Three Essays on Repeated Games and Games with Incomplete Information

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Digest

Game theory is designed to help people understand the situations in which individuals in the same segment make decisions that affect not only their gains and losses but also the other of them. This dissertation studies the strategic interaction between players acting in infinitely repeated games, auctions and cheap talk games.

In Chapter 2, we theoretically analyze players' payoffs in infinitely repeated games when coalitional deviations are allowed. Subgame perfect Nash equilibria which concern only individually deviations are generally exerted to analyze players' payoffs in infinitely repeated games. However, it is sometimes possible for players to arrange mutually beneficial deviations in the subgame perfect Nash equilibrium. Hence, in this chapter, we introduce a notion of degree-K subgame perfect Nash equilibria in infinitely repeated n-player games. In a degree-K subgame perfect Nash equilibrium, deviations by any coalition whose size is up to K are allowed and payoffs of players in a coalition are transferable in a coalition in infinitely repeated n-player games. If we only assume that players' actions are observable, a coalitional deviation with hidden deviators who play as in the equilibrium may not be detected by the other players. Hence we consider two models in which the hidden deviators can and cannot be detected, respectively.

In the first model, there is an observer who can detect any coalitional deviation and report it to all players. We show an extension of the standard folk theorem by extending the notion of minmax payoff to the coalitions. All feasible payoff vectors in which any feasible coalition's sum of payoffs is strictly larger than its minmax payoff arise as a degree-K subgame perfect Nash equilibrium if the players are sufficiently patient. In the second model where the hidden deviators cannot be distinguished, we show that the degree-K subgame perfect equilibria under patience achieve all feasible payoff vectors in which any feasible coalition's sum of payoffs is strictly larger than

its best response total payoff from an action profile which is called simultaneous punishment action profile. Finally, we characterize degree-n subgame perfect Nash equilibrium payoff vectors in the first model. The result is not a special case of the above folk theorem, because any degree-n subgame perfect Nash equilibrium payoff vector does not belong to the set of payoff vectors concerned by the Folk theorem.

Chapter 3 considers an auctioneer who has a non-monotonic utility function with a unique maximizer. The auctioneer is able to reject all bids over some amount by using rejection prices. The aim of this chapter is to study the expected utilities of the auctioneer and bidders who have private values in first-price and second-price sealed-bid auctions with rejection prices.

In a second-price sealed-bid auction with a rejection price, we show that the optimal bidding strategy for a bidder is bidding the lower one between his private value and the rejection price. Further, based on bidders' bidding strategies, we find that the optimal rejection price for the auctioneer is equal to that maximizer. In a first-price auction, we find that in the optimal bidding strategy there exists a jump point below which bidders bid the same as the standard model and above which bidders bid the rejection price. And we show that the optimal rejection price for the auctioneer is lower than the maximizer. Moreover, in each auction, we characterize a necessary and sufficient condition that by using the optimal rejection price not only the auctioneer but also bidders can be better off, compared to a standard auction. Finally, we find that the auctioneer strictly prefers a first-price sealed-bid auction if he is risk-averse when his revenue is lower than the maximizer or if the distribution of revenues which are lower than the maximizer in a standard first-price sealed-bid auction is first-order stochastic dominant over the one in a standard second-price sealed-bid auction. And we also find some cases that the auctioneer strictly prefers a second-price sealed-bid auction with the optimal rejection price.

Chapter 4 develops Crawford and Sobel (1982)'s cheap talk model by considering that a sender and a receiver are unaware of different parts of the type space. In our model, we assume that when the sender observes a type of which he was unaware, he will be aware of this type and becomes ambiguity averse. We first define the equilibrium concept in this model by extending Bayesian Nash equilibria. In equilibrium, the sender and the receiver play a Bayesian Nash equilibrium with the opponent who is believed by them based on their current awareness. Without unawareness, we show the existence of equilibria and characterize them by extending Crawford and Sobel (1982)'s result to a model in which the type space is a union of finite disjoint intervals. Finally, we show the existence of equilibria and characterize them in a model with unawareness. We find that in the equilibrium we characterized, the sender and the receiver essentially play a Bayesian Nash equilibrium when the sender observes a type of which he was aware. And when the sender observes a type of which he was unaware, the sender reveals his type and the receiver maximizes his utility.

References

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica: Journal of the Econometric Society*, pages 1431–1451.