Level-set methods applied to the kinematic wave equation governing surface water flows

Sovanna Mean [a,*]
[a] Graduate School of Agriculture, Kyoto University,
Kitashirakawa-Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan
E-mail: sovannamean@gmail.com

Koichi Unami [b]
[b] Graduate School of Agriculture, Kyoto University,
Kitashirakawa-Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan
E-mail: unami@adm.kais.kyoto-u.ac.jp

Masayuki Fujihara [c]
[c] Graduate School of Agriculture, Kyoto University,
Kitashirakawa-Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan
E-mail: fujihara@kais.kyoto-u.ac.jp

ABSTRACT

Critical issues arising from the governing nonlinear equations in surface water hydrodynamic include discontinuities in water surface levels, blow-up of water surface gradient, and treatment of dry beds or zero water depths, involving mathematical problems related to functional regularities of unknown variables such as the water depth. The level-set method is a powerful approach to relax requirements for functional regularities of unknowns in nonlinear partial differential equations of first order. In this study, the level-set method is applied to the one-dimensional kinematic wave equation, resulting in a linear level-set equation of the first order in a two-dimensional space to tackle dry beds. The zeros of the level-set function represent the water depths. Hypothesizing that the level-set function is continuous in the domain, it is numerically computed with a characteristic method. The development of overturning is regulated with singular viscosity regularization (SVR), whose effect is to relocate the zeros of the level-set function close to the exact positions of the shock fronts in dam-break problems. The method is firstly verified with the explicitly known exact solutions of primitive dam-break problems, optimizing a parameter of SVR. Then, abrupt water release from Chan Thnal Reservoir, Kampong Speu Province, Cambodia into an initially dry bed of its irrigation canal system is simulated as a practical demonstrative example. In contrast to most of the available software tools using either the shallow water equations with some artificial viscosity or the diffusion wave approximation, the proposed method turns out to be free from spurious diffusive deformation of water surfaces even if relatively coarse computational mesh is used.

Keywords

Level-set method; surface water flow; dry bed; functional regularity; singular viscosity regularization; kinematic wave equation.

1. Introduction

[*] Corresponding author. Tel: +81 70 1075 8955
E-mail address: mean.sovanna.86c@st.kyoto-u.ac.jp, sovannamean@gmail.com

Analysis of surface water flows is essential in understanding hydrodynamic phenomena such as flash floods, surge propagation and inundation resulting from dam-breaks, tsunami, flows resulting from operation of hydraulic structures such as gate and weir (Khan, 2000). Those hydrodynamic phenomena influence on humans, environment, and economics (Alvarez et al., 2019). For instance, the sudden release of water stored in a reservoir could lead to severe environmental issues, risks to human life and economical damage (Castro-Orgaz and Chanson, 2017). The more vulnerable areas that would be at risk due to such flooding caused by dam failure are the downstream dry bed terrains occupied by humans, infrastructures, industries, and agricultural lands. Frequency of extreme precipitation events and thus floods are expected to increase due to global climate change (Berndtsson et al., 2019). Flood simulation outputs play pivotal roles in flood risk management (Mohanty et al., 2020). Disaster risk reduction is better achieved if a socio-hydrological approach is well linked with hydrodynamic modelling (Abebe et al., 2019). On the other hand, abrupt operation of irrigation facilities is commonly practiced in order to increase water efficiency in agriculture.

The surface water flows have been well comprehended in the context of the shallow water equations (SWEs) with the assumption that the vertical scale is much smaller than the horizontal scale. They consist of two conservation laws: mass and momentum in analogy to the Navier-Stokes equations for incompressible fluids. The SWEs have been applied to various fields, such as coastal, environmental, and water resources engineering. The coastal engineering deals with the problems of tsunami-wave propagation, tide-currents, and storm-surges, etc. Liu et al. (2009) compared linear and nonlinear SWEs in describing tsunami-wave propagation over China Sea. Zhu et al. (2017) estimated tidal currents and residual currents by using the SWEs to analyze their generation mechanisms. Akbar and Aliabadi (2013) used hybrid finite element and finite volume techniques to solve two-dimensional (2D) SWEs for dealing with hurricane induced storm surge flow problem. The SWEs have been consistently used in flood propagation, flood inundation modelling, and river flooding in the field of water resources engineering and pollution transport problems in the environmental engineering. Cozzolino et al. (2019) considered a simplification of the complete SWEs called the Local Inertia Approximation, which is derived by neglecting the advective term in the momentum equation. This model is generally applied to simulate slow flooding propagation at moving wetting-drying area on even and uneven beds. A set of equations was derived from 1D SWEs to be used for simulating 2D flood inundation (Bates et al., 2010). Audusse and Bristeau (2003) computed the transport of a passive pollutant with the SWEs using a finite volume kinetic method. Kuriqi and Ardiclioglu (2018) investigated the hydraulic regime of the Loire River in France using HEC-RAS, which is a widely adapted simulation software based on the 1D SWEs. Ardiclioglu and Kuriqi (2019) applied HEC-RAS to discuss the channel roughness of a natural river in six different flow regimes.

The complexity of the SWEs is attributed to that they model dynamics of water with all aspects including local acceleration term, convection acceleration term, pressure force term, gravity force term, and friction force term. There are major two approximation methods for the 1D SWEs. The first approximation method is diffusion wave approximation, in which the local acceleration term and convection acceleration term are not considered. The second one is to consider only gravity force term and friction force term as force terms, resulting in the kinematic wave equation. The model is originally introduced and specifically described in Lighthill and Whitham (1955). In the kinematic wave equation, several terms in the equation of motion such as local acceleration term, convective acceleration term, and pressure force term are assumed to be insignificant; hence, the equation of motion is simply expressed that the bed slope is equivalent to friction slope (Miller, 1984). The kinematic wave equation has been employed to a number of hydraulic processes of subsurface flow, surface flow, sediment transport, solute transport and glacier hydrology. Singh (2001) presented the history of the kinematic wave theory and its applications in water resources. Singh (2017) discussed the general concept of kinematic wave for overland flows such as mathematical formulation and validity of the concept. The mixed runoff generation model and 2D kinematic wave model were introduced for overland flow routing in the upper Kongjiapo basin in the Qin River (Bao et al., 2017). Huang et al. (2015) combined rainfall-runoff and snowmelt modules, which were derived from the kinematic wave equation and the energy budget method, respectively, to estimate the surface water resources in the semiarid area of Heilongjiang Province, China. Yomota and Islam (1992) used the kinematic wave equation for calculating the flood runoff discharge from the inclined upland field in Hiroshima Prefecture, Japan, concluding that the application of the kinematic wave equation with Manning's roughness produced the better results than the other resistance law of Darcy and laminar.

In mathematical viewpoints, an initially dry bed for simulation of surface water flows using the kinematic wave equation is a challenging issue, involving the deformation of the domain. Most of conventional flow models assume sufficiently small positive water depths in the domain for a well-posed hyperbolic problem to avoid failure in producing solutions. The level-set method is powerful in relaxing requirements for functional regularities of unknowns in nonlinear partial differential equations of first order. The method is one of the approaches to track the motion of propagating fronts or surfaces, which are considered as the zero level-set of higher level-set functions (Caselles et al., 1993). Originally introduced for curvature flows by Osher and Sethian (1988), the overview of level-set method to solve Hamilton-Jacobi equations has been described in the review paper by Gibou et al. (2018). The method has been seen in various fields such as image segmentation (Li et al., 2010), computed tomography (Malladi et al., 1995), and geometry optimization (Osher and Santosa, 2001). Li et al. (2011) successfully applied the level-set method to solving image segmentation which had faced the difficulty of intensity inhomogeneity in real world images. In structural engineering, the level-set method is applied for structural optimization to minimize the structural load while satisfying the constraint (Sethian and Wiegmann, 2000). The method has been applied to fluid mechanics for both incompressible and compressible flows as well, in terms of ship hydrodynamics, image segmentation, and shape or topology optimization. For instance, a coupled level-set and volume-of-fluid method has been used for the computation of interfacial flows in the ship hydrodynamics (Wang et al., 2009). Sethian and Smereka (2003) tracked fluid interfaces. Duan et al. (2008) proposed the variational level-set function to optimize the shape-topology in the Navier-Stokes problem by maintaining the smooth evolution without re-initialization and topology change. Yue et al. (2003) presented a numerical method to simulate free surface flows by solving the 3D incompressible Navier-Stokes equation with the level-set method. However, the kinematic wave equation has not yet been tackled with the level-set method in the literature.

This study aims at clarifying advantages and limitations of regarding the kinematic wave equation as a Hamilton-Jacobi equation. The level-set method is firstly applied to the continuity equation of one-dimensional (1-D) open-channel flows, resulting in a nonlinear level-set equation of first order in a two-dimensional (2-D) space governing a level-set function whose zeros represent the water depths. Prior to the analysis of surface water flows, the Eikonal equation is considered as a primitive but an important example to comprehend the general idea of the level-set method. The Eikonal equation is often used for delineating the first-arrival time problems, and it has various practical applications including computational geometry, computer vision, and material science, etc. (Fomel et al., 2009). Transmit times for 3D seismic waves can be computed numerically from the Eikonal equation using the finite difference method (Vidale, 1988). However, calculation of expanding wave fronts requires the notion of characteristics (Qin et al., 1992). Therefore, we numerically compute the level-set function for the kinematic wave equation with a characteristic method, and the numerical solutions are verified with the analytical solutions of relevant dam-break problems. The analytical weak solutions of the dam-break problems are obtained from theoretical celerity, which becomes the speed of propagating shock front on a dry bed. However, it turns out that the problems related to overturning phenomena appear. In order to control the development of overturning, singular viscosity regularization (SVR) is employed (Tsai et al., 2003). Then, the computational results with and without SVR are compared with the analytical weak solution representing time evolution of the shock front propagating downstream, optimizing a parameter of SVR. Finally, abrupt water release from Chan Thnal Reservoir, Kampong Speu Province, Cambodia into an initially dry bed of its irrigation canal system is simulated as a practical demonstrative example.

2. Methods

Mathematical models for surface water flows involve nonlinear partial differential equations. In this section, the kinematic wave equation is derived from the SWEs, and the level-set method is briefed with the Eikonal equation. Inclusion relations and dependency among those concepts are delineated in Figure 1.

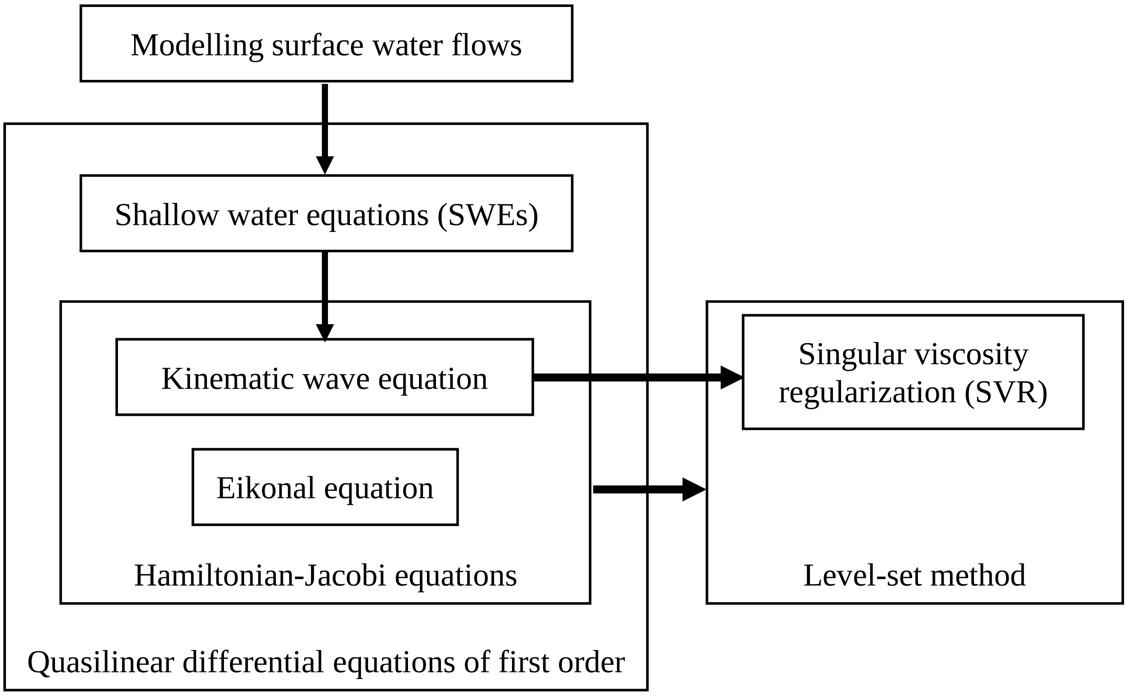

Figure 1. Inclusion relations and dependency among the concepts appearing in the methods

2.1. Governing equation of surface water flows

In one-dimensional surface water flows of hydrostatic pressure distribution, the conservation laws of mass and momentum of the SWEs are written as

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ Q \end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix} Q \\ \dfrac{\beta Q^2}{A} \end{pmatrix} = \begin{pmatrix} q \\ -gA\dfrac{\partial \eta}{\partial x} - gAS_f \end{pmatrix} \tag{1}$$

where t is the time, A is the cross-sectional area, Q is the discharge, x is the local curvilinear abscissa along the channel bed, β is the momentum coefficient, η is the water level, q is the lateral inflow discharge per unit length, g is the acceleration due to gravity, and S_f is the friction slope which is given by the Manning's formula (Unami and Alam, 2012). The kinematic wave equation assumes local equilibrium of momentum and negligible changes in water depths, to reduce (1) to

$$\begin{cases} \dfrac{\partial A}{\partial t} + \dfrac{\partial (AV)}{\partial x} = q \\ V = \dfrac{1}{n} R^{2/3} S_0^{1/2} \end{cases} \tag{2}$$

where V is the cross-sectional averaged velocity, n is the Manning's roughness, R is the hydraulic radius, and S_0 is the bed slope. When a dry bed takes place, the cross-sectional averaged velocity V simply vanishes.

2.2. Analytical solution of kinematic wave equation

Analysis of the kinematic wave equation stems from regarding the system (2) as quasilinear differential equation of first order expressed as

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = r \tag{3}$$

where u is a generic unknown variable, F is the flux which is a nonlinear function of u, and r is a source term. For a piecewise continuous weak solution having a jump from u_l to u_r, the local celerity is given by

$$C(u_l, u_r) = \frac{F(u_l) - F(u_r)}{u_l - u_r} \tag{4}$$

provided that $F'(u_l) > C(u_l, u_r) > F'(u_r)$ (Osher and Fedkiw, 2001). In the continuous case, the celerity approaches to

$$C(u, u) = \frac{\partial F(u)}{\partial u} \tag{5}$$

which is known as the Kleitz-Seddon law in the context of the kinematic wave equation. The analytical solutions will be employed to verify the level-set method with SVR appropriately working.

2.3. Level-set method

The level-set method firmly relies on the Hamilton-Jacobi equation, which often appears in variational calculus. The conservative form (3) is formally rewritten as the Hamilton-Jacobi equation (6)

$$\frac{\partial u}{\partial t} + H(t, x, u, u_x) = 0 \tag{6}$$

where H is the Hamiltonian. The notion of viscosity solution is commonly applied to Hamilton-Jacobi equations. The level-set function $\varphi = \varphi(t, x, z)$ is a function of t, x, and another secondary independent variable *z*, such that its zeros represents *u*. The governing equation of φ is the level-set equation

$$\frac{\partial\varphi}{\partial t}-\frac{\partial\varphi}{\partial z}H\left(t,x,z,-\frac{\varphi_x}{\varphi_z}\right)=0 \tag{7}$$

which must be treated in the viscosity sense, as it is formally derived from $\varphi_t+\varphi_z u_t=0$ and $\varphi_x+\varphi_z u_x=0$ (Tsai et al., 2003).

2.4. Example of the level-set method applied to Eikonal equation

A primitive but important application of the level-set method is to the Eikonal equation. The unsteady form of the Eikonal equation is

$$\frac{\partial u}{\partial t}+\left|\frac{\partial u}{\partial x}\right|=1, \tag{8}$$

whose Hamiltonian H is

$$H(t,x,u,p)=H(p)=|p|-1, \tag{9}$$

and the corresponding level-set equation is

$$\frac{\partial\varphi}{\partial t}-\frac{\varphi_z}{|\varphi_z|}\left|\frac{\partial\varphi}{\partial x}\right|+\frac{\partial\varphi}{\partial z}=0. \tag{10}$$

The level-set function (10) with the initial and boundary condition $\varphi(0,x,z)=z$ and $\varphi(t,\pm 1,z)=z$ is numerically computed in the *t*-*x*-z-domain $(0,6)\times(-1,1)\times(-2,2)$, using an upwind differencing discretization scheme in the *x*-*z*-space and the fourth-order Runge-Kutta method in time *t*. With meshes of $\Delta t=0.1$, $\Delta x=0.1$, and $\Delta z=0.1$, the computed results at *t* = 0, 1, 2, and 3 are presented in Figure 2, where the zeros are highlighted with different colors for different times. As the solution *u* to the Eikonal equation (8) represents the minimum first exit time from the domain, there is a steady-state

$$u=1-|x|, \tag{11}$$

which is achieved within a finite time. The computational results well reproduce the solution, as can be seen from the transient state at *t* = 1 followed by the identical values at *t* = 2 and 3. However, the case of the kinematic wave equation is not straightforward, due to the vanishing Hamiltonian for dry beds.

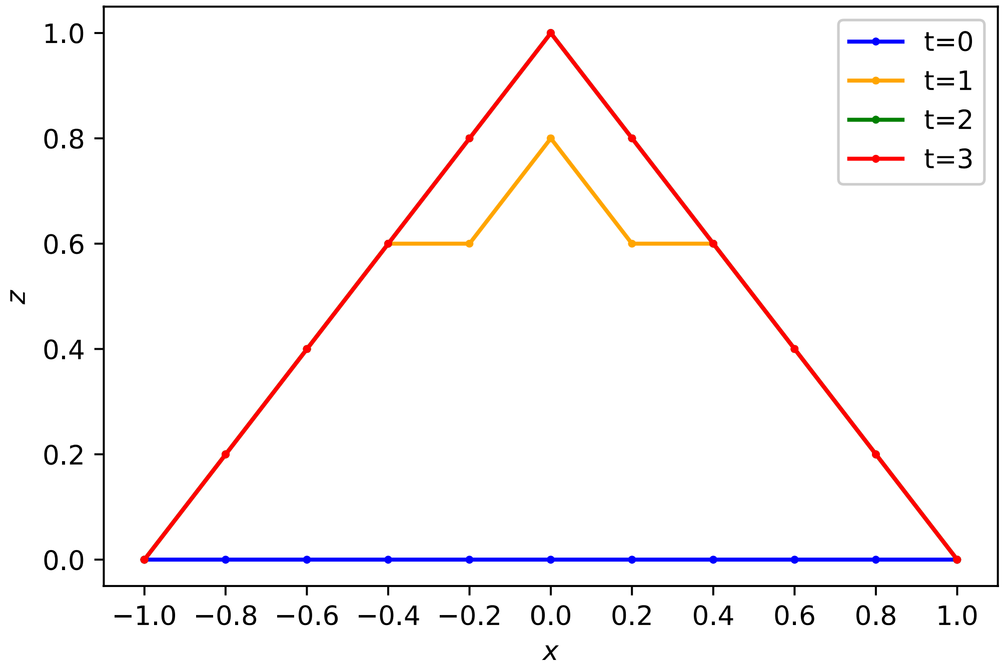

Figure 2. Numerical solutions to the level-set equation for the Eikonal equation

2.5. Derivation of level-set equation for kinematic wave equation

For the sake of simplicity, unit width of an open channel having very broad rectangular cross-section without any lateral flow is considered in the kinematic wave equation (2), implying that $R=h$, $u=h$, $F(u)=\left(S_0^{1/2}/n\right)u^{5/3}$, and $F'(u)=\left(5S_0^{1/2}/3n\right)u^{2/3}$, where h is the water depth. The bed slope S_0 and the Manning's roughness n are assumed piecewise constant. Then, the Hamiltonian H becomes

$$H(t,x,u,p)=\frac{5S_0^{1/2}}{3n}u^{2/3}p \tag{12}$$

and then the level-set equation becomes

$$\frac{\partial\varphi}{\partial t}+\frac{5S_0^{1/2}}{3n}z^{2/3}\frac{\partial\varphi}{\partial x}=0 \tag{13}$$

which governs φ almost everywhere in the t-x-z-domain $(0,\infty)^3$. The initial condition is imposed as

$$\varphi(0,x,z)=z \tag{14}$$

to represent the initial dry bed. Then, assuming dam-break or sudden operation of a hydraulic structure at the upstream end, the boundary condition

$$\varphi(t,0,z)=z-h^{\mathrm{up}} \tag{15}$$

is imposed to specify the upstream water depth as h^{up}. Although u is a function of bounded variation (BV function) allowing discontinuities, the level-set function φ is possibly continuous in the domain but not up to the boundary. Since there is no term of φ_z, (13) is solved as it is an advection equation in the *t*-*x*-domain $(0,\infty)^2$.

2.6. Computational method with SVR

The domain is discretized into meshes of equal size Δt by Δx by Δz, to compute the approximate values of φ. The notation $\varphi_{k,i,j}$ represents the approximated $\varphi(k\Delta t, i\Delta x, j\Delta z)$ for $k,i,j \in \mathbb{Z}$. The characteristic method analogous to the one solving a Bellman equation in dynamic programming is employed (Unami et al., 2019). The piecewise linear interpolation is applied to the *x*-direction as

$$\hat{\varphi}(k\Delta t, x, j\Delta z) = \varphi_{k,i,j} + \left(\varphi_{k,i+1,j} - \varphi_{k,i,j}\right)\frac{x - i\Delta x}{\Delta x} \tag{16}$$

where $\hat{\varphi}$ is the interpolated φ, and $i\Delta x \le x < (i+1)\Delta x$. Then, the level-set equation (13) is approximately solved as

$$\varphi_{k+1,i,j} = \hat{\varphi}(k\Delta t, \xi, j\Delta z) \tag{17}$$

where

$$\xi = i\Delta x - \frac{5S_0^{1/2}}{3n}(j\Delta z)^{2/3}\Delta t\ . \tag{18}$$

In practical implementation of (17), “overturning” may develop due to violation of the minimum principle $\varphi_z(t,x,z) \ge 0$ for $t \ge 0$, and adding a singular viscosity term to (13) serves as regularization. The singular viscosity term $\psi_{k,i,j}$ at the discretized stage is given as

$$\psi_{k,i,j} = M\left|(\nabla\varphi)_{k,i,j}\right| \frac{\tanh\left(\gamma\dfrac{\varphi_{k,i,j+1} - \varphi_{k,i,j}}{\Delta z}\right) - \tanh\left(\gamma\dfrac{\varphi_{k,i,j} - \varphi_{k,i,j-1}}{\Delta z}\right)}{\Delta z} \tag{19}$$

where

$$\left|(\nabla\varphi)_{k,i,j}\right| = \sqrt{\left(\frac{\varphi_{k,i+1,j} - \varphi_{k,i-1,j}}{2\Delta x}\right)^2 + \left(\frac{\varphi_{k,i,j+1} - \varphi_{k,i,j-1}}{2\Delta z}\right)^2} \tag{20}$$

with parameters M to be optimized and $\gamma = 1/\Delta z$, as recommended in the paper.

The zeros of the computed level-set function at each time stage $k\Delta t$, which are represented by $(\xi_{k,i,j}, j\Delta z)$ or $(i\Delta x, \zeta_{k,i,j})$, solve

$$\varphi_{k,i,j} + \left(\varphi_{k,i+1,j} - \varphi_{k,i,j}\right)\frac{\xi_{k,i,j} - i\Delta x}{\Delta x} = 0 \tag{21}$$

with $i\Delta x \le \xi_{k,i,j} < (i+1)\Delta x$ or

$$\varphi_{k,i,j} + \left(\varphi_{k,i,j+1} - \varphi_{k,i,j}\right)\frac{\zeta_{k,i,j} - j\Delta z}{\Delta z} = 0 \tag{22}$$

with $j\Delta z \le \zeta_{k,i,j} < (j+1)\Delta z$, respectively, for each i and j.

3. Results

3.1. Effect of SVR in dam-break problems

The level-set method for the kinematic wave equation is now applied to the dam-break problems with the dry bed initial condition. Without loss of generality, $S_0^{1/2}/n = 1$ is assumed, and the local celerity of dam-break flows over the dry bed becomes $C(u,0) = u^{2/3}$ (m/s) in the model. Numerical experiments for the level-set equation are performed over the subset $(0,100]\times(0,500]\times(0,12)$ of $(0,\infty)^3$, with meshes of $\Delta t = 0.01$ (s), $\Delta x = 1$ (m), and $\Delta z = 0.5$ (m). The boundary $x = 0$ is considered as the position of a dam, separating upstream and downstream areas. At the initial time, the downstream area is set to be a dry bed. Setting a boundary condition to specify a water depth h^{up} at $x = 0$ formulates a dam-break problem for the kinematic wave equation. Firstly, the dam-break problem for h^{up} = 2 (m) is numerically solved by the level-set method without SVR. Figure 3 compares the computed zeros of the level-set function (circular dots, noSVR) with the exact positions of the shock fronts obtained from the analytical solution (lines, Exact). The different colors show the time stages every 10 (s) as the shock front propagates downstream. The computed zeros constitute the upstream water surface $h^{\mathrm{up}} = 2$ (m) and the propagating shock front for each *t*. However, the overturning phenomena occur, which cause unwanted shock front motion. Hence, SVR is introduced with an optimized $M = 0.01$. The results are similarly depicted in Figure 4 where the triangular dots indicate the zeros of level-set function with SVR (SVR). The overturning still remains in the zeros of level-set function with SVR, however, they better approximate the analytical solution with reasonable reproduction of celerity.

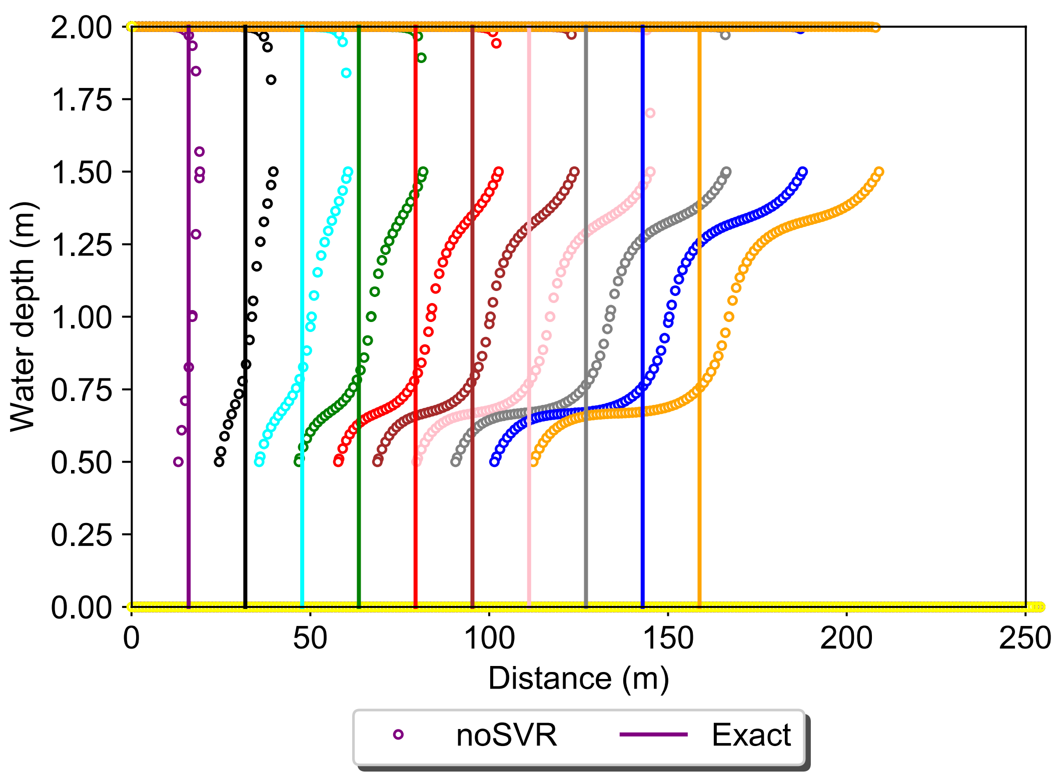

Figure 3. Zeros of the computed level-set function without SVR (noSVR) for the dam-break over the dry bed in comparison with the exact positions of the shock front (Exact) for the case h^{up}=2 (m)

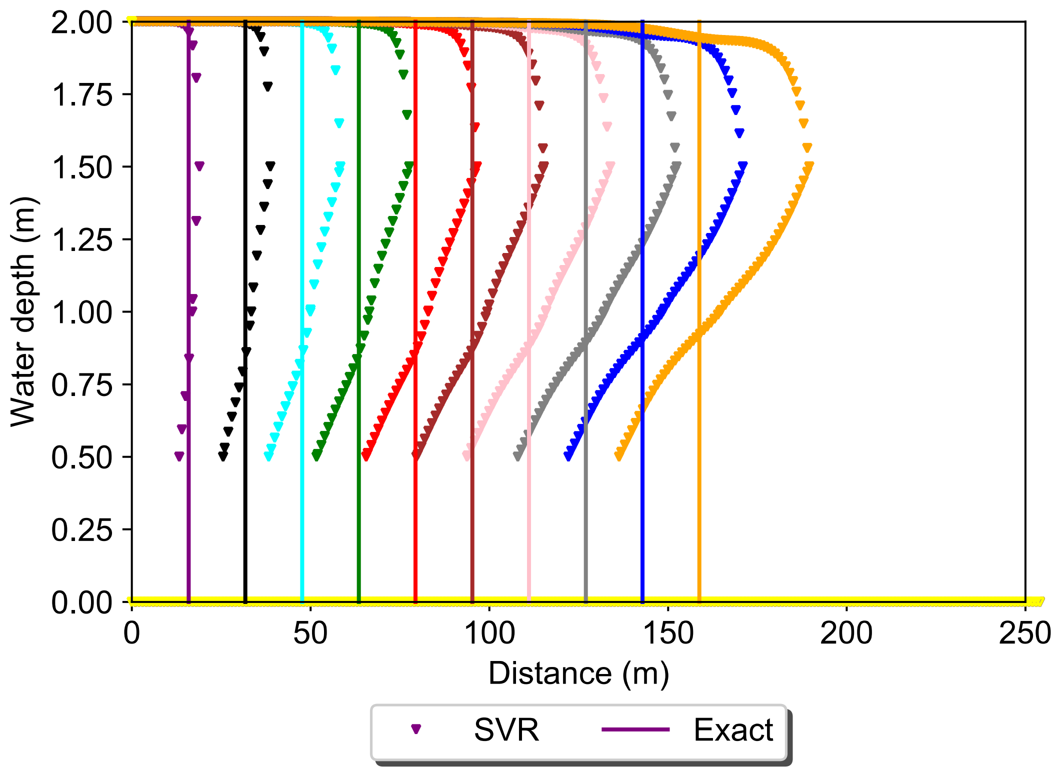

Figure 4. Zeros of the computed level-set function with SVR (SVR) for the dam-break over the dry bed in comparison with the exact positions of the shock front (Exact) for the case h^{up}=2 (m)

3.2. Optimal values of coefficient M

Coefficient M and γ are two parameters that are required to be specified when SVR is included in the level-set method. According to Tsai et al. (2002), γ was set to be $1/\Delta z$; however, M was stated to be a sufficiently large value. Such a case the coefficient M has not well defined in the paper. Thus, this study examines the optimal values of the coefficient M by implementing different cases of boundary conditions: h^{up} = 1, 2,…, 10. The subset and meshes where numerical experiments are performed are the same as in the subsection 3.1. For each case of h^{up}, different values of M in the range $(0,1)$ are specified to produce the corresponding zeros of the level-set function with SVR. For each value of M, the minimum distance between the zeros of the level-set function and the exact solution are calculated to obtain the optimal value of the coefficient M. The optimized results of the coefficient M for the different cases of h^{up} are summarized in Figure 5. The higher upstream water depth is, the greater the coefficient M is needed. The relation is approximated by the quadratic function of $M = 0.0063h^{up2} - 0.0134h^{up} + 0.0098$ with $R^2 = 0.99$.

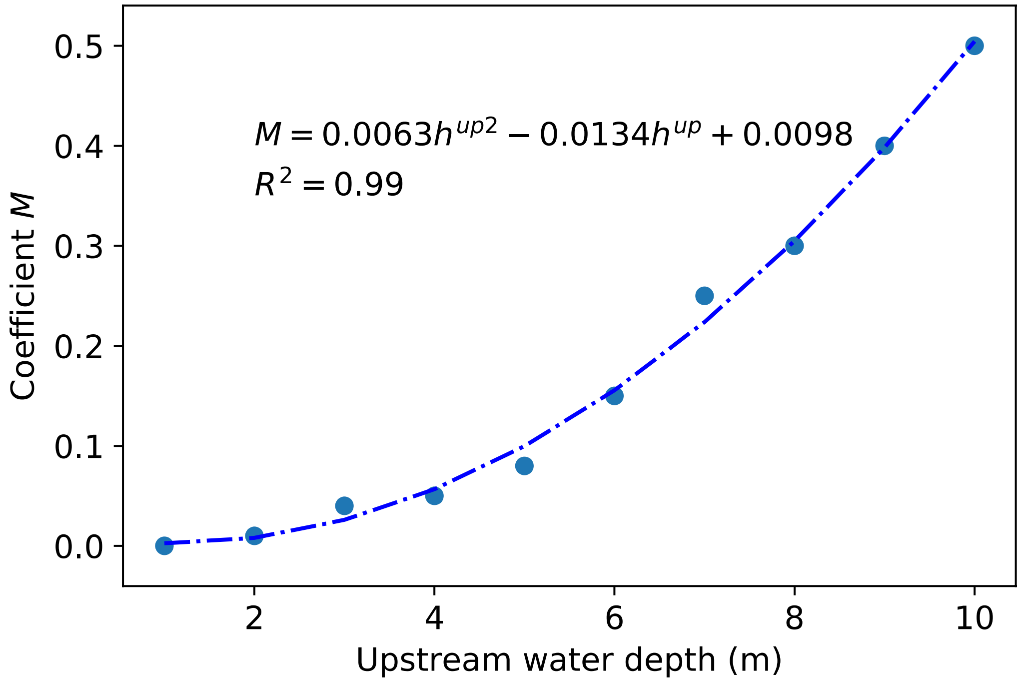

Figure 5. Optimal coefficient M for different cases of upstream water depth

To visualize the difference between the cases with and without the optimized SVR, the error in each case is determined by the maximum distance between the zeros of the level-set function and the exact positions of shock fronts, as illustrated in Figure 6. The blue dots and the red dots represent the errors in the cases without and with the optimized SVR, respectively. Figure 6 implicates that the effect of SVR is so significant that it bounds the error without depending on h^{up}.

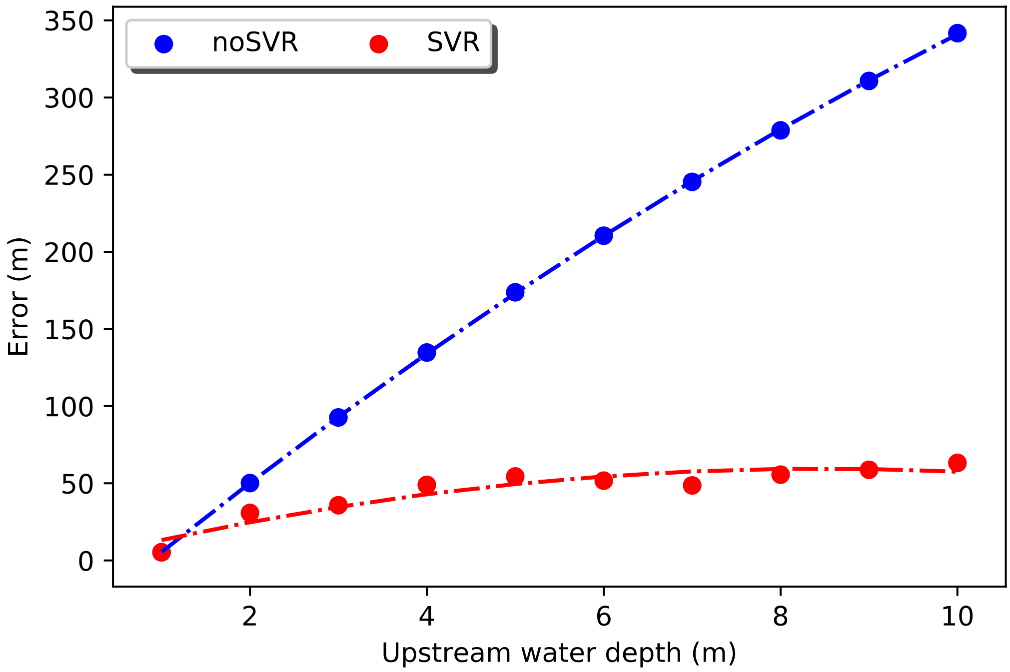

Figure 6. The errors in the zeros of the level-set functions with the optimized SVR (red dots) and without SVR (blue dots)

3.3. Practical demonstrative example

Chan Thnal Reservoir is located in Kampong Speu Province of Cambodia, having the maximum capacity of 3 million cubic meters. It collects rainwater from a catchment area of 268 km^2, supplying the stored water to the irrigation canal system from the main gate at the coordinates 11°34'13"N and 104°31'26"E (Perera et al., 2007). As one of the central low lands of Cambodia, the average annual rainfall in Kampong Speu Province is about 1400 mm (Thoeun, 2015). Tropical Monsoon climate usually shows the characters of a unimodal rainfall intensity curve with a specific long dry spell (Alam et al., 2018), and the catchment area and the command area of Chan Thnal Reservoir are not an exception. The reservoir is operated both in the rainy seasons (May to October) for 1000 ha of agricultural area and in the dry seasons (November to April) for 115 ha, often encountering the problem of abrupt water release to an initially dry bed of the irrigation canal system. Such operation may aim at increasing water efficiency in agriculture as well as enhancing the flood retention function of the reservoir (McMinn et al., 2010). The main canal of the system, having the total length of 7320 m, as shown in Figure 7, is modeled as an open channel with varied bed slopes, which are 0.0089, 0.0018, 0.0016, 0.0020, and 0.0005 for the five reaches divided by the points 610 m, 2070 m, 3670 m, and 5060 m distant from the reservoir. A constant Manning's roughness $n = 0.03$ ($\mathrm{m}^{-1/3}\mathrm{s}$) is applied to the model since the canal is earthen type with vegetation (Chow, 1959).

The ability of the level-set method with SVR applied to the kinematic wave equation is demonstrated in the practical problem of abrupt water release from Chan Thnal Reservoir into an initially dry bed of the main canal. The level-set equation is numerically solved for the dam-break problem for h^{up} = 2 (m) over the subset $(0,3600]\times(0,7320]\times(-1,3)$ of $(0,\infty)^3$, with meshes of $\Delta t = 0.1$ (s), $\Delta x = 10$ (m), and $\Delta z = 0.1$ (m), considering the varied bed slopes. As we have observed in the primitive test cases, strong overturning occurs as the fronts propagate downstream when M is small. If M is large, then artificial diffusion takes place so that the upper parts of the fronts tend to move slower than the lower parts. With an optimized value of $M = 0.003$, surface water flows are computed and delineated in Figure 8. The computed zeros of the level-set function are plotted every 100 (s) with different colors. It is clearly seen that the fronts propagate downstream with varied celerity according to the bed slope during the computational period of 3600 (s).

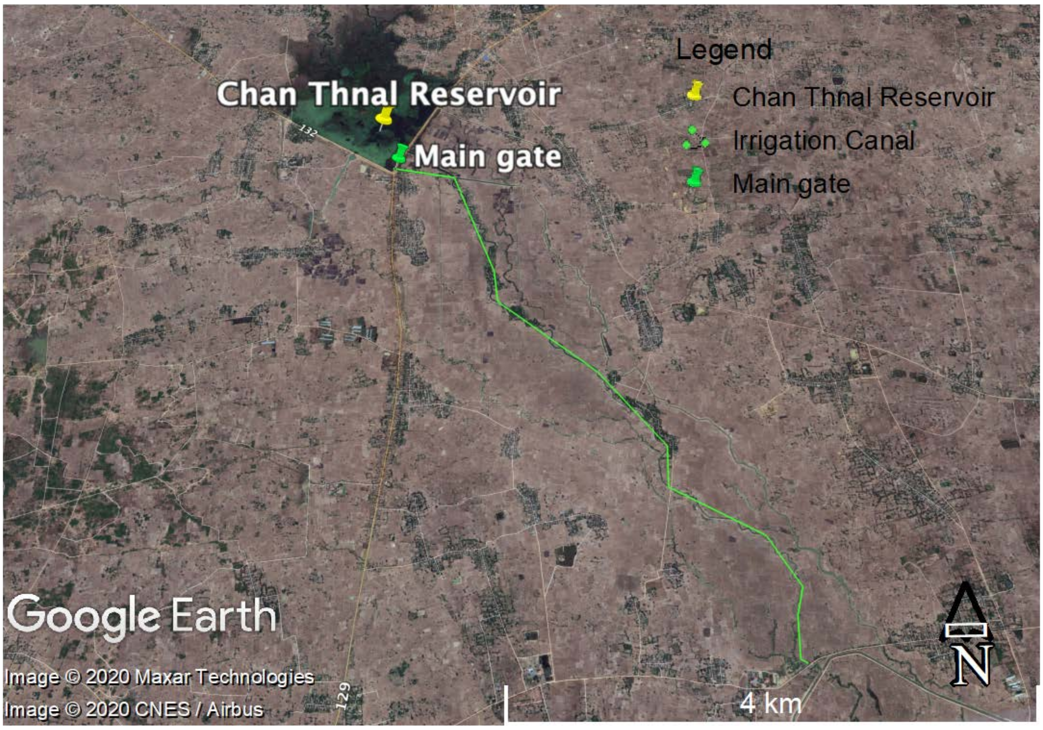

Figure 7. Satellite image of Chan Thnal Reservoir and its irrigation command area with the main canal (green line) (Google Earth image taken on January 12, 2014, accessed on April 08, 2020)

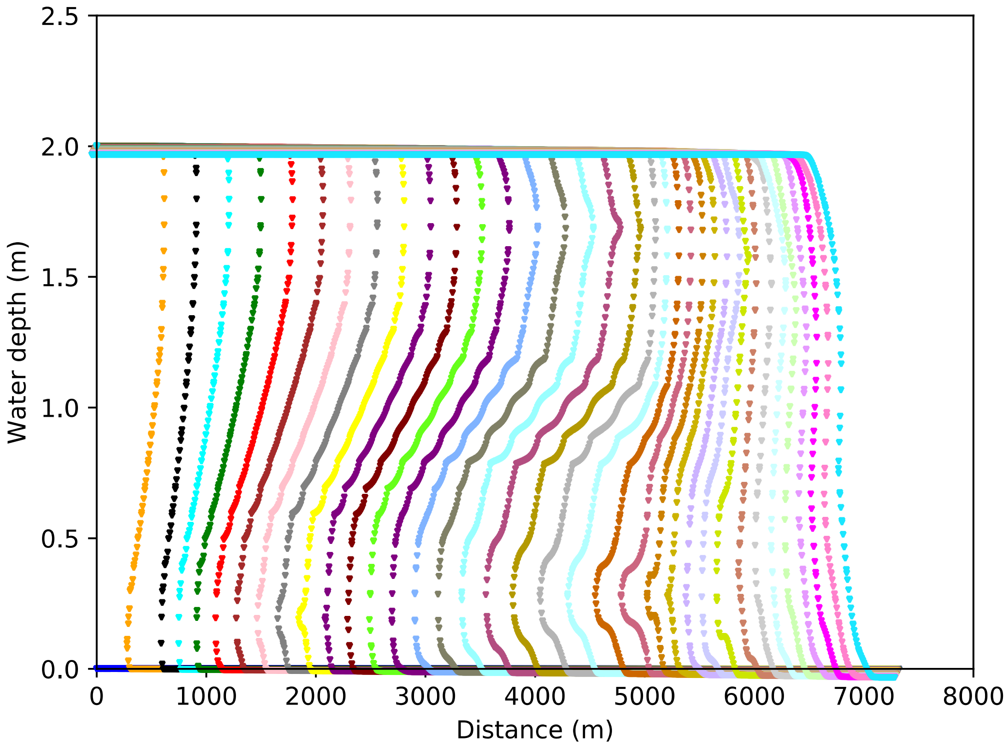

Figure 8. Zeros of the computed level-set function with SVR over the dry bed of Chan Thnal irrigation canal for $h^{up} = 2$ (m)

4. Discussion

The standpoint that regards the kinematic wave equation as a Hamilton-Jacobi equation has been almost untracked, but the model development and the numerical experiments conducted here have revealed its advantages and limitations.

The kinematic wave equation remains hyperbolic even if the water depth becomes zero and does not involve any well-posedness issue when dealing with the initially dry bed problems. While, the dam-break problems imply discontinuities in the water depths. The level-set method is powerful in relaxing requirements for functional regularities of unknowns in nonlinear partial differential equations of first order including Hamilton-Jacobi equations. Thanks to this property, the computational method successfully worked in the practical demonstrative example of Chan Thnal irrigation canal with the varied bed slopes as well. However, inhomogeneous slight overturning phenomena can be seen in the propagating shock front mostly in the midstream of the canal, implicating that the constant coefficient M was not the optimal for the varied bed slopes. The Froude numbers achieved after the arrival of the shock front were 1.1272, 0.5069, 0.4779, 0.5343, and 0.2672 for the five reaches divided by the points 610 m, 2070 m, 3670 m, and 5060 m distant from the reservoir, respectively, implying that there was a hydraulic jump in terms of the SWEs at the point 610 m from the reservoir. This incapability of detecting hydraulic jumps is one of the limitations of the level-set method for the kinematic wave equation. While, in contrast to most of the available software tools using either the 1-D SWEs with some artificial viscosity or the diffusion wave approximation, the proposed method turns out to be free from spurious diffusive deformation of water surfaces. As demonstrated in the practical example with $\Delta x = 10$ (m) and $\Delta z = 0.1$ (m), the use of relatively coarse mesh admits the efficiency of the method despite the computation is implemented in the 2-D space.

The critical point of applying the level-set method to the kinematic wave equation is the requirement of SVR. The applications of level-set method to the kinematic wave equation with and without SVR have been compared and verified with analytical solutions of dam-break problems. These results clearly indicated the importance of SVR in improving the numerical solutions. The formal deduction of (7) tentatively assuming the continuous differentiability of u without treatment in the viscosity sense causes the overturning phenomena. We conjecture that the level-set method with SVR for φ in (13) is consistent with the vanishing viscosity method for u in (7) with (12) and thus regulates the development of overturning. However, examining that conjecture with mathematical rigor is beyond the scope of this paper.

5. Conclusions

This paper discussed the applicability of the level-set method to the kinematic wave equation for reproduction of propagating discontinuous water surface caused by dam-break over an initially dry bed in the downstream side. Unlike the Eikonal equation, overturning is intrinsic to the kinematic wave equation whose Hamiltonian vanishes on the dry bed. The introduction of SVR was effective for relocating the zeros of the level-set function close to the correct positions of the shock front. However, that effect was sensitive to the coefficient M, which was optimized to produce a better numerical solution of the level-set function for each case of upstream water depth. The relation between upstream water depth and coefficient M is approximated by the quadratic function of $M = 0.0063h^{up2} - 0.0134h^{up} + 0.0098$ with $R^2 = 0.99$. The maximum distance between the zeros of the level-set function and the exact positions of shock fronts was used for determining the error. An important outcome of this study is to implicate that SVR can uniformly suppress the overturning phenomena which might linearly grow as the upstream water depth is increased. Finally, the practical application understandably shows that the level-set method with SVR applied to the kinematic wave equation is versatile for dry beds with varied bed slopes, although the coefficient *M* still needs adjustment depending on the varied bed slopes for better performance of the method.

The level-set method for the full SWEs over common digital elevation mesh shall be tackled in the follow-up study to develop methodologies for better understanding the practical hydrodynamic phenomena. Future works shall also deal with technical issues such as treatment of more irregular channel topography and roughness, as well as inclusion of lateral flows and then channel junctions.

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Nomenclature

Symbols

A	Cross-sectional area
$C(u_l, u_r)$	Local celerity at jump from u_l to u_r
F	Flux
g	Acceleration due to gravity
H	Hamiltonian
h	Water depth
h^{up}	Upstream water depth
M	Parameter controlling singular viscosity regularization
n	Manning's roughness
p	u_x as an argument of Hamiltonian
Q	Discharge
q	Lateral inflow discharge per unit length
R	Hydraulic radius
r	Source term
S_f	Friction slope
S_0	Bed slope
t	Time
u	Generic unknown variable
u_l	u at the left hand side of a jump
u_r	u at the right hand side of a jump

V	Cross-sectional averaged velocity
x	Space coordinate
$\mathbb{Z}$	The set of integers
z	Secondary coordinate for level-sets
β	Correction coefficient for velocity distribution
γ	Parameter to approximate singular diffusion
η	Water level
ξ	x at the previous time stage
$\varphi = \varphi(t,x,z)$	Level-set function as a function of t, x, and z
$\hat{\varphi}$	Interpolated φ
ψ	Singular viscosity term
Subscripts	
k,i,j	Integers indexing discretized t,x,z
x	Partial derivative with respect to x
z	Partial derivative with respect to z
Prefix	
Δ	Increment

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