Studies on Discrete-Valued Vector Reconstruction from Underdetermined Linear Measurements

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Abstract

Reconstruction of an unknown discrete-valued vector from its linear measurements is a common problem in communication systems. When the number of measurements is greater than or equal to the dimension of the unknown vector, the low complexity linear methods, such as minimum mean-square-error (MMSE) method, might achieve satisfactory reconstruction performance. In the underdetermined case with an insufficient number of measurements, however, their performance is severely degraded. On the other hand, although the maximum likelihood (ML) approach with the exhaustive search can achieve an excellent performance, it requires huge computational complexity in large-scale problems. This thesis proposes an efficient algorithm for the discrete-valued vector reconstruction and provides asymptotic performance analyses for some reconstruction methods.

Chapter 1 describes the discrete-valued vector reconstruction and its application in communication systems. Moreover, the conventional methods are briefly reviewed. Finally, the outline of this thesis is explained.

In Chapter 2, we focus on the reconstruction of the binary vector as the simplest example of the discrete-valued vector reconstruction. We extend the conventional sum of absolute values (SOAV) optimization to the weighted SOAV (W-SOAV) optimization so that we can use the prior information of the unknown vector. We then propose an iterative approach named iterative weighted SOAV (IW-SOAV), where we iterate the W-SOAV optimization and the update of the weight parameters in the objective function. The W-SOAV optimization can be efficiently solved with proximal splitting methods for convex optimization. Simulation results show that the reconstruction performance of the proposed IW-SOAV is better than several conventional methods in massive overloaded multiple-input multiple-output (MIMO) signal detection and the decoding of non-orthogonal space-time block codes.

In Chapter 3, we propose an algorithm for the reconstruction of a complex discrete-valued vector. The proposed method can be considered as an extension of the conventional SOAV optimization in the real-valued domain. The proposed approach in the complex-valued domain can utilize the dependency between the real part and the imaginary part of the unknown vector. It is shown that an optimization algorithm based on alternating direction method of multipliers (ADMM) can provide a sequence converging

to the solution of the optimization problem. We have shown via computer simulations that the proposed method can achieve good performance in MIMO signal detection and channel equalization.

Chapter 4 proposes a possibly nonconvex optimization problem for the discrete-valued vector reconstruction. The proposed sum of sparse regularizers (SSR) optimization problem can be regarded as a generalization of the convex SOAV optimization. For the proposed SSR optimization, two optimization algorithms based on ADMM and primal-dual splitting are proposed. Simulation results show that the proposed algorithms using nonconvex optimization can achieve better reconstruction performance than several conventional approaches using convex optimization.

In Chapter 5, we analyze the asymptotic performance of the SOAV optimization. We firstly propose the Box-SOAV optimization by adding a box constraint to the conventional SOAV optimization. By using convex Gaussian min-max theorem (CGMT), we evaluate the asymptotic performance of the estimate obtained by the Box-SOAV optimization. We also propose an approach to optimize the parameters of the Box-SOAV optimization on the basis of the theoretical result.

In Chapter 6, we analyze the performance of the SOAV optimization from a different perspective. We firstly propose a message passing-based algorithm using the idea of the SOAV optimization. Although the proposed method requires some assumptions on the measurement matrix, it can achieve good performance with low computational complexity. Moreover, we evaluate the asymptotic performance of the proposed algorithm on the basis of the state evolution. We also obtain the required measurement ratio for the perfect reconstruction in the noise-free case.

In Chapter 7, we present the conclusion of the thesis.