ON A PREDATOR-PREY SYSTEM WITH NONLOCAL DISPERSAL

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Abstract. We are concerned with the propagation dynamics of a predator-prey system with nonlocal dispersal. We obtain a threshold phenomenon for the invasion of the predator into the habitat of the aborigine prey. It turns out that this threshold is the so-called spreading speed of the predator as well as the minimal wave speed of traveling wave solutions connecting the predator-free state to a nontrivial state.

1. Introduction

In this paper, we consider the following predator-prey system:

$$\begin{cases} U_t(x,t) = d_1\mathcal{N}_1[U(\cdot,t)](x) + F\left(U(x,t),V(x,t)\right) \\ V_t(x,t) = d_2\mathcal{N}_2[V(\cdot,t)](x) + G\left(U(x,t),V(x,t)\right), \end{cases} \tag{1.1}$$

in which U and V stand for the densities of prey and predator, respectively, and

$$F(U,V) := r_1U(1-U) - aUV,\ G(U,V) := bUV - r_2V\left(1+\mu V\right),$$

$$\mathcal{N}_i[\varphi](x) := (J_i * \varphi)(x) - \varphi(x) = \int_{\mathbb{R}} J_i(x-y)\varphi(y)dy - \varphi(x),$$

where J_1 and J_2 are probability kernel functions. Here $d_1, d_2, r_1, r_2, a, b, \mu$ are positive constants. The dynamics of the prey population follows a logistic growth with a normalized (to one) carrying capacity and r_1 denotes its intrinsic growth rate. The parameter r_2 denotes the death rate for the predator. The parameter μ describes the intensity of the intra-specific competition in the predator population, and the constants a and b denote the predation rate and the biomass conversion rate, respectively.

The main purpose of this work is to study the propagation phenomenon of this predator-prey system. For propagation in predator-prey systems, one typical problem is to study the spatial invasion process of the predator when it was introduced into the habitat of a prey (cf. [7, 20]). In this work, we characterize the features of the predator invasion process by the asymptotic spreading and traveling wave solutions.

Note that a nonlocal dispersal is involved in the system (1.1). In fact, many nonlocal dispersal models have been derived from material science and other fields to model long distance effects and nonadjacent interactions, see, e.g., [10, 9, 4]

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for the physical background. The classical diffusion originated from Fick's law of diffusion formulated by certain elliptic operators could be thought as a limit case of the nonlocal dispersal. There are plentiful propagation dynamics of nonlocal models due to the nonlocal effect ([1]). Nonlocal models also arise in population dynamics to describe long distance dispersal of individuals, see, e.g., [18] and the references cited therein.

Some difficulties are encountered when we are dealing with system (1.1). For example, the nonlocal dispersal model often admits lower regularity than the classical diffusion one (see [2]). Also, our predator-prey system (1.1) is a nonmonotone system and so that the theory related to monotone semiflows (e.g., [8]) cannot be applied.

Before stating our main results, we first give the following definition on the kernels.

Definition 1.1. *Let $\bar{\lambda} \in (0, \infty]$ be given. We say that the kernel function $J : \mathbb{R} \to \mathbb{R}$ belongs to the class $\mathcal{T}(\bar{\lambda})$ if it satisfies the following properties:*

(J1) *The kernel J is nonnegative and continuous in $\mathbb{R}$;*

(J2) *it holds that*

$$\int_{\mathbb{R}} J(y)dy = 1 \text{ and } J(y) = J(-y) \text{ for all } y \in \mathbb{R};$$

(J3) *it holds that $\int_{\mathbb{R}} J(y)e^{\lambda y}dy < \infty$ for any $\lambda \in (0, \bar{\lambda})$ and*

$$\int_{\mathbb{R}} J(y)e^{\lambda y}dy \to \infty \text{ as } \lambda \uparrow \bar{\lambda}.$$

Throughout this paper, we assume that

$$\mu = 1 (\text{replacing } \mu V \text{ by } V), \quad b > r_2, \quad a(b - r_2) < r_1 r_2. \tag{1.2}$$

Also, for kernels, we assume

$$\text{for } i = 1, 2, \ \exists \, \bar{\lambda}_i \in (0, \infty] \text{ such that } J_i \in \mathcal{T}\left(\bar{\lambda}_i\right). \tag{1.3}$$

Define the quantity

$$c^* := \inf_{0 < \lambda < \bar{\lambda}_2} \frac{d_2 \left[\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy - 1\right] + b - r_2}{\lambda}. \tag{1.4}$$

Note that due to properties (J1)-(J3) for J_2, it is easy to see that c^* is well-defined and $c^* > 0$ since $b > r_2$. Then we have the following theorem on the spreading speed of the predator.

Theorem 1.2 (Predator's spreading)**.** *Under the assumption* (1.2)*, the constant c^*, defined in* (1.4)*, corresponds to the spreading speed of the predator for system* (1.1) *with initial condition*

$$U(x, 0) = 1, \quad V(x, 0) = v_0(x), \quad x \in \mathbb{R}, \tag{1.5}$$

as long as v_0 is a non-zero compactly supported continuous function with $0 \le v_0 \le b - r_2$. This means that the density of the predator $V = V(x, t)$ satisfies

$$\lim_{t\to\infty} \sup_{|x|>ct} V(x, t) = 0 \text{ for any } c > c^*;$$

$$\liminf_{t\to\infty} \inf_{|x|<ct} V(x, t) > 0 \text{ for any given } c \in (0, c^*).$$

Theorem 1.2 is proved by deriving some delicate a priori estimates that are combined with known results on scalar equations with nonlocal dispersal. Our arguments strongly relies on the regularity of the solutions of (1.1) and more particularly their uniform continuity properties. For mathematical results on scalar equations with nonlocal dispersal, we refer the reader to, for examples, [12, 13, 15] and the references cited therein.

Next, we study the traveling wave solutions of (1.1). A solution (U, V) to (1.1) is called a traveling wave solution of (1.1), if there exist a constant $c \in \mathbb{R}$, *the wave speed*, and a function pair (Φ, Ψ), *the wave profile*, such that $(U, V)(x, t) = (\Phi, \Psi)(\xi)$, $\xi := x + ct$, for any $(x, t) \in \mathbb{R} \times \mathbb{R}$ and with $\Phi > 0$ and $\Psi > 0$. We are interested in the traveling waves connecting the predator-free state $(1, 0)$ at $\xi = -\infty$ to a nontrivial state at $\xi = \infty$ in the sense that

$$\liminf_{\xi\to\infty} \Phi(\xi) > 0, \quad \liminf_{\xi\to\infty} \Psi(\xi) > 0. \tag{1.6}$$

We have the following theorem on the minimal wave speed in the weaker sense.

Theorem 1.3. *Suppose, in addition to* (1.2)*, that* $ab < r_1 r_2$*. Then the following holds true:*

(i) *For any speed* $c > c^*$*, system* (1.1) *admits a traveling wave solution connecting* $(1, 0)$ *at* $\xi = -\infty$ *to a nontrivial state at* $\xi = \infty$*.*
(ii) *If we furthermore assume that* J_2 *is compactly supported then* (1.1) *admits a traveling wave solution for* $c = c^*$*.*

Moreover, under the assumption (1.2)*, there is no traveling wave solution to* (1.1) *connecting* $(1, 0)$ *to a nontrivial state with speed* $c \in (0, c^*)$*.*

Finally, with some extra conditions, the constant c^* is actually the minimal wave speed to the system (1.1) in the usual sense.

Corollary 1.4. *In addition to* (1.2)*, assume that* $ab < r_1 r_2$ *and* $d_2 < b - r_2$*, and that* J_2 *has a compact support. Then,* (1.1) *admits traveling wave solution connecting* $(1, 0)$ *to a nontrivial state with speed* $c \in \mathbb{R}$ *if and only if* $c \geq c^*$*.*

The existence of traveling wave solutions is proved by constructing some suitable upper and lower solutions with the help of Schauder's fixed point theorem, cf. e.g., [19, 11, 14, 17, 16, 5]. The nonexistence of traveling wave solutions is proved by applying the theory of asymptotic spreading for scalar equations ([13]).

In the rest of this paper, we shall only provide some details of the method of upper-lower solutions in the derivation of traveling wave solutions.

2. Method of upper-lower solutions

For convenience, we set

$$u(x, t) = 1 - U(x, t), \;\; v(x, t) = V(x, t), \; \bar{a} = \frac{a}{r_1}, \; \bar{b} = \frac{b}{r_2}.$$

Dropping the bar, problem (1.1) is re-written as

$$\begin{cases} u_t(x, t) = d_1 \mathcal{N}_1[u(\cdot, t)](x) + r_1(1 - u)(av - u)(x, t), \\ v_t(x, t) = d_2 \mathcal{N}_2[v(\cdot, t)](x) + r_2 v(b - 1 - v - bu)(x, t). \end{cases} \tag{2.1}$$

Note also that condition (1.2) becomes

$$b > 1, \quad a(b-1) < 1, \tag{2.2}$$

and the condition $ab < r_1 r_2$ becomes $ab < 1$.

Traveling wave solution for (1.1) connecting the predator-free equilibrium at $\xi = -\infty$ and a nontrivial state at $\xi = \infty$ is equivalent to a solution of system (2.1) in the form

$$u(x,t) = \phi(\xi), \quad v(x,t) = \psi(\xi), \text{ where } \xi := x + ct,$$

such that $(\phi, \psi)(-\infty) = (0,0)$ and

$$\limsup_{\xi\to\infty} \phi(\xi) < 1, \quad \liminf_{\xi\to\infty} \psi(\xi) > 0. \tag{2.3}$$

This yields the following nonlocal system of equation for the profile function (ϕ, ψ)

$$\begin{cases} c\phi'(\xi) = d_1\mathcal{N}_1[\phi](\xi) + r_1(1-\phi)(a\psi - \phi)(\xi), \xi \in \mathbb{R}, \\ c\psi'(\xi) = d_2\mathcal{N}_2[\psi](\xi) + r_2\psi(b - 1 - \psi - b\phi)(\xi), \xi \in \mathbb{R}, \end{cases} \tag{2.4}$$

where the linear operators $\mathcal{N}_i$ are defined by

$$\mathcal{N}_i[\varphi](\xi) := \int_{\mathbb{R}} J_i(y)\varphi(\xi - y)dy - \varphi(\xi), \ i = 1, 2.$$

In the sequel, we shall use the following notation.

$$u \le v \Leftrightarrow u_i \le v_i, i = 1, 2, u = (u_1, u_2), v = (v_1, v_2);$$
$$X_d := \{w \text{ is uniformly continuous on } \mathbb{R}, 0 \le w \le d\},$$
$$X_b^2 := X_1 \times X_\alpha, \ \alpha := b - 1.$$

Definition 2.1. *A pair of functions $(\overline{\phi}, \overline{\psi}), (\underline{\phi}, \underline{\psi}) \in X_b^2$ is called a pair of upper and lower solutions of (2.4) if $(\underline{\phi}(\xi), \underline{\psi}(\xi)) \le (\overline{\phi}(\xi), \overline{\psi}(\xi))$ for all $\xi \in \mathbb{R}$ and the following inequalities*

$$c\overline{\phi}'(\xi) \ge d_1\mathcal{N}_1[\overline{\phi}](\xi) + r_1[1 - \overline{\phi}(\xi)][a\overline{\psi}(\xi) - \overline{\phi}(\xi)], \tag{2.5}$$

$$c\overline{\psi}'(\xi) \ge d_2\mathcal{N}_2[\overline{\psi}](\xi) + r_2\overline{\psi}(\xi)[b - 1 - \overline{\psi}(\xi) - b\underline{\phi}(\xi)], \tag{2.6}$$

$$c\underline{\phi}'(\xi) \le d_1\mathcal{N}_1[\underline{\phi}](\xi) + r_1[1 - \underline{\phi}(\xi)][a\underline{\psi}(\xi) - \underline{\phi}(\xi)], \tag{2.7}$$

$$c\underline{\psi}'(\xi) \le d_2\mathcal{N}_2[\underline{\psi}](\xi) + r_2\underline{\psi}(\xi)[b - 1 - \underline{\psi}(\xi) - b\overline{\phi}(\xi)] \tag{2.8}$$

hold for all $\xi \in \mathbb{R} \setminus E$, where E denotes some finite set $E \subset \mathbb{R}$.

It should be noted that $\{(\overline{\phi}, \overline{\psi}), (\underline{\phi}, \underline{\psi})\}$ is NOT super-sub-solution in the usual sense. It is rather an upper bound and a lower bound for the space to be used in applying Schauder's fixed point theory.

2.1. **General framework.**

The following lemma is proved by applying Schauder's fixed point theorem.

Lemma 2.2. *Let $c > 0$ be given. Let $(\overline{\phi}, \overline{\psi}), (\underline{\phi}, \underline{\psi})$ be a pair of upper and lower solutions of (2.4). Then system (2.4) admits a solution (ϕ, ψ) such that*

$$(\underline{\phi}(\xi), \underline{\psi}(\xi)) \le (\phi(\xi), \psi(\xi)) \le (\overline{\phi}(\xi), \overline{\psi}(\xi)), \ \xi \in \mathbb{R}.$$

The proof of Lemma 2.2 can be carried out in the following steps.

First, we introduce the integral operator $P = (P_1, P_2) : X_b^2 \to X^2$:

$$\begin{cases} P_1(\phi,\psi)(\xi) = \frac{1}{c}\int_{-\infty}^{\xi} e^{-\frac{\beta(\xi-s)}{c}} F_1(\phi,\psi)(s)ds, \\ P_2(\phi,\psi)(\xi) = \frac{1}{c}\int_{-\infty}^{\xi} e^{-\frac{\beta(\xi-s)}{c}} F_2(\phi,\psi)(s)ds, \end{cases} \quad \forall(\phi,\psi) \in X_b^2,$$

where

$$\begin{aligned} F_1(\phi,\psi)(\xi) &:= \beta\phi(\xi) + d_1\mathcal{N}_1[\phi](\xi) + r_1[1-\phi(\xi)][a\psi(\xi)-\phi(\xi)], \\ F_2(\phi,\psi)(\xi) &:= \beta\psi(\xi) + d_2\mathcal{N}_2[\psi](\xi) + r_2\psi(\xi)[b-1-\psi(\xi)-b\phi(\xi)] \end{aligned}$$

for $(\phi,\psi) \in X_b^2$ with β some large positive constant.

Secondly, define a set $\Gamma \subset B_\mu(\mathbb{R},\mathbb{R}^2)$ by

$$\Gamma = \{(\phi,\psi) \in X_b^2 : (\underline{\phi},\underline{\psi}) \le (\phi,\psi) \le (\overline{\phi},\overline{\psi})\},$$

where for a suitable positive constant μ the set

$$B_\mu(\mathbb{R},\mathbb{R}^2) = \left\{(\phi,\psi) \in X^2 : |(\phi,\psi)|_\mu := \sup_{\xi\in\mathbb{R}} \max\left(|\phi(\xi)|, |\psi(\xi)|\right) e^{-\mu|\xi|} < \infty\right\}.$$

is a Banach space.

Finally, we verify that $P(\Gamma) \subset \Gamma$ and P is completely continuous with respect to the norm $|\cdot|_\mu$. Then P has a fixed point by Schauder's fixed point theorem. □

2.2. **Construction of upper-lower solutions.** Set

$$\Delta(\lambda,c) := d_2\left[\int_{\mathbb{R}} J_2(y)e^{\lambda y}dy - 1\right] - c\lambda + r_2(b-1).$$

Recall $b > 1$ and the definition of c^* in (1.4). Then

(1) For any given $c > c^*$, the equation $\Delta(\lambda,c) = 0$ admits two positive roots $\lambda_1(c) < \lambda_2(c) < \bar{\lambda}_2$ such that $\Delta(\lambda,c) < 0$ if and only if $\lambda \in (\lambda_1(c), \lambda_2(c))$.

(2) There exists $\lambda^* \in \left[0, \bar{\lambda}_2\right)$ such that $\Delta(\lambda^*, c^*) = 0$, $\Delta(\lambda, c^*) > 0$ for all $\lambda \in [0, \bar{\lambda}_2) \setminus \{\lambda^*\}$ while

$$\left.\frac{\partial\Delta(\lambda,c)}{\partial\lambda}\right|_{(\lambda,c)=(\lambda^*,c^*)} = d_2\int_{\mathbb{R}} J_2(y)ye^{\lambda^* y}dy - c^* = 0. \tag{2.9}$$

(3) For any given $c \in (0, c^*)$, one has $\Delta(\lambda,c) > 0$ for all $\lambda \in [0, \bar{\lambda}_2)$.

For $c > c^*$, we define the following continuous functions:

$$\begin{aligned} &\overline{\phi}(\xi) = \min\{1, Ke^{\lambda\xi}\}, \quad \underline{\phi}(\xi) = 0, \\ &\overline{\psi}(\xi) = \min\{b-1, e^{\lambda_1\xi}\}, \quad \underline{\psi}(\xi) = \max\{0, e^{\lambda_1\xi} - qe^{\eta\lambda_1\xi}\}, \end{aligned}$$

where $\lambda_1 = \lambda_1(c)$, $\lambda \in (0, \min\{\bar{\lambda}_1, \lambda_1(c)\})$ such that

$$A(\lambda) = d_1\left[\int_{\mathbb{R}} J_1(y)e^{\lambda y}dy - 1\right] - c\lambda < 0,$$

and the constants K, η, q shall be chosen in order below.

Define ξ_1 by $e^{\lambda_1\xi_1} = b-1$ and we choose a constant $\xi_0 < \min\{0, \xi_1\}$ such that $ae^{(\lambda_1-\lambda)\xi_0} < 1$. Set $K = e^{-\lambda\xi_0}$ and observe that $K > 1$ and

$$ae^{\lambda_1\xi} < e^{\lambda\xi}, \ \forall\, \xi < \xi_0. \tag{2.10}$$

Choose $\eta \in (1,2)$ such that

$$\eta\lambda_1 < \min\{\lambda_2, \lambda_1 + \lambda\}. \tag{2.11}$$

For $q > 1$ define $\xi_2 = \xi_2(q) < 0$ by $e^{(\eta-1)\lambda_1\xi_2} = 1/q$. Since $\xi_2(q) \to -\infty$ as $q \to \infty$, one can choose a constant $q > 1$ large enough such that $\xi_2 \leq \xi_1$ and

$$q > \frac{r_2 + r_2 bK}{-\Delta(\eta\lambda_1, c)} + 1. \tag{2.12}$$

Then we can verify that $\{(\overline{\phi}, \overline{\psi}), (\underline{\phi}, \underline{\psi})\}$ is a pair of upper-lower solutions.

Applying Lemma 2.2, we obtain

Theorem 2.3. *Let $c > c^*$ be given and fixed. Then* (2.4) *admits a nonnegative solution (ϕ, ψ) such that $\lim_{\xi\to-\infty}(\phi(\xi), \psi(\xi)) = (0,0)$.*

The case for $c = c^*$ is more involved, we only state the theorem here.

Theorem 2.4. *Assume that the function J_2 is compactly supported. Then* (2.4) *with $c = c^*$ admits a nonnegative solution (ϕ, ψ) such that $\lim\limits_{\xi\to-\infty}(\phi(\xi), \psi(\xi)) = (0,0)$.*

2.3. **Existence of traveling waves.** With Theorems 2.3 and 2.4, we prove the following theorem on the existence of traveling waves.

Theorem 2.5. *Let $c \geq c^*$ and let (ϕ, ψ) be any solution provided by Theorem 2.3 or Theorem 2.4. Then it holds that $\limsup_{\xi\to\infty}\phi(\xi) < 1$. Moreover, $\liminf_{\xi\to\infty}\psi(\xi) > 0$, if we further assume that $ab < 1$.*

To prove this theorem, we need the following two classical results from [13] for scalar equation

$$\begin{cases} w_t = d\mathcal{N}[w(\cdot,t)](x) + rw(s-w), \; x \in \mathbb{R}, \, t > 0, \\ w(x,0) = \chi(x), \; x \in \mathbb{R}, \end{cases} \tag{2.13}$$

where r, s are positive constants and $\mathcal{N}[w] := J * w - w$.

Proposition 2.6 (Comparison principle)**.** *Let w be a solution of* (2.13) *with $w(\cdot,t) \in X_s$ for all $t > 0$ for a given $\chi \in X_s$. If $z(\cdot,0) \in X_s$ and $z(x,t)$ satisfies*

$$\begin{cases} \dfrac{\partial z(x,t)}{\partial t} \geq d\mathcal{N}[z(\cdot,t)](x) + rz(x,t)\left[s - z(x,t)\right], \; x \in \mathbb{R}, \, t > 0, \\ z(x,0) \geq \chi(x), \; x \in \mathbb{R}, \end{cases}$$

then $z(x,t) \geq w(x,t)$ for all $x \in \mathbb{R}$, $t > 0$. Similar result holds for the reverse inequality.

Next, we define the quantity $\bar{c}$ by

$$\bar{c} := \inf_{0<\lambda<\bar{\lambda}} \frac{d\left[\int_{\mathbb{R}} J(y)e^{\lambda y}dy - 1\right] + rs}{\lambda}.$$

Then $\bar{c}$ is well-defined and $\bar{c} > 0$ since $rs > 0$. Moreover, $\bar{c}$ is the spreading speed of the solution w to (2.13) (as introduced by Aronson and Weinberger [3]) as follows.

Proposition 2.7 (Spreading speed)**.** *Let w be a solution of* (2.13) *with $w(\cdot,t) \in X_s$ for all $t > 0$ for a given $\chi \in X_s$. Assume that χ has a nonempty compact support. Then we have*

$$\lim_{t\to\infty} \inf_{|x|<ct} w(x,t) = s \quad \textit{for any } c \in (0,\bar{c}), \tag{2.14}$$

$$\lim_{t\to\infty} \sup_{|x|>ct} w(x,t) = 0 \quad \textit{for any } c > \bar{c}. \tag{2.15}$$

Proof of Theorem 2.5. Set $(U,V)(x,t) := (1-\phi,\psi)(x+ct)$. Since $V \le b-1$, $U = U(x,t)$ satisfies

$$U_t \ge d_1\mathcal{N}_1[U] + r_1U\{[1-a(b-1)] - U\},\ x \in \mathbb{R},\ t > 0.$$

Applying Propositions 2.6 and 2.7, we obtain that

$$\limsup_{\xi\to\infty} \phi(\xi) \le a(b-1) < 1.$$

This proves the first statement of Theorem 2.5.

Next, we claim that

$$B := \sup_{\xi\in\mathbb{R}} \phi(\xi) \le a(b-1). \tag{2.16}$$

Using (2.16), the function $V(x,t) = \psi(x+ct)$ satisfies

$$\begin{cases} V_t \ge d_2\mathcal{N}_2[V] + r_2V[(1-ab)(b-1) - V],\ x \in \mathbb{R},\ t > 0, \\ V(x,0) = \psi(x) > 0. \end{cases}$$

Then a similar argument we deduce that

$$\liminf_{\xi\to\infty} \psi(\xi) \ge (1-ab)(b-1) > 0$$

and this completes the proof of the theorem. □

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