMS Kôkyûroku 2120*

*Profinite monodromy, Galois representations,  
and Complex functions*

*May 21 ~23, 2018*

*edited by Masanobu Kaneko*

*July, 2019*

*Research Institute for Mathematical Sciences*

*Kyoto University, Kyoto, Japan*

This is a report of research done at the Research Institute for Mathematical Sciences,  
an International Joint Usage/Research Center located in Kyoto University.  
The papers contained herein are in final form and will not be submitted for publication elsewhere.

講究録

Kôkyûroku

RIMS Kôkyûroku was started in 1964 as the proceedings of symposia, colloquia and workshops supported by RIMS, the Research Institute for Mathematical Sciences, Kyoto University. It was the next year of the establishment of RIMS as one of the Nationwide Cooperative Research Centers. For half a century since then, several dozen volumes have been issued each year, and the 2,000th volume was issued in 2016. The volumes of Kôkyûroku from the 1st through the 2,000th, containing enormous 29,265 articles and 342,960 pages, not only deliver the latest research activities in mathematics and mathematical sciences but also constitute valuable and incomparable collections of articles that pass down history of progress of mathematics and mathematical science in Japan.

Articles in Kôkyûroku are available on the websites of RIMS and Kyoto University Research Information Repository. They are very frequently accessed on the internet, with a total of as many as 1,380,032 accesses in 2017.

The authors choose the languages to write articles, and many are written in Japanese, which is one of the characteristics of Kôkyûroku. As a result, Kôkyûroku is regarded as a significant and important literature which allows easy access to the latest specialized knowledge in the large fields of mathematics and mathematical sciences written in native language for Japanese readers, while more and more research papers are being written in English in recent years.

We are deeply grateful to many of those who have participated in cooperative research activities of RIMS and greatly developed Kôkyûroku. We heartily ask for your continuous participation in research activities at RIMS as an International Joint Usage/Research Center(\*) and your warm support and cooperation for the fruitful development of Kôkyûroku.

\* RIMS was certified as an International Joint Usage/Research Center on Nov. 13, 2018.

## まえがき

この講究録は 2018 年 5 月 21 日（月）から 23 日（水）にかけて京都大学数理解析研究所において開催された研究集会（RIMS 共同研究（公開型））“Profinite monodromy, Galois representations, and Complex functions”（邦題は「副有限モノドロミー，ガロア表現，および複素関数」）の報告集で，講演して下さいましたそれぞれの方からご寄稿頂きました。

この研究集会は伊原康隆先生の傘寿のお祝いも兼ね，長年にわたる先生のお仕事に関わりの深い研究テーマを中心に，概説講演，最先端の研究発表を織り交ぜ，また先生ご自身にもご講演頂いて，大変実りある集会となりました。集会のタイトルは先生の代表作の一つ，1986 年の Annals 論文 “Profinite braid groups, Galois representations and complex multiplications” のもじりで，若干とってつけた感じがするのはその故ですが，先生のお仕事同様，幅広い話題についての興味深い講演が並びました。本報告集からまた集会の熱気を感じ取って頂けるのではないかと思います。

大変な盛況のうちに三日間の研究集会を無事終わられましたこと，この講究録を滞りなく発行できますこと，講演者の皆さんはじめ，出席下さった多くの方々，名前は挙げませんが，お世話になりました数理研の事務の方々と懇親会等でお手伝いをして下さった何人かの方々，そして最後になりますが伊原先生に，世話人（伊吹山知義，中村博昭，玉川安騎男，金子）を代表して心より感謝申し上げます。

研究代表者 金子昌信（九州大学）

## Preface

This volume is the proceedings of the conference “Profinite monodromy, Galois representations, and Complex functions” held from May 21st to 23rd in 2018 at the RIMS, Kyoto University, and consists of contributions by all the speakers.

The conference took place on the occasion of, and to celebrate, Professor Yasutaka Ihara’s 80th birthday. As such, topics of the talks were directly or indirectly related to his long-standing, influential work. We were also very happy to have his own talk(s). The title of the conference was taken (and modified) from that of Professor Ihara’s monumental Annals paper “Profinite braid groups, Galois representations and complex multiplications”. We hope it did not sound too weird, and believe in any event the conference was most fruitful and successful, and wish this volume gives a proof of this.

On behalf of the organizers (Tomoyoshi Ibukiyama, Hiroaki Nakamura, Akio Tamagawa, and myself), I would like to express our sincere gratitude to all the speakers, all those who attended the conference, the staff of the RIMS who helped us in various stages, and those who supported us in holding the party and the photo session etc. Last but certainly not least, we offer our heartfelt thanks to Professor Ihara for his guidance over the past several decades.

Representative Organizer Masanobu Kaneko (Kyushu University)

# Profinite monodromy, Galois representations, and Complex functions

## RIMS 共同研究 (公開型)

京都大学数理解析研究所の共同研究事業の一つとして、下記のように研究集会を開催しますので、ご案内申し上げます。

研究代表者 金子昌信 (九州大学)

日時：2018年5月21日(月) 13:30 — 5月23日(水) 15:50

場所：京都大学数理解析研究所 4階 420号室 (京都市左京区北白川追分町)

### 5月21日(月)

13:30 — 14:20 Henri Darmon (Mcgill University)

The  $p$ -adic uniformisation of modular curves by  $p$ -arithmetic groups

14:50 — 15:35 伊原 康隆 (東京大学, 京都大学)

On  $(\infty \times p)$ -adic uniformization of curves mod  $p$  with assigned many rational points

(I) some reviews; Shimura's and Igusa's works as points of departure

16:10 — 17:00 佐藤 巖 (小山工業高専)

Ihara zeta function and quantum walk

### 5月22日(火)

10:00 — 10:50 中村 博昭 (大阪大学)

Arithmetic and combinatorics in Galois fundamental groups

11:10 — 12:00 南出 新 (京都大学)

The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

14:00 — 14:50 森下 昌紀 (九州大学)

Arithmetic topology in Ihara theory

— Milnor invariants, Heisenberg covers and triple power residue symbols

15:15 — 16:05 Romyar Sharifi (University of California, Los Angeles)

Modular symbols and arithmetic

16:30 — 17:20 Francis Brown (Oxford University)

The projective line minus 3 points: past, present and future

18:30 — 20:30 懇親会 (京都大学カフェレストラン「カンフォーラ」)

### 5月23日(水)

10:00 — 10:50 山ノ井 克俊 (大阪大学)

Kobayashi hyperbolicity of the complements of ample divisors in abelian varieties

11:10 — 12:00 松本 耕二 (名古屋大学)

On the theory of  $M$ -functions

14:00 — 14:45 伊原 康隆 (東京大学, 京都大学)

On  $(\infty \times p)$ -adic uniformization of curves mod  $p$  with assigned many rational points

(II) existence of uniformization is an abelian problem

15:00 — 15:50 伊吹山 知義 (大阪大学)

Ihara lifts and conjectural correspondences between symplectic automorphic forms of genus two

# Profinite monodromy, Galois representations, and Complex functions

## RIMS Workshop

Masanobu Kaneko (Rep. Organizer, Kyushu University)

**Date :** May 21 (Mon) — May 23 (Wed), 2018

**Place :** Room 420, Research Institute for Mathematical Sciences, Kyoto University

### May 21 (Mon)

13:30 — 14:20 Henri Darmon (Mcgill University)

The  $p$ -adic uniformisation of modular curves by  $p$ -arithmetic groups

14:50 — 15:35 Yasutaka Ihara (Tokyo University, Kyoto University)

On  $(\infty \times p)$ -adic uniformization of curves mod  $p$  with assigned many rational points  
(I) some reviews; Shimura's and Igusa's works as points of departure

16:10 — 17:00 Iwao Sato (National Institute of Technology, Oyama College)

Ihara zeta function and quantum walk

### May 22 (Tue)

10:00 — 10:50 Hiroaki Nakamura (Osaka University)

Arithmetic and combinatorics in Galois fundamental groups

11:10 — 12:00 Arata Minamide (Kyoto University)

The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

14:00 — 14:50 Masanori Morishita (Kyushu University)

Arithmetic topology in Ihara theory

— Milnor invariants, Heisenberg covers and triple power residue symbols

15:15 — 16:05 Romyar Sharifi (University of California, Los Angeles)

Modular symbols and arithmetic

16:30 — 17:20 Francis Brown (Oxford University)

The projective line minus 3 points: past, present and future

18:30 — 20:30 Banquet (Camphora, Kyoto University)

### May 23 (Wed)

10:00 — 10:50 Katsutoshi Yamanoi (Osaka University)

Kobayashi hyperbolicity of the complements of ample divisors in abelian varieties

11:10 — 12:00 Kohji Matsumoto (Nagoya University)

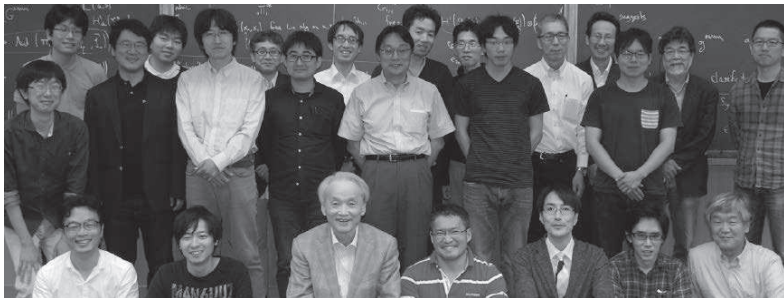
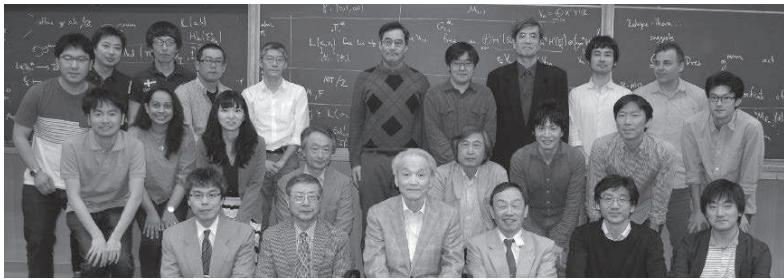
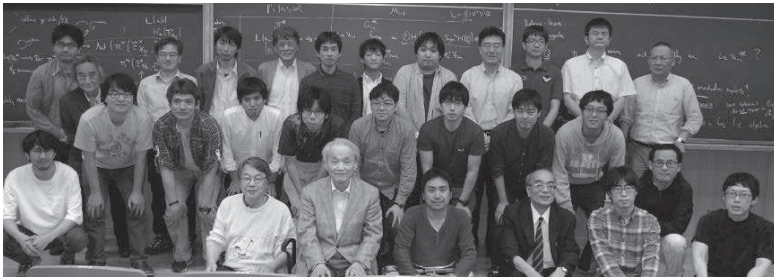
On the theory of  $M$ -functions

14:00 — 14:45 Yasutaka Ihara (Tokyo University, Kyoto University)

On  $(\infty \times p)$ -adic uniformization of curves mod  $p$  with assigned many rational points  
(II) existence of uniformization is an abelian problem

15:00 — 15:50 Tomoyoshi Ibukiyama (Osaka University)

Ihara lifts and conjectural correspondences between symplectic automorphic forms of genus two





副有限モノドロミー, ガロア表現, および複素関数  
Profinite monodromy, Galois representations, and Complex functions  
RIMS 共同研究 (公開型) 報告集

2018 年 5 月 21 日~5 月 23 日  
研究代表者 金子 昌信 (Masanobu Kaneko)

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