

Examining Trustworthiness in Canadian and Japanese Cultural Context: How to quantify the distance between two groups, each consisting of 3-dimensionally estimated objects?

Yuichiro Kanazawa, Catherine Kwantes, Shinichiro Watanabe
International Christian University, University of Windsor, University of Tsukuba

1 Introduction

Quantifying trustfulness goes back to Rotter (1967) ([Rotter, 1967]) in which a new type of likert-like scale for the measurement of interpersonal trust was developed and refined on the basis of item analysis. Here interpersonal trust was defined to be the generalized expectancy that the verbal statements of others can be relied upon. Trustfulness has been actively investigated by many researchers for a long time in a different context and using questionnaire surveys and/or experiments. We first review this development.

Trustfulness through questionnaire survey

By carefully separating the concept of *trust* from the concept of *assurance*, Yamagishi and Yamagishi (1994) ([Yamagishi and Yamagishi, 1994]) tried to explain a puzzle of cross-national differences between the United States and Japan in the levels of trustfulness through a cross-national questionnaire survey that challenges the prevailing wisdom at the time. Through a US-Japan comparative questionnaire survey, Yamagishi and

Komiyama (1995) ([Yamagishi and Komiyama, 1995]) tested a series of predictions involving US-Japan contrasts derived from Yamagishi's theory of trust that emphasizes the role trust plays in “emancipating” people from commitment relation. They found that Americans are higher in general trust, Japanese have higher expectations of benefit from maintaining relations with specific others, Americans consider information role of reputation more important, and Americans consider honesty and fairness more important.

Trustfulness through experiments

Knowledge on many aspects of trustfulness has been accumulated over the years through carefully designed experiments as well. It started out as an investigation into somewhat different but related topic of **cooperation measured by the contribution to a public good**, members of five-person groups decided whether they would contribute their resource money to the provision of a public good when expectation of other members' cooperativeness was manipulated in an experiment in Sato and Yamagishi ([Sato and Yamagishi, 1986]). They found, for instance, trust in other members and the motivation of free-riding had more effects upon subjects' cooperation when subjects believe that other members' cost for cooperation (the amount of resource money they were required to contribute) was high. Yamagishi and Sato ([Yamagishi and Sato, 1986]) experimentally investigated two motivational bases for not contributing to a public good—desire to “free ride”(or greed) and fear of being a “sucker.” They found fear would have a strong effect but greed would not when a public good is produced *conjunctively*, that is, by individuals working in union and every member' s contribution is essential. Also the greater mutual trust existing among friends would make them contribute more than strangers would in this conjunctive condition. On the other hand, they

found greed would have a strong effect but fear would not when a public good is produced *disjunctively* or by one or a few member's cooperative action.

On the validity and raison d'etre of a prisoner's dilemma (henceforth PD) game and its variants in differentiating trust, assurance, and cooperation, Hayashi et al. (1999) ([Hayashi et al., 1999]), arguing that many participants treat a prisoner's dilemma game as an *assurance* game and respond in a reciprocal manner to the choice or expected choice of their partner, examined two bases—*general trust* and *a sense of control*—for the expectation of a partner's cooperation in one-shot prisoner's dilemma games and why they expect general trust and a sense of control to play different roles in different societies. They found that a sense of control plays a relatively more important role as a foundation for expectations in Japanese society and general trust plays the more important role in American society. Yamagishi et al. (2005) ([Yamagishi et al., 2005]) summarized the major findings discovered by experimental studies using a prisoner's dilemma games with choice of dependence (henceforth PD/D) that have been published previously only in Japanese and discussed some of their implications. They define *trustfulness*, or *an act of trustfulness*, as an act that *voluntarily exposes oneself to greater positive and negative externalities by the actions of the other(s)*, while *cooperation* as an act that increases the welfare of the other(s) at some opportunity cost where the former is greater than the latter. PD/D allows each player separately to choose the level of trust she wants to place in the other player, and the behavioral choice to cooperate or defect with the other. That is, it allows the players to make separate decisions to trust and to cooperate, and thus allows the researchers to study the two processes separately. In PD/D, it is possible for players to choose to cooperate with the others without trusting them. They found that it is cooperation which leads to trust, not the other way around and demonstrated that separating *trust* from *cooper-*

ation is critical for building trust relations. Matsuda and Yamagishi (2001) ([Matsuda and Yamagishi, 2001]) examined the relationship between *coop-eration* and *trust* in interpersonal trust formation through a PD/D and revealed a more nuanced result obtained through a simple iterated prisoner's dilemma games. That is, participants in the PD/D adopted “cautious and unconditional cooperation strategy” rather than tit-for-tat strategy. Terai et al. (2003) ([Terai et al., 2003]) conducted two experiments using the PD/D and found that the cooperation rate was extremely high (95.1%) while the game was being repeated, but only half of the subjects cooperated in the final game, suggesting “shadow of the future” (Axelrod, 1984) ([Axelrod, 1984]) in play in forming cooperation. Nonetheless, most subjects trusted a partner who had behaved cooperatively toward them in the repeated games (i.e. under an incentive structure that encouraged such behavior), even in the final game, in which such an incentive basis was absent, indicating that the subjects failed to distinguish the two bases of expecting benign behavior from interaction partners—trust based on the inferred personal traits of the partner and assurance of cooperation based on the nature of the incentive structure.

On the conditions under which trust is being formed, Yamagishi and Komiyama (1995) ([Yamagishi and Komiyama, 1995]) through a series of laboratory experiments successfully confirmed: the most standard solution to problems caused by social uncertainty is *commitment formation*, i.e., formation of stable and durable relations with reliable partners; high trusters (those who have a high level of trust in people in general) would be easier to leave a commitment relation that had been formed in order to reduce social uncertainty. Yamagishi et al. ([Yamagishi et al., 1995]) distinguishing two kinds of trust—*general* and *particularistic trust*, and in an experiment simulating buyer-seller relations, found support for the following: social uncertainty promoted commitment formation; commitment formation promoted one partner's trust in the other (which they

called particularistic trust); as a result of the above two effects, social uncertainty promoted the general level of particularistic trust in a group; general trust, which was defined as a general belief in human benevolence, suppressed commitment formation. Kakiuchi and Yamagishi (1997) ([Kakiuchi and Yamagishi, 1997]) examined the development of trusting relationships as investments in relation-specific assets through a new experimental game called “the dilemma of variable interdependency” based on iterated prisoner's dilemma game. This game will be called later as a prisoner's dilemma games with choice of dependence (PD/D) as mentioned in Yamagishi et al. (2005) ([Yamagishi et al., 2005]). It was invented independently of Van Lange and Visser (1999) ([Van Lange and Visser, 1999]) and was first published in Kakiuchi and Yamagishi (1997) a couple of years before theirs. Mashima et al. (2004) ([Mashima et al., 2004b]) examined the role of trust in relationships between temporary partners through PD/D and PD and found that trust serves as a signal of the player's intention, which in turn promotes mutual cooperation. Cook et al. (2005) ([Cook et al., 2005]) arguing that a series of risk-taking behaviors is indispensable to building a trust relation, conducted experiments in Japan and the United States to examine the independent and cross-cultural effects of risk taking on trust building. The results of these experiments indicate that the American participants took more risks than did the Japanese, supporting the general claim that Americans are inclined toward risk taking and trust building. Kiyonari et al. (2006) ([Kiyonari et al., 2006]) provided an answer to whether trusting begets trustworthiness. In two experimental games with Japanese and American participants, respectively in Trust and Faith Games and found that trust does not beget trustworthiness, at least in one-shot games. Yamagishi et al. (2015) ([Yamagishi et al., 2015]), tackling the problem that attitudinal measures of general trust often fail to predict actual trusting behavior in laboratory testing, proposed that measures of attitudinal trust is more successful in predicting behavioral trust

when they tap both the responder's belief that his/her trust will be honored and his/her preference to be a trusting person. They demonstrated that the newly constructed measure including these two responder's belief better predicted behavioral trust in a trust game.

On the characteristics and contrasts of high and low trusters, Kikuchi et al. ([Kikuchi et al., 1997]) tested and supported a hypothesis that subjects who are more trusting of others in general (high-trusters) are more accurate than low-trusters in judging trustworthiness of others as Rotter (1980) ([Rotter, 1980]) claims through an experiment, indicating high-trusters are shown to be prudent people who pay careful attention to information potentially revealing other's lack of trustworthiness. Yamagishi et al. ([Yamagishi et al., 1998]) tested and confirmed the prediction that social uncertainty promotes commitment formation between particular partners and high trusters tend to form committed relations less frequently than would low trusters when facing social uncertainty through two experiments conducted in the United States and Japan. Kosugi and Yamagishi (1998) ([Kosugi and Yamagishi, 1998]) demonstrated that trustful people are more sensitive to information that indicates lack of trustworthiness in other people through two experiments. Yamagishi et al. ([Yamagishi et al., 1999]) conducted a series of experiments in Japan showing that high trusters (as measured with a general trust scale) are more sensitive than low trusters to information potentially revealing lack of trustworthiness in others and judge other people's choice in a one-shot prisoner's dilemma more accurately.

On ingroup versus outgroup trust formation, Mashima et al. (2004) ([Mashima et al., 2004a]) in a PD/D experiment participated by American and Japanese students, found no evidence of ingroup bias in terms of trust and cooperation. However they found that American participants were more sensitive than Japanese counterparts, to information regarding the past trust behavior of other players when they were de-

cluding whom they trust and whether or not they reciprocate another's trust. Suzuki et al. (2007) ([Suzuki et al., 2007]) tested that expectations of generalized reciprocity within one's own group are responsible for in-group trust through an allocator choice game in the minimal group situation. Participants' in-group trust is derived from the general belief that people treat in-group members more favorably than out-group members—a belief about generalized reciprocity within groups. Kiyonari et al. (2007) ([Kiyonari et al., 2007]) conducted experiments called Trust and Faith games. In both Trust and Faith Games, one person plays a role of the distributor of 2,500JPY and the other person plays a role of the receiver with two choices, receiving 1,000JPY for certainty, or receiving whatever the amount the distributor decides. In the Trust game, both the distributor and the receiver make the decision simultaneously and the distributor does not have the information on receiver's decision, while in the Faith game, the distributor makes the decision after learning receiver's decision. In other words, in Faith game, the receiver can expect rewarding and possibly mutually beneficial behavior on the part of the distributor for his/her decision. They tested and confirmed that a prediction that people show stronger trust of in-group members than of out-group members in the Faith game but not in the Trust game. In Foddy et al. (2009) ([Foddy et al., 2009]), when offered participants a choice between an unknown monetary allocation made by an in-group (university or major) or an out-group allocator, both of whom had total control over the distribution of an identical sum of money, participants strongly preferred the in-group allocator. This preference occurred regardless of whether the stereotype of the in-group was relatively more positive or more negative than that of the out-group. However, this preference did not persist when participants believed that the allocator was unaware of their group membership. Platow et al. (2012) ([Platow et al., 2012]) evaluated the role that common knowledge of a shared social group membership between self and a

to-be-trusted stranger provides as a basis for trusting this stranger in two experiments. This common knowledge emerges when the truster knows the group membership of the to-be-trusted other, and believes that this other also knows the group membership of the truster. They show that people are more likely to trust an in-group member over an out-group member under conditions of common group-membership knowledge rather than private group-membership knowledge (i.e. other does not know truster's group).

Yuki et al. (2005) ([Yuki et al., 2005]), **on cross-cultural trust and cooperation**, explored differences in depersonalized trust (trust toward a relatively unknown target person) across cultures. Based on a theoretical framework that postulates predominantly different bases for group behaviors in Western cultures versus Eastern cultures, it was predicted and supported by two experiments that Americans would tend to trust people primarily based on whether they shared category memberships, while Japanese was expected to be based on the likelihood of sharing direct or indirect interpersonal links. Kuwabara et al. (2007) ([Kuwabara et al., 2007]) conducted a variation of the Trust game in two different experimental conditions: a “flags-on” condition in which everyone's nationality was publicly identified during the session, and a “flags-off” condition in which participants did not know who was Japanese or American using a Web-based “virtual lab” to study trust and trustworthiness between Japanese and Americans in real-time interaction. They found a support for Yamagishi 's structural theory of trust, in that Japanese will form more durable exchange relations compared to Americans. However, they found less support for explanations that focus on cultural differences in trust and trustworthiness, and for cognitive explanations that point to the effects of a shared social identity between participants and partners with common nationality. University students from Japan, China, and Taiwan participated in experiments involving participants from their own society and another society in real time using an intercultural trust paradigm derived from a game

theoretic and evolutionary approach to social exchange in Takahashi et al. (2008) ([Takahashi et al., 2008]). They found that Japanese collectivism is based more on long-term assurance networks, whereas Chinese collectivism provides a more expansive, guanxi-based approach to building new social networks. Japanese also showed less in-group favoritism in both trust and trustworthiness (or conditional fairness) at the national-level compared to cultural Chinese. Liu et al. (2011) ([Liu et al., 2011]), using a dyadic game theory paradigm, conducted three experiments—China vs Japan, China vs Taiwan, Taiwan vs Japan—on the social dilemma of trust over the Internet in real time, involving real money. It was predicted and found that ingroup favoritism in trusting behavior was contingent on historical relationships between societies. They found Japanese were unique in not displaying ingroup favoring behavior at all, whereas both Chinese and Taiwanese were context specific in their in- group favoritism. In Yamagishi (2011) ([Yamagishi, 2011]), he wrote that collectivist societies produce security, but destroy trust: In collectivist societies, people are connected through networks of strong personal ties where the behavior of all agents is constantly monitored and controlled; As a result, individuals in collectivist networks are assured that others will abide by social norms, and gain a sense of security erroneously thought of as “trust;” However, this book argues that this security is not truly trust, based on beliefs regarding the integrity of others, but assurance, based on the system of mutual control within the network. When the political tide in many countries has shifted inward, with skepticism and reluctance to cooperate with other countries, Romano et al. (2017) ([Romano et al., 2017]) conducted an experimental study in 17 countries designed to test several theories that explain why, who, and where people trust and cooperate more with ingroup members, compared with outgroup members. In addition to the standard finding that participants trust and cooperate more with ingroup than outgroup members, they obtained findings that reputational concerns play a decisive role

for promoting trust and cooperation universally across societies.

Trustworthiness

Perhaps because trustfulness have been investigated so extensively as described above, it is natural to focus on the receiving end of that trustfulness. We observe upsurge of interest in the constructs of trustworthiness in the social science research literature recently. Along with it, a recognition of the importance of developing more contextual understandings of these constructs ([Li, 2007]; [Doney et al., 1998]) has emerged, with some theorists explicitly asserting that trust always depends on context (e.g. [Hardin, 2002]). For that end, we formed in April 2018 the *Trustworthiness Research Alliance* at <https://www.trustworthiness.ca/index.htm>, an international and multidisciplinary group of researchers who investigate trustworthiness and the role that perceptions of the trustworthiness of individuals, organizations, and institutions plays in decisions to take the leap of faith we know of as “trust.”

Societal culture has been identified as one important contextual factor in determining expectations of trustworthiness as “*simply put, culture is a shared agreement (usually implicit) about how to approach the world and each other*” ([Whitener et al., 2000], p.4). We already described how societal culture may matter to trustfulness above, the same can be said for trustworthiness. *What remains lacking, however, is an explicit, empirical examination of the context within which decisions related to trust are made and direct empirical assessments of trustworthiness.* This research explicitly examines the bases for determining trustworthiness in the Canadian and Japanese sociocultural context.

Trustfulness and Trustworthiness

While related, the two constructs of trustfulness and trustworthiness are distinctly different. *Trustfulness*, an abstract construct, is a *willingness to place ones' self in a vulnerable position to another* ([Saunders et al., 2014]); Trustworthiness comprises an *assessment of another's likelihood of honoring that vulnerability*. Implicit in the willingness to trust is an appraisal of whether or not another individual is worthy of that trust.

This calculation is based on a *number of assessments, such as a determination of the extent to which the other party is able to, willing to and responsible enough to be trusted*. [Mayer et al., 1995] cite three important characteristics of a potential trustee — the *ABI framework*.

Ability A *degree of trustee competence* in the realm in which trust will be extended.

Benevolence An assumption that the *potential trustee has the potential trustor's best interests at heart*.

Integrity A perception that the potential trustee is a person who *upholds moral standards that are deemed appropriate by the potential trustor*.

Trustworthiness, as an assessment of the extent to which one is willing to be vulnerable, *focuses on the other party* and implicitly suggests that some current or future interaction will occur between the trustor and the trustee. Being willing to trust another implicitly reflects social decision making and *incorporates a social exchange ideology* ([van't Wout and Sanfey, 2008]). While each party to a social exchange brings expectations based on that individual's personality (e.g. level of dependence versus independence), expectations are also based on examples of role exchanges that the individual has been exposed to (e.g. relationships within a specific societal culture).

Social exchanges do not, however, occur in a vacuum—each party to a social exchange brings expectations that are based on personality as well

as expectations that are based on examples of role exchanges that an individual has been exposed to. These expectations are based, at least in part, on the context that these decisions are made within ([Hardin, 2002]; [Li, 2012]). It has been argued that all situational contexts within which people operate are given meaning by culture ([Matsumoto, 2007]). *Societal culture has a profound impact in individuals, as individuals are socialized early on to internalize a culture's values and norms of behavior.* Culture therefore functions as a critical context for interactions, as *individual behavior is shaped by what that person has learned is rewarded or punished within that cultural context* ([Bond and Smith, 2018]), and has a fundamental role in shaping how trust develops.

As Markus and Kitayama (1991) ([Markus and Kitayama, 1991]) suggested for culture and the self, we are interested in observing, for instance, *“on average, relatively more individuals in Canadian culture will hold the integrity to be the dominant components in the ABI framework than will individuals in Japanese culture.* Within a given culture, however, we are ready to accept that individuals will vary in the extent to which they are good cultural representatives and consider trustworthiness in the mandated way.”

2 Data

Undergraduate students (n= 238) at a university in Ontario, Canada and similarly undergraduate students (n=237) at a university in Ibaraki, Japan, were recruited for this study. Of those, two from Canadian sample and three from Japanese sample were listwisely-deleted because of their extensive missing values. Therefore our sample consist of 236 Canadian and 234 Japanese undergraduate students. Although students generally have limited organizational experience, their perception of social culture on trustworthiness is within their realm of experience and were considered

an appropriate population for the study.

Participants were asked to describe a trustworthy person, friend, family member, colleague, and supervisor by responding and writing down in their own words to statements such as “A trustworthy person (friend, family member, colleague, or supervisor) is someone who is or does ____.”

For Japanese sample, the original English questionnaire items were translated into Japanese and another independently *back-translate* the Japanese-translation of the original items back into English to *evaluate the quality of the items translated into Japanese*. The Japanese participants were asked to respond and write down *in Japanese* to statements such as above. One professional experienced in the field then translate the responses into English, and another independently *back-translate* the English-translation of the original responses back into Japanese to *compare the validity of English translation of the original responses in Japanese*.

Participants' responses, *all in English*, were then *coded in Canada* by two raters independently, and then reconciled to reflect the extent to which the response reflected the attribute of Ability (A), Benevolence (B), Integrity (I) or something else. This last category was added to the ABI framework, expanding it to include “Other (O)” and thus responses were coded according to an “*ABIO*” framework. These ratings ranged from “0” (the attribute is not present in the response) to “5” (the attribute is emphasized in the response). The ratings for “1” (the attribute is slightly present in the response) and “2” (the attribute is somewhat present in the response) were very small, suggesting that there was no meaningful difference between those ratings, and therefore these categories were collapsed, and the final possible ratings were 0, 1, 2, 3, and 4. Thus, for each role—*trustworthy person, friend, family member, colleague, and supervisor*, there was a distribution of the relative importance of ability, benevolence, integrity, and other attributes in the response.

A distribution of 0440 (ABIO) reflects the same distribution as 0220

(ABIO) since in both cases B and I are rated as equally important, and A and O are absent. These patterns will be turned into proportionate responses (or probability) so that the total probability of ABI&O adds up to “1.” An ABIO pattern of 0440 would become $(0, 0.5, 0.5, 0)$ and the ABIO pattern of 0220 would also become $(0, 0.5, 0.5, 0)$. Similarly, a pattern of 0140 would become $(0, 0.2, 0.8, 0)$.

We present two 3-dimensional (Ability, Benevolence, Integrity) scatter-plots, one for Canadian and the other for Japanese sample with additional data dimension “Other” represented by a colored vertical bar to the right in Figures 1 and 2.

Since there are overwhelming number of points clustered at the $(A, B, I, O) = (0, 0, 1, 0)$ location and to a lesser degree at the $(A, B, I, O) = (0, 0, 0, 1)$ location in the 3-dimensional plot, we “jittered” or added a small amount of noise, to all the data to break ties. We immediately notice that both Canadian and Japanese sample has a many $(0, 0, 1, 0)$ for ABIO pattern. There are 137 Canadian respondents in dark blue clustered around $(A, B, I, O) = (0, 0, 1, 0)$ in Figure 1, while there are almost as many (125) Japanese respondents (in dark blue) with the same response pattern in Figure 2.

On the other hand, there are many more Japanese samples with $(A, B, I, O) = (0, 0, 0, 1)$ than Canadian samples: there are 33 Japanese respondents in dark brown clustered around $(A, B, I, O) = (0, 0, 0, 1)$, while there are only 5 Canadian respondents (in dark brown) with the same response pattern. This group is *potentially troublesome* because the ABI framework may not hold for respondents belonging to this subgroup and applicability/universality of the ABI framework is in doubt at least for this subgroup.

3 Methods

Note that for each respondents, we originally record the extent to which the response reflected the attribute of Ability (A), Benevolence (B), In-

Figure 1: Four dimensional “jittered” scatterplot of (Ability, Benevolence, Integrity) for trustworthy person in Canadian sample with “Other” represented by a colored vertical bar to the right.

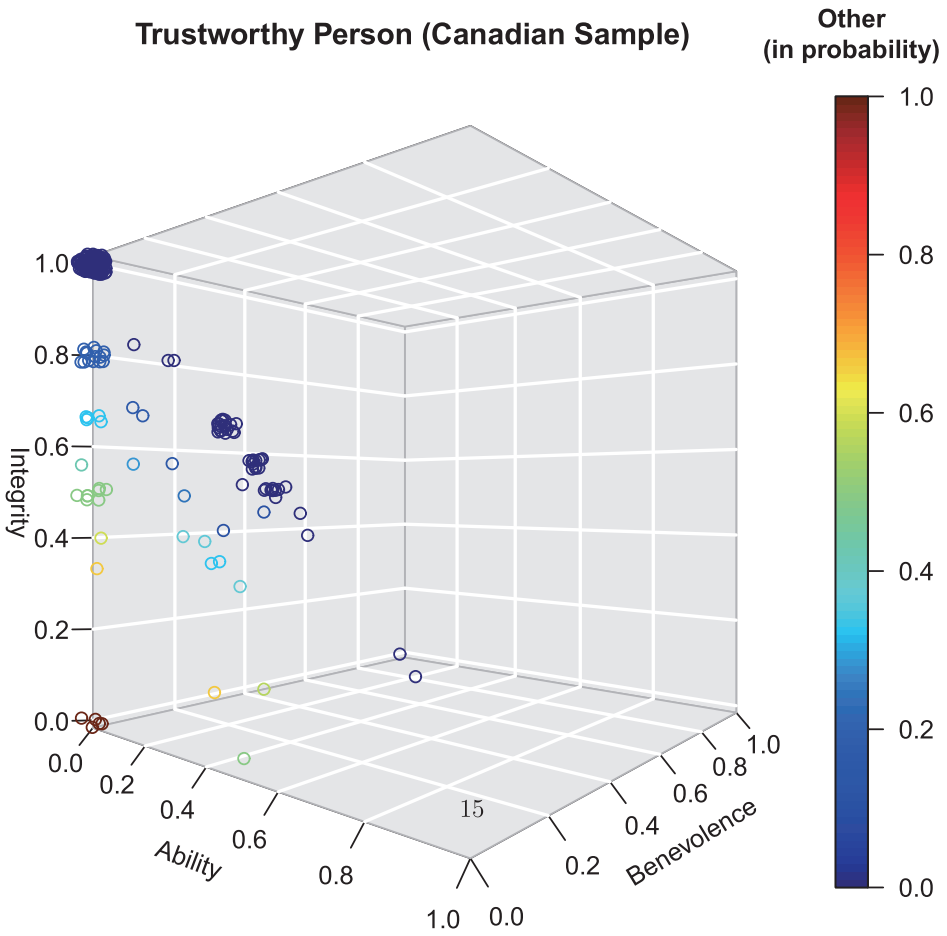
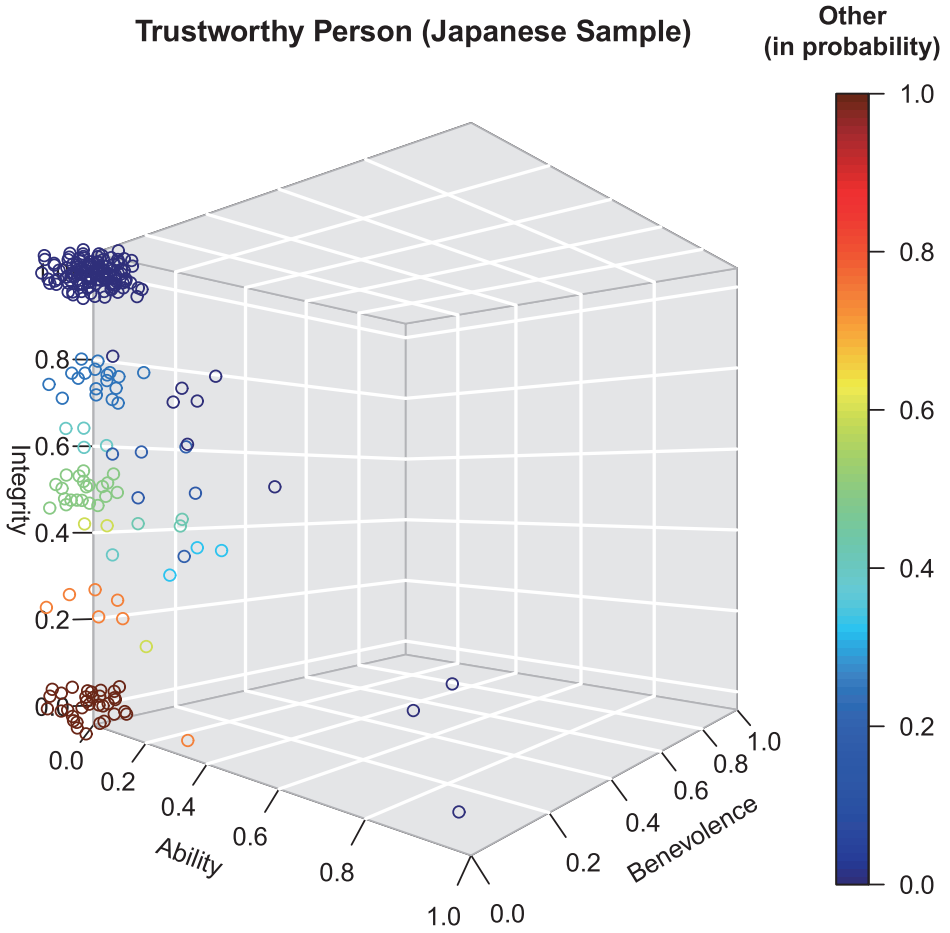


Figure 2: Four dimensional “jittered” scatterplot of (Ability, Benevolence, Integrity) for Trustworthy Person in Japanese sample with “Other” represented by a colored vertical bar to the right.



tegrity (I) or something else (O) by ratings of 0, 1, 2, 3, and 4, then we convert them into four category proportionate responses (or probability) so that the total probability of ABI&O adds up to 1. In other words, our data are quantitative descriptions of the parts of some whole, conveying relative information, or all relevant information in this type of data is contained in ratios between components. This type of data are thus called “compositional data.”

Compositional Data

The use of standard statistical methods for the analysis of compositional data that use the usual Euclidean geometry is not coherent. Rather, compositional data follow the so-called Aitchison geometry on the simplex.

A compositional data point (or composition for short) $\mathbf{x} = (x_1, x_2, \dots, x_D)$ with D component can be represented by a positive real vector. The sample space of compositional data is a D -part simplex:

$$\mathcal{S}^D = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_D) \in \mathbb{R}^D \mid x_i > 0, i = 1, 2, \dots, D; \sum_{i=1}^D x_i = \kappa \right\},$$

where κ is a prescribed sum constraint. In other words, sample space of a composition consists of the set of all complete rays from the origin, such that the parts are *strictly positive*.

According to Egozcue (2009) [Egozcue, 2009], compositional data analysis should respect the following principles:

scale invariance The information in a composition does not depend on the particular units in which the composition is expressed;

permutation invariance Permutation of parts of a composition does not alter the information conveyed by the compositional vector;

subcompositional coherence Information conveyed by a composition of D parts should not be in contradiction with that coming from a subcomposition containing d parts, $d < D$.

In Aitchison geometry, in order to maintain such desirable properties as *scale invariance*, *permutation invariance*, and *subcompositional coherence*, the so-called “Aitchison distance” $d_A(\mathbf{x}, \mathbf{y})$ between D -component composition $\mathbf{x} = (x_1, x_2, \dots, x_D)$ and $\mathbf{y} = (y_1, y_2, \dots, y_D)$ is defined as

$$d_A(\mathbf{x}, \mathbf{y}) = \sqrt{\frac{1}{2D} \sum_{i=1}^D \sum_{j=1}^D \left(\ln \frac{x_i}{x_j} - \ln \frac{y_i}{y_j} \right)^2}, \quad (1)$$

which supports the concept of relative scale. If one compares the Aitchison distance in (1) with the usual Euclidean distance $d_E(\mathbf{x}, \mathbf{y})$ of

$$d_E(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^D (x_i - y_i)^2}, \quad (2)$$

one immediately notices the difficulty of defining the Aitchison distance when at least one of x_i or y_i is zero, because $\ln(0) = -\infty$. This is why the sample space of a composition consists of the set of all *strictly positive* complete rays from the origin.

Structural or essential zero in compositional data analysis

According to the nature of values, *rounded* or *structural/essential zeros* need to be considered. Rounded zeros occur when either small values of components are rounded zeros, or a measurement device has incorporated a detection limit or a threshold value that automatically sets values below this limit/threshold to zero. Therefore, their replacement by a small positive value is reasonable. For rounded zeros, model-based algorithms have been developed and are available in such statistical package as *R*.

However, structural or essential zero are result of a structural process, and thus imputing them does not make sense. Aitchison and Kay (2003) [Aitchison and Kay, 2003] points out:

By an essential zero we mean a component which is truly zero, not something recorded as zero simply because the experimental design or the measuring instrument has not been sufficiently sensitive to detect a trace of the part. Such essential zeros occur in many compositional situations. ... In household budget patterns, where some households may spend nothing on such commodity groups as tobacco, alcohol, entertainment, over the period of observation.

Structural or essential zeros cannot be analyzed directly within the well-established logratio methodology for compositional data. One approach discussed in Aitchison and Kay (2003) [Aitchison and Kay, 2003] is “to interpret structural or essential zeros in a certain part as indicators of two different subgroups: one group containing observations with a value of zero in a certain part as indicators of two different subgroups. This implicitly assumes that the observations originate from two populations, with and without zero in the specific component, with possibly different distributions of the non-zero parts (Filzmoser, Hron and Templ, 2018, p.266)” [Filzmoser et al., 2018]. If this is indeed the case, both groups of observations can be analyzed separately. Such an approach makes sense only when there are a very simply zero structures, however. When the zero structure is more complex, one would have to split the data set into many number of subgroups for all possible patterns of zeros in the data. Consequently, there will be insufficient sample size for analyzing any one of such a subgroup. From Figures 1 and 2, it is obvious that we are facing situation of structural or essential zeros with our Canadian and Japanese samples.

The Dirichlet distribution

Before the logratio methodology was introduced, a standard approach for modeling compositional data was based on the Dirichlet distribution. The Dirichlet distribution is a probability distribution as well—but it is not sampling from the space of real numbers. Instead it is sampling over a probability simplex. A probability simplex is a bunch of numbers that add up to 1. For example: (0.65, 0.35), (0.05, 0.25, 0.7), (0.05, 0.2, 0.1, 0.15, 0.35, 0.15). These numbers represent probabilities over D distinct categories. In the examples above, D is 2, 3, and 6 respectively. When we are dealing with categorical distributions and we have some uncertainty over what that distribution is, the simplest way to represent that uncertainty as a probability distribution is the Dirichlet.

A D -dimensional Dirichlet distribution has D parameters. These parameters can be any positive number. For example, a 4-dimensional Dirichlet may look like this: (0.23, 0.06, 0.32, 0.39). The probabilities just happen to be the mean value of the Dirichlet. So, *all samples from it will center around that simplex*.

The Dirichlet distribution of order $D \geq 2$ with positive parameters $\alpha_1, \dots, \alpha_K > 0$ for $\mathbf{x} = (x_1, x_2, \dots, x_D) \in \mathcal{S}^D$ has a probability density function with respect to Lebesgue measure on the Euclidean space \mathbb{R}^{D-1} given by

$$f(x_1, \dots, x_K; \alpha_1, \dots, \alpha_D) = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^D x_i^{\alpha_i - 1}$$

where $\{x_k\}_{k=1}^{k=D}$ belong to the standard $D - 1$ simplex, or in other words: $\sum_{i=1}^D x_i = 1$ and $x_i \geq 0$ for all $i \in [1, D]$. The normalizing constant is the multivariate beta function, which can be expressed in terms of the gamma function:

$$B(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^D \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^D \alpha_i\right)}, \quad \boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_D).$$

The Dirichlet distribution has many advantages: its marginal distributions are again Dirichlet distributions; widely applied in Bayesian statistics as prior and posterior; *it allows structural or essential zeros in the composition*. However, the approach based on the Dirichlet distribution is *not scale invariant*. It is also known that this shortcoming cannot be overcome even by redefining it with respect to the Aitchison geometry.

Mixture models

In a mixture model, each observed data point is assumed to belong to a cluster. Mixture models are used for understanding the group structure of a data set and for flexibly estimating the distribution of a population. The traditional mixture modeling approach to clustering requires *the number of clusters to be specified in advance* of analyzing the data. Most statisticians address them by first fitting several models, with different numbers of clusters or factors, and then selecting one using model comparison metrics ([Claeskens and Hjort, 2008]). K-means clustering is the most commonly used clustering (or unsupervised machine learning) algorithm for partitioning a given data set into a set of k groups (i.e. k clusters), where k represents the number of groups pre-specified by the analyst. It classifies objects in multiple groups (i.e., clusters), such that *objects within the same cluster are as similar as possible* (i.e., *high intra-class similarity*), whereas *objects from different clusters are as dissimilar as possible* (i.e., *low inter-class similarity*). The most common method to determine the number of clusters is to use *Elbow Method* that plots within-sum-of-squares in y -axis against the number of clusters in x -axis and find the optimal number of clusters as it appears to be the bend in the knee (or elbow) as seen in Figure 3. We present the result of the optimal 7 clusters by K-means in Figure 4. *Conditional on the seven clusters from the Elbow Method chosen by K-means algorithm*, Canadian and Japanese respondents form the

Figure 3: Elbow Method: The plot of within-sum-of-squares against the number of clusters for trustworthy person in Canadian and Japanese samples combined

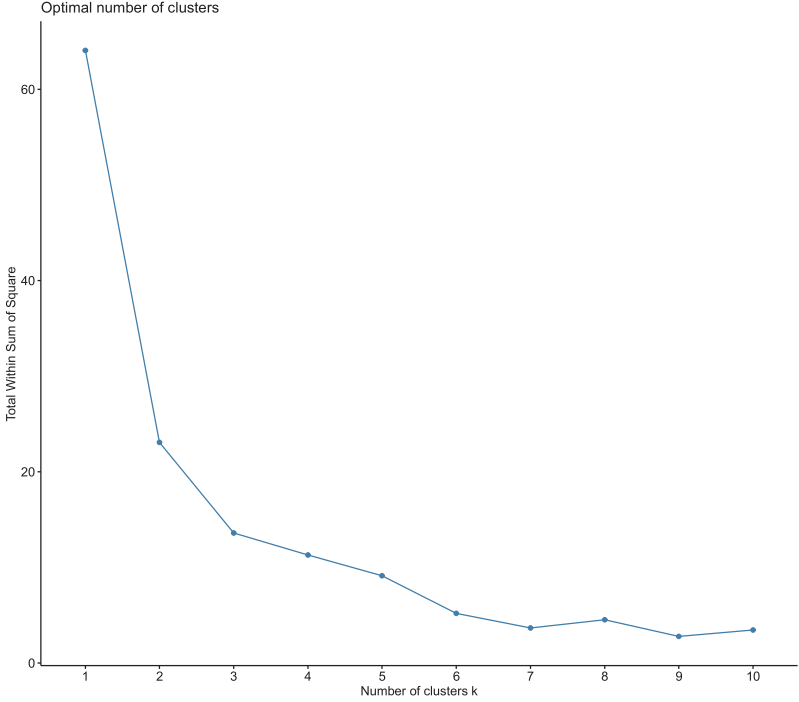


Table 1: How many Canadian and Japanese subjects fall in the 7 clusters given by K-means

Sample \ Cluster	#1	#2	#3	#4	#5	#6	#7	Row Sum
Canada	12	137	6	48	3	5	25	236
Japan	33	125	41	5	2	8	20	234
Column Sum	45	262	47	53	5	45	13	470

following multinomial samples within each cluster as seen Table 1.

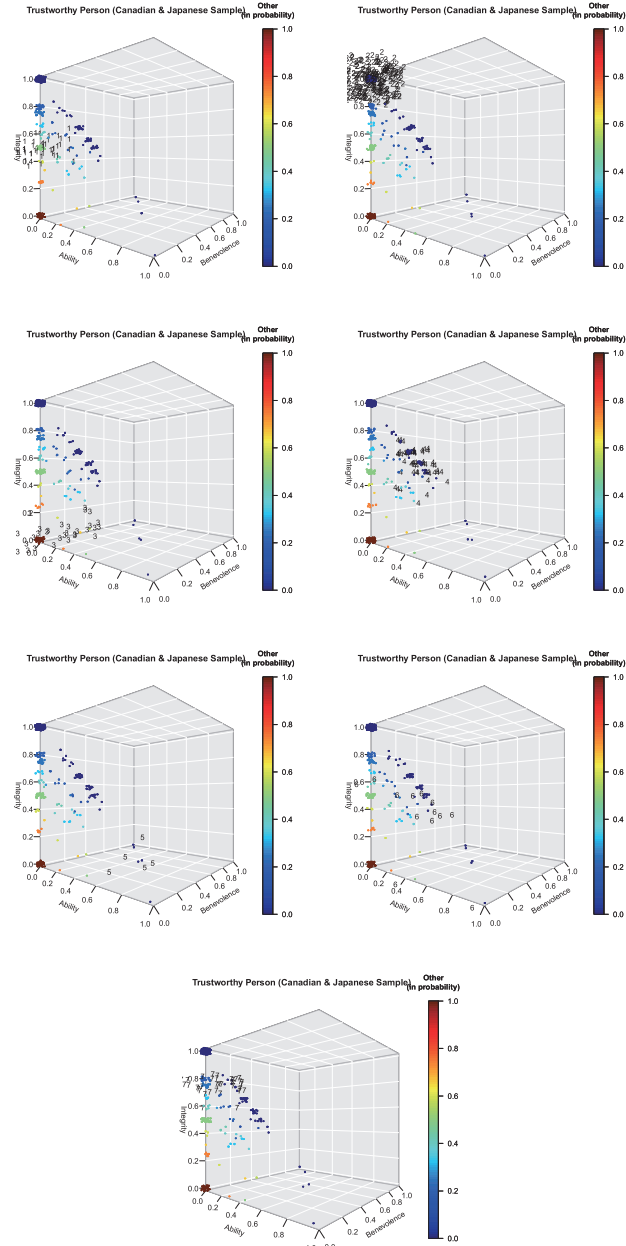
Conditional on the seven clusters given by K-means algorithm, we reject the hypothesis that the two multinomial samples, one consisting of respondents from Canada and the other comprising of respondents from Japan, come from the same distribution because: $X\text{-squared} = 72.741$, $df = 6$, $p\text{-value} = 1.119e-13$.

Bayesian nonparametric clustering

One potential disadvantage of K-means clustering is that *it requires us to pre-specify the number of clusters*. *Bayesian nonparametric* (BNP) models provide a different approach to this problem ([Hjort et al., 2010]). Rather than comparing models that vary in complexity, the BNP approach is to fit a *single model that can adapt its complexity to the data*. In other words, BNP models allow the complexity (*the number of clusters in our case*) to grow as more data are observed.

For example, consider the problem of clustering data. The Bayesian nonparametric approach estimates how many clusters are needed to model the observed data and allows future data to exhibit previously unseen clusters. A finite mixture model assumes that there are K clusters, each associated with a parameter θ_k . Each observation y_n is assumed to be generated by first choosing a cluster c_n according to $\Pr\{c_n\}$ and then generating the

Figure 4: Four dimensional “jittered” scatterplots of (Ability, Benevolence, Integrity) for trustworthy person in Canadian and Japanese samples combined with “Other” represented by a colored vertical bar to the right according to the 7 clusters by K-means.



observation from its corresponding observation distribution $\Pr\{y_n|\theta_{c_n}\}$ parameterized by θ_{c_n} .

Finite mixtures can accommodate many kinds of data by changing the data generating distribution. In this study, the data—conditional on knowing their cluster assignments—are assumed to be drawn from the Dirichlet distributions. Bayesian mixture models further contain a prior over the mixing distribution $\Pr(c)$, and a prior over the cluster parameters: $\theta \sim G_0$. A convenient choice for the distribution on the mixing distribution $\Pr(c)$ in our case is also a Dirichlet.

BNP clustering addresses this problem by assuming that *there is an infinite number of latent clusters*, but that *a finite number of them is used to generate the observed data*. The BNP approach finesses the problem of choosing the number of clusters by assuming that it is infinite, while specifying the prior over infinite groupings $\Pr\{\mathbf{c}\}$ in such a way that it favors assigning data to a small number of groups.

BNP clustering with Chinese Restaurant process

Our BNP clustering model uses the *Chinese restaurant process* (CRP) in an infinite-capacity mixture model. Each table k is associated with a cluster and with a cluster parameter θ_k , drawn from a prior G_0 . Each data point is a “customer,” who sits at a table T_n and then draws its observed value from the distribution $\Pr(x_n|\theta_{T_n})$.

The concentration parameter α^1 controls the prior expected number of clusters (i.e., occupied tables) $K+$. In particular, this number grows logarithmically with the number of customers N :

$$E[K+] = \alpha \log N \quad \text{for } \alpha < N/\log N$$

When we analyze data with a CRP, we form an approximation of the joint posterior over all latent variables and parameters. We use this posterior to

¹If α is treated as unknown, one can put a hyperprior over it

Table 2: How many Canadian and Japanese subjects fall in the 7 clusters given by CRP mixture BNP clustering with with $\alpha = 0.3$

Sample\Cluster	#1	#2	#3	#4	#5	#6	#7	Row Sum
Canada	152	25	43	8	6	1	1	236
Japan	131	83	8	4	5	3	0	234
Column Sum	283	108	51	12	11	4	1	470

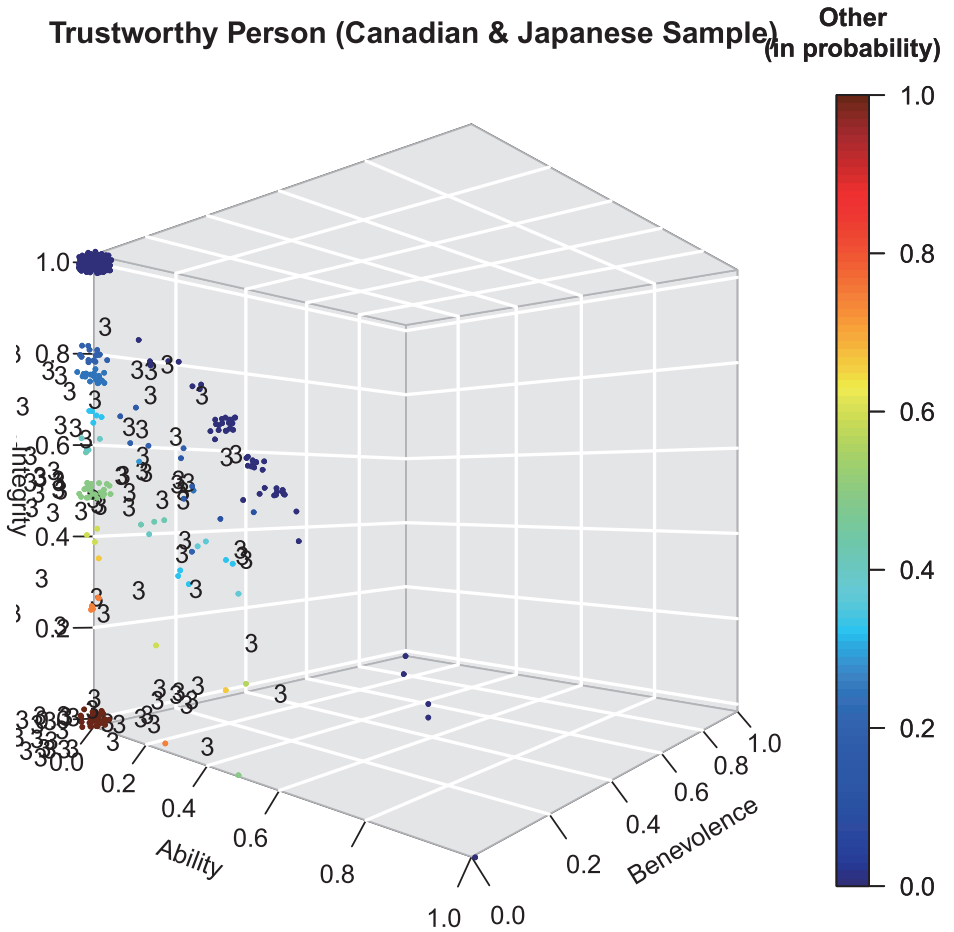
examine the likely partitioning of the data: This gives us a sense of how data are grouped, and how many groups the CRP model chose to use.

Conditional on the seven clusters given by Chinese Restaurant process mixture BNP clustering with $\alpha = 0.3$, respondents from Canada and Japan form the following multinomial samples as seen in Table 2: Conditional on the seven clusters given by Dirichlet process Mixture BNP, we reject the hypothesis that the two multinomial samples, one respondents from Canada and the other from Japan, come from the same distribution because: $X\text{-squared} = 60.143$, $df = 6$, $p\text{-value} = 4.21e-11$. However, we are concerned about 108 respondents belonging to cluster #2 in Figure 5 because, as it turns out, this cluster contains 5 Canadian and 33 Japanese respondents with $(A, B, I, O) = (0, 0, 0, 1)$ and it seems this cluster is too broad.

4 Conclusion and Discussion

Employing two clustering methods— K -means and BNP with CP—we learned that Canadian and Japanese samples do seem to differ significantly, even though a majority in both samples have—137 Canadian out of 236 ($\approx 58.1\%$) and 125 Japanese out of 234 ($\approx 53.4\%$) samples responded with clear and overwhelming importance of integrity shown in Figures 1 and 2. It should be noted that BNP with CP clustering tends

Figure 5: Four dimensional “jittered” scatterplot of (Ability, Benevolence, Integrity) for trustworthy person in Canadian and Japanese samples combined with “Other” represented by a colored vertical bar to the right according to the #2 of 7 clusters by CRP mixture BNP clustering with $\alpha = 0.3$.



to produce many small clusters regardless of whether they are needed to accurately characterize the data. Interpretability, parsimony, data storage and communication costs all are hampered by having overly many clusters, however. See, for instance, Lu et al. (2018) ([Lu et al., 2018]).

If one is asked if the proportion of Japanese respondents who think integrity is the only characteristic that matters for trustworthiness different from the corresponding proportion of Canadian respondents, we have $(0.534 - 0.581) / \sqrt{0.581(1 - 0.581)/236} \approx -0.04632/0.03212$, which is about 1.44 via the normal approximation of the binomial distribution. The answer is clearly no and we cannot reject the hypothesis that “*about the same proportion of individuals in Canadian as well as Japanese culture will hold the integrity to be the dominant components in the ABI framework.*” Since we are ready to accept that individuals will vary in the extent to which they are good cultural representatives and consider trustworthiness in the mandated way within a given culture, however, we need to find out where the differences come from between these two samples.

For many applications involving compositional data, it is necessary to establish a valid measure of distance, yet when essential zeros are present traditional distance measures for compositional data are problematic. Recently Stewart (2017) ([Stewart, eros]) compared measures of distance for compositional data capable of handling zeros, but not satisfying some of the well-accepted principles of compositional data analysis. She found that the chi-square (CS) measure of distance between $\mathbf{x} = (x_1, x_2, \dots, x_D)$ and $\mathbf{y} = (y_1, y_2, \dots, y_D)$ with D component as

$$\text{CS}(\mathbf{x}, \mathbf{y}) = \sqrt{2D} \left(\sum_{i=1}^D r_i \right)^{1/2},$$

where

$$r_i = \begin{cases} 0 & \text{if } x_i = y_i = 0, \\ \frac{\left(\frac{x_i}{\sum_{k=1}^D x_k} - \frac{y_i}{\sum_{k=1}^D y_k} \right)^2}{\frac{x_i}{\sum_{k=1}^D x_k} - \frac{y_i}{\sum_{k=1}^D y_k}} & \text{otherwise.} \end{cases}$$

allows for essential zeros, and **scale** and **permutation invariant**, though **not subcompositionally dominant**, but the last property may be approximately satisfied through simulation studies. We may wish to investigate the possibility of employing this distance measure for comparing responses from Canadian and Japanese respondents for future research.

References

- [Aitchison and Kay, 2003] Aitchison, J. and Kay, J. (2003). Possible solution of some essential zero problems in compositional data. In Thió-Henestrosa, S. and Martín-Fernández, editors, *Proceedings of the Co-DaWork '03, The 1st Compositional Data Analysis Workshop*, pages CD-ROM. University of Girona, Girona.
- [Axelrod, 1984] Axelrod, R. (1984). *The Evolution of Cooperation*. Basic Books, New York.
- [Bond and Smith, 2018] Bond, M. H. and Smith, P. B. (2018). Discovering culture’s influence in studies of individual behavior in organizational settings: A challenging proposal. *International Studies of Management & Organization*, 48(4):419–434.
- [Claeskens and Hjort, 2008] Claeskens, G. and Hjort, N. (2008). *Model selection and model averaging*. Cambridge University Press.
- [Cook et al., 2005] Cook, K. S., Yamagishi, T., Cheshire, C., Cooper, R., Matsuda, M., and Mashima, R. (2005). Trust building via risk taking: A cross-societal experiment. *Social Psychology Quarterly*, 68(2):121–142.
- [Doney et al., 1998] Doney, P., Cannon, J., and Mullen, M. (1998). Understanding the influence of national culture on the development of trust. *The Academy of Management Review*, 23(3):601–620.
- [Egozcue, 2009] Egozcue, J. J. (2009). Reply to “on the harker variation diagrams:...” by j.a. cortés. *Mathematical Geosciences*, 41:829834.
- [Filzmoser et al., 2018] Filzmoser, P., Hron, K., and Templ, M. (2018). *Applied Compositional Data Analysis With Worked Examples in R*. Springer.

- [Foddy et al., 2009] Foddy, M., Platow, M., and Yamagishi, T. (2009). Group-based trust in strangers: The roles of stereotypes and expectations. *Psychological Science*, 20(4):419–422.
- [Hardin, 2002] Hardin, R. (2002). *Trust and Trustworthiness*. Russell Sage Foundation, New York, NY.
- [Hayashi et al., 1999] Hayashi, N., Ostrom, E., Walker, J., and Yamagishi, T. (1999). Reciprocity, trust, and the sense of control: A cross-societal study. *Rationality and Society*, 11(1):27–46.
- [Hjort et al., 2010] Hjort, N., Holmes, C., M'uller, P., and Walker, S. G. (2010). *Bayesian nonparametrics: principles and practice*. Cambridge University Press.
- [Kakiuchi and Yamagishi, 1997] Kakiuchi, R. and Yamagishi, T. (1997). General trust and the dilemma of variable interdependency. *Japanese Journal of Social Psychology*, 12(3):212–221. (in Japanese).
- [Kikuchi et al., 1997] Kikuchi, M., Watanabe, Y., and Yamagishi, T. (1997). Judgment accuracy of other's trustworthiness and general trust: an experimental study. *Japanese Journal of Experimental Social Psychology*, 37(1):23–36. (in Japanese).
- [Kiyonari et al., 2007] Kiyonari, T., Foddy, M., and Yamagishi, T. (2007). Effects of direct and indirect exchange on trust of in-group members. *The Japanese Journal of Psychology*, 77(6):519–527 (in Japanese).
- [Kiyonari et al., 2006] Kiyonari, T., Yamagishi, T., Cook, K. S., and Cheshire, C. (2006). Does trust beget trustworthiness? trust and trustworthiness in two games and two cultures: A research note. *Social Psychology Quarterly*, 69(3):270–283.

- [Kosugi and Yamagishi, 1998] Kosugi, Y. and Yamagishi, T. (1998). General trust and judgments of trustworthiness. *The Japanese Journal of Psychology*, 69(5):349–357. (In Japanese).
- [Kuwabara et al., 2007] Kuwabara, K., Willer, R., Macy, M., Mashima, R., Terai, S., and Yamagishi, T. (2007). Culture, identity, and structure in social exchange: A web-based trust experiment in the u.s. and japan. *Social Psychology Quarterly*, 70(4):461–479.
- [Li, 2007] Li, P. P. (2007). Towards an interdisciplinary conceptualization of trust: A typological approach. *Management and Organization Review*, 3(3):421–455.
- [Li, 2012] Li, P. P. (2012). When trust matters the most: The imperatives for contextualising trust research. *Journal of Trust Research*, 2(2):101–106.
- [Liu et al., 2011] Liu, J., Yamagishi, T., Wang, F., Schug, J., Lin, Y., Yu, S., Takahashi, C., and Hang, L. (2011). Unbalanced triangle in the social dilemma of trust: Internet studies of real-time, real money social exchange between china, japan, and taiwan. *Asian Journal of Social Psychology*, 14(4):246–257.
- [Lu et al., 2018] Lu, J., Li, M., and Dunson, D. (2018). Reducing over-clustering via the powered chinese restaurant process. *arXiv*, 1802.05392v1:1–9.
- [Markus and Kitayama, 1991] Markus, H. and Kitayama, S. (1991). Culture and the self: Implications for cognition, emotion, and motivation. *Psychological Review*, 98(2):224–253.
- [Mashima et al., 2004a] Mashima, R., Yamagishi, T., and Macy, M. (2004a). Trust and cooperation: A comparison of ingroup preference

- and trust behavior between american and japanese students. *Japanese Journal of Psychology*, 75(4):308–315. (in Japanese).
- [Mashima et al., 2004b] Mashima, R., Yamagishi, T., and Matsuda, M. (2004b). Trust as a signal in relations with temporary partners. *Japanese Journal of Social Psychology*, 19(3):175–183. (in Japanese).
- [Matsuda and Yamagishi, 2001] Matsuda, M. and Yamagishi, T. (2001). Trust and cooperation: An experimental study of pd with choice of dependence. *The Japanese Journal of Psychology*, 72(5):413–421. (In Japanese).
- [Matsumoto, 2007] Matsumoto, D. (2007). Culture, context, and behavior. *Journal of Personality*, 75(6):1285–1320.
- [Mayer et al., 1995] Mayer, R. C., Davis, J. H., and Schoorman, F. D. (1995). An integrative model of organizational trust. *The Academy of Management Review*, 20(3):709–734.
- [Platow et al., 2012] Platow, M. J., Foddy, M., Yamagishi, T., Lim, L., and Chow, A. (2012). Two experimental tests of trust in in-group strangers: The moderating role of common knowledge of group membership. *European Journal of Social Psychology*, 42(1):30–35.
- [Romano et al., 2017] Romano, A., Balliet, D., Yamagishi, T., and Liu, J. (2017). Parochial trust and cooperation across 17 societies. *Proceedings of the National Academy of Sciences of the United States of America*, 114(48):12702–12707.
- [Rotter, 1980] Rotter, J. (1980). Interpersonal trust, trustworthiness, and gullibility. *American Psychologist*, 35:1–7.
- [Rotter, 1967] Rotter, J. B. (1967). A new scale for the measurement of interpersonal trust. *Journal of Personality*, 35(4):651–665.

- [Sato and Yamagishi, 1986] Sato, K. and Yamagishi, T. (1986). Two psychological factors in the problem of public goods. *THE JAPANESE JOURNAL OF EXPERIMENTAL SOCIAL PSYCHOLOGY*, 26(1):89–95. (in Japanese).
- [Saunders et al., 2014] Saunders, M. N. K., Dietz, G., and Thornhill, A. (2014). Trust and distrust: Polar opposites, or independent but co-existing? *Human Relations*, 67(6):639–665.
- [Stewart, eros] Stewart, C. (An approach to measure distance between compositional diet estimates containing essential zeros). 2017. *Journal of Applied Statistics*, 44(7):1137–1152.
- [Suzuki et al., 2007] Suzuki, N., Konno, Y., and Yamagishi, T. (2007). In-group bias in trusting behavior: A choice of allocator experiment with minimal groups. *The Japanese Journal of Psychology*, 78(1):17–24. (in Japanese).
- [Takahashi et al., 2008] Takahashi, C., Yamagishi, T., Liu, J. H., Wang, F., Lin, Y., and Yu, S. (2008). The intercultural trust paradigm: Studying joint cultural interaction and social exchange in real time over the internet. *International Journal of Intercultural Relations*, 32:215–228.
- [Terai et al., 2003] Terai, S., Morita, Y., and Yamagishi, T. (2003). Trust and assurance of cooperation in ongoing relations: An experimental study using the prisoner’s dilemma with variable dependence. *Japanese Journal of Social Psychology*, 18(3):172–179. (In Japanese).
- [Van Lange and Visser, 1999] Van Lange, P. A. M. and Visser, K. (1999). Locomotion in social dilemma: How people adapt to cooperative, tit-for-tat, and noncooperative partners. *Journal of Personality and Social Psychology*, 77:762–773.

- [van't Wout and Sanfey, 2008] van't Wout, M. and Sanfey, A. (2008). Friend or foe: the effect of implicit trustworthiness judgments in social decision-making. *Cognition*, 108(3):796–803.
- [Whitener et al., 2000] Whitener, E. M., Maznevski, M. L., Hua, W., Saebo, S. R., and Ekelund, B. Z. (2000). Testing the cultural boundaries of a model of trust: Subordinate-manager relationships in china, norway and the united states. <http://escholarship.org/uc/item/8th9x1fn>. UCLA Asia Institute.
- [Yamagishi, 2011] Yamagishi, T. (2011). *Trust: The Evolutionary Game of Mind and Society*. Springer The science of mind, Springer Japan.
- [Yamagishi et al., 2015] Yamagishi, T., Akutsu, S., Cho, K., Inoue, Y., Li, Y., and Matsumoto, Y. (2015). Two-component model of general trust: Predicting behavioral trust from attitudinal trust. *Social Cognition*, 33(5):436–458.
- [Yamagishi et al., 1998] Yamagishi, T., Cook, K. S., and Watabe, M. (1998). Uncertainty, trust and commitment formation in the united states and japan. *American Journal of Sociology*, 104(1):165–194.
- [Yamagishi et al., 2005] Yamagishi, T., Kanazawa, S., Mashima, R., and Terai, S. (2005). Separating trust from cooperation in a dynamic relationship: Prisoner's dilemma with variable dependence. *Rationality and Society*, 17(3):275–308.
- [Yamagishi et al., 1999] Yamagishi, T., Kikuchi, M., and Kosugi, M. (1999). Trust, gullibility and social intelligence. *Asian Journal of Social Psychology*, 2:145–161.
- [Yamagishi and Komiyama, 1995] Yamagishi, T. and Komiyama, H. (1995). Significance and the structure of trust: Theoretical and empiri-

- cal research on trust and commitment relations. *INSS Journal*, 2:1–59. (in Japanese).
- [Yamagishi and Sato, 1986] Yamagishi, T. and Sato, K. (1986). Motivational bases of the public goods problem. *Journal of Personality and Social Psychology*, 50(1):67–73.
- [Yamagishi and Yamagishi, 1994] Yamagishi, T. and Yamagishi, M. (1994). Trust and commitment in the united states and japan. *Motivation and Emotion*, 18(2):129–166.
- [Yamagishi et al., 1995] Yamagishi, T., Yamagishi, M., Takahashi, N., Hayashi, N., and Watabe, M. (1995). Trust and commitment formation. *Japanese Journal of Experimental Social Psychology*, 35(1):23–34. (in Japanese).
- [Yuki et al., 2005] Yuki, M., Maddux, W. W., Brewer, M. B., and Take-mura, K. (2005). Cross-cultural differences in relationship- and group-based trust. *Personality and Social Psychology Bulletin*, 31:4862.

E-mail address: yukanazawa@icu.ac.jp