Determination of the Second Derivatives of the Gravitational Potential on the Jaluit Atoll.

By

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CONTENTS.

I. Introduction.

The Jaluit Atoll is situated at the southern end of the Ralick Chain of the Marshal Islands. The coral reef of this atoll has a mean width of about 450 meters and runs, like a long belt, along the sides of an irregular rhombic lagoon of mean depth of 40 meters. The outside ocean is about 4000 meters deep at a distance of 10000 meters from the atoll. The top of the reef is just at the level of low tide, except that the middle part is heaped up by fragments of coral reef and coral sand. The height of the land thus formed is from 1 to 2 meters above high water. The land is generally 150 meters wide and forms a chain of discontinuous islets.

Among the scientific problems connected with coral atolls, the gravitational condition at such an isolated island of very simple construction is one of the most interesting, especially when we consider the state of mass distribution under the atoll. For the study of this and allied problems, the late H. Kaneko and the author left Yokosuka on the 20th of January, 1915, for the Jaluit Atoll, arriving there on the 9th of February. Leaving the atoll on the 12th of March, they returned to Yokosuka on the 17th of April. During their sajourn of about one month they determined the values of the second derivatives of the gravitational potential at various parts of the atoll by Eötvös' gravity-variometer; and the present paper gives the results of observations and studies on the gravitational condition and the depth of the coral reef of this atoll.

On board the transports to and from Jaluit, Lieutenants Kawara and Ogata kindly took precaution for the careful transportation of our delicate instruments. In the Jaluit Atoll we were indebted to the hospitality of Commander Tachikawa and his associates, to whom hearty thanks are due for the unexpected measure of success attending our efforts.

II. The Instrument.

(*a*). *Principle of the Gravity-variometer.*

The instrument used was the Eötvös' gravity-variometer of duplicate form. Its essential part is a kind of torsion balance with its two weights at different heights. By means of telescope and scale the rotation of the arm can be observed. Equating the rotationary moment due to the gravitational field with the torsional moment of the torsion wire AO, we obtain the following equation[1] of equilibrium of the arm:

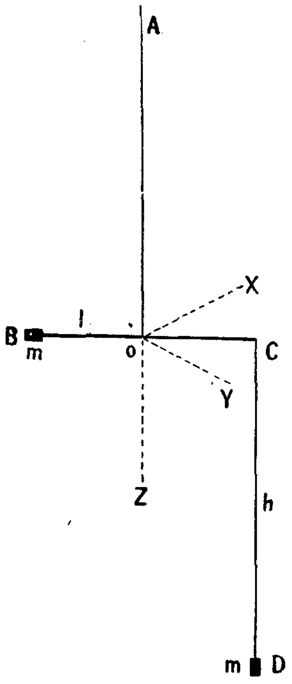
$$(1)\quad \tau\frac{n-n_0}{2D} = K\frac{1}{2}\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)\sin 2\alpha + K\frac{\partial^2 U}{\partial x\partial y}\cos 2\alpha - mhl\frac{\partial^2 U}{\partial x\partial z}\sin\alpha + mhl\frac{\partial^2 U}{\partial y\partial z}\cos\alpha,$$

where τ is the torsional coefficient, n the scale reading, n_0 that in

[1] R. v. Eötvös, Untersuchungen über Gravitation und Erdmagnetismus, Ann. d. Phys. 355, **59**, 1896.

torsionless state, D the scale distance, K the moment of inertia of the moving part about the vertical axis through O, U the gravitational potential and α the angle between the x-axis and the arm. This equation contains five unknown factors so that for a complete solution we need to observe n at five different azimuths of the arm.

Fig. 1.



For a second balance, with arm in reverse direction to that of the first, we have the following equation:—

$$(2)\quad \tau' \frac{n'-n_0'}{2D'} = K' \frac{1}{2}\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right) \sin 2\alpha + K' \frac{\partial^2 U}{\partial x \partial y} \cos 2\alpha + m'h'l' \frac{\partial^2 U}{\partial x \partial z} \sin \alpha - m'h'l' \frac{\partial^2 U}{\partial y \partial z} \cos \alpha,$$

where α is the azimuth of the first balance. Combining this equation with equation (1), we see that with a gravity-variometer of double system[1] we need, for a complete solution, to observe the equilibrium positions of the arms at three different azimuths.

(*b*) *The Instrumental Constants.*

The instrumental constants are contained in the above equations in the combinations

$$(3)\quad \begin{cases} a = 2D \dfrac{K}{\tau}, & b = 2D \dfrac{mhl}{\tau}, \\ a' = 2D' \dfrac{K'}{\tau'}, & b' = 2D' \dfrac{m'h'l'}{\tau'}. \end{cases}$$

Among the constants, the following numerical values were given by Dr. Desider Pekar at the instrument-maker's in Budapest.

1 R. v. Eötvös, Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hülfe der Drehwage. Verh. 15 allg. Conf. d. Intn. Erdmessung, 337, 1906.

Instrument	No. I.	No. II.
D	1252	1252 s.d.
K	24825	24767 c.g.s
m	29.848	29.737 gr.
h	68.44	68.44 cm.
l	20.0	20.0 cm.
temp. coeff.	+0.074	+0.075 s. d.

The abbreviation s.d. stands for a scale division which is equal to 0.5 millimeter.

Beside these, we must have the values of the torsional coefficient τ which may be obtained by repeating the Cavendish's method. Let a lead sphere of mase M be brought to a distance ρ from the axis of the hanging cylindrical mass, whose length is λ and radius r. Then the mutual attraction will be

$$f = GM\frac{\partial}{\partial\rho}\int_0^r\int_0^{2\pi}\int_{-c}^{\lambda-c}\frac{\sigma r dr d\theta d\lambda}{\{(\rho^2+r^2)+r^2-2r\sqrt{\rho^2+r^2}\cos\theta\}^{\frac{1}{2}}}$$

where c is the height of the center of the sphere from the base of the cylinder. Neglecting small quantities and considering the case when the center of the sphere is at the same level as the center of the cylinder, we obtain

$$f = G\frac{Mm}{\rho^2}\frac{1+\frac{3}{8}\left(\frac{r}{\rho}\right)^2}{\left\{1+\frac{1}{4}\left(\frac{\lambda}{\rho}\right)^2\right\}^{\frac{1}{2}}}.$$

If $n-n_0$ be the change of equilibrium-position of the arm due to this force, we have

$$\tau = G\frac{Mm}{\rho^2}\frac{2D}{n-n_0}l\frac{1+\frac{3}{8}\left(\frac{r}{\rho}\right)^2}{\left\{1+\frac{1}{4}\left(\frac{\lambda}{\rho}\right)^2\right\}^{\frac{1}{2}}}. \tag{4}$$

A lead sphere of 11.609 kgr weight was repeatedly brought in front of, and behind, the hanging mass. The center of the sphere was always 10.50 cm. from the axis of the cylindrical tube in which the weight hung. After several trials, the complete determination of τ was effected from June 29 till July 11, after the return from the Jaluit Atoll, and the observed data are given in Table 1.

Now in observations with this instrument, it was always noticed that the position of equilibrium gradually changed. In determination of τ, special observations for this elastic aftereffect[1] were made. The instrument was set in a definite azimuth and, after the hanging system was set free, hourly observations were made, the instrument being turned by 360°, but not arrested after each observation. The results are given in Pl. I.

In Table 1, T_a is the atmospheric temperature, T and T′ the temperature of the balances No. I and No. II, n and n' the scale-readings corrected for the temperature and elastic after-effect.

Table 1. Determination of τ

Condition	Time	T_a	T	n	$n-n_0$	T′	n'	$n'-n'_0$
Series I for instrument I. June 29–30, 1915.								
	h m	°	°	°		°	°	
set free	7 20 p.	24·2	24·1	(T=23·0)		24·1	(T=23·0)	
stand only	8 20	24·0	24·0	165·7		24·0	446·9	
sphere in front	9 20	24·0	24·0	186·2	+20·4	24·0	446·8	−0·4
„ behind	10 20	23·9	33·9	144·4	−21·4	24·0	447·3	+0·1
„ in front	11 20	23·8	23·8	185·9	+20·1	23·9	446·9	−0·3
„ behind	0 20 a.	23·7	23·7	144·3	−21·5	23·8	447·3	+0·1
„ in front	1 20	23·6	23·6	186·1	+20·3	23·6	447·2	0·0
„ behind	2 20	53·4	23·4	144·8	−21·0	23·4	447·4	+0·2
„ in front	3 20	23·2	23·2	186·2	+20·4	23·3	447·2	0·0
„ behind	4 30	23·2	23·2	144·4	−21·4	23·2	447·4	+0·2
„ in front	5 30	23·1	23·1	186·2	+20·4	23·1	447·0	−0·2
stand only	6 35	23·0	23·0	165·9		23·0	447·4	
Series II for instrument II. July 1–2, 1815.								
set free	7 0 p.	24·0	24·5	(T=23·6)		24·6	(T=2·40)	
stand only	9 45	23·3	23·4	166·9		23·5	448·1	
sphere in front	10 45	23·1	23·2	166·9	+0·2	23·2	427·7	−20·2
„ behind	11 45	23·0	23·1	166·3	−0·4	23·2	468·5	+20·6
„ in front	0 45 a.	23·0	22.9	166·9	+0·2	23·0	427·3	−20·6
„ behind	1 45	22·8	22·9	166·2	−0·5	22·9	468·6	+20·7
„ in front	2 45	22·7	22·8	166·8	+0·1	22·9	427·5	−20·4
„ behind	3 45	22·5	22·7	166·2	−0·5	22·8	468·7	+20·8
„ in front	4 45	22·4	22·5	166·7	00	22·6	427·5	−20·4
stand only	5 45	22·3	22·4	166·4		22·5	447·6	

1 R. v. Eötvös, Bericht über Geodetische Arbeiten in Ungarn, besonders über Beobachtungen mit der Drehwage. Verh. 16 allg. Conf. d. Intn. Erdmessung, 319, 1909.

Condition	Time	T_a	T	n	$n-n_0$	T′	n'	$n'-n_0'$
			Series III for instrument I. July 7–8, 1915.					
set free	10 h 10 h p.	22·5	22·6°	(T=21·0°)		22·6°	(T=21·0°)	
stand only	11 10	22·2	22·3	166·0		22·3	446·0	
sphere in front	0 10 a.	22·0	22·1	186·5	+20·4	22·2	445·9	−0·6
,, behind	1 10	21·8	22·0	145·4	−20·7	22·0	446·6	+0·1
,, in front	2 10	21·7	21·8	186·8	+25·7	21·9	446·4	−0·1
,, behind	3 20	21·5	21·5	145·1	−21·0	21·8	446·9	−0·4
,, in front	4 15	21·4	21·4	186·4	+20·3	21·6	446·4	−0·1
,, behind	5 10	23·1	21·4	145·1	−21·0	21·4	447·2	+0·7
,, in front	6 5	21·3	21·4	186·5	+20·4	21·4	446·5	0·0
stand only	7 0	21·4	21·4	166·2		21·4	446·9	
			Series IV for instrument II. July 10–11, 1915.					
set free	7 0 p.	24·9	25·0	(T=24·0)		25·0	(T=24·0)	
stand only	10 0	24·2	24·2	166·7		24·2	447·6	
sphere in front	11 0	24·2	24.2	166·8	+0·3	24·2	427·6	−19·8
,, behind	0 0 a.	24·1	24·1	166·0	−0·5	24·0	467.8	+20·4
,, in front	1 0	24·0	24·0	166·5	0·0	24·0	427·2	−20·2
,, behind	2 0	24·0	24·0	166·1	−0·4	24·0	468·2	+20·8
,, in front	3 0	24·0	24·0	166·4	−0·1	24·0	427·5	−19·9
,, behind	4 0	24·0	24·0	165·9	+0·6	24·0	467·9	+20·5
,, in front	5 0	24·0	24·0	166·3	+0·2	24·0	42·76	−19·8
stand only	6 0	24·0	24·0	166·2		24·0	447·2	

The mean deviations of the equilibrium-positions of the balances produced by the lead sphere are as follows:—

	$n-n_0$		$n'-n_0'$	
Sphere	in front	behind	in front	behind
Series I	+20.32	−21·33	−0·18	+0·15
,, II	0·00	−0·50	−19·93	+20·57
,, III	+20.45	−20·90	−0·20	+0·40
,, IV	+0.13	−0·47	−20·40	+20·70
Mean	+20·39	−21·12	−20·17	+20·64

The center of the lead sphere was always 10.50 cm. from the axis of the cylindrical tube of the instrument. The actual distance of the sphere from that of the hanging weight differed from this in two ways. First, the center of the weight, free from the influence of the sphere, might not coincide with the axis of the tube, and secondly, the weight changed its position by the attraction of the lead sphere. If δ be the initial deviation, the actual distance of the sphere from the weight will be

$$\rho = 10{\cdot}50 \pm \delta - \frac{n-n_0}{2D}\, l,$$

the double sign being introduced for the cases when the sphere is in front of, or behind, the hanging weight. In calculating τ we may use the mean distance and the mean deviation without making any appreciable error, thus

$$\rho = 10{\cdot}50 - \frac{n-n_0}{2D}\, l$$

Using this distance and the mean deviations

$$n - n_0 = 20{\cdot}76 \text{ s. d.},$$
$$n' - n'_0 = 20{\cdot}41 \text{ s. d.},$$

we obtain the following values of the torsional coefficient:—

$$\tau = 0{\cdot}5139 \text{ c. g. s.},$$
$$\tau' = 0{\cdot}5215 \text{ c. g. s.}$$

From these values of the torsional coefficient and the values of the other instrumental constants we obtain

$$a = 1{\cdot}2108 \times 10^8, \quad b = 1{\cdot}9920 \times 10^8,$$
$$a' = 1{\cdot}1893 \times 10^8, \quad b' = 1{\cdot}9545 \times 10^8.$$

(*c*) *Formulae for Calculation.*

We now proceed to derive the numerical formulae for calculation of the second derivatives of the gravitational potential. The equations of equilibrium have been given in (1) and (2). Since the increasing direction of the scales in the instrument are in the negative direction of the angular measurements in the *xy*-plane, we must write $n_0 - n$ and $n_0' - n'$ in the left-hand sides of these equations. Giving α the values 0°, 120°, 240° successively, we obtain six linear equations, from which we get

$$(5)\qquad \begin{cases} n_0 = \dfrac{1}{3}\left(n_1+n_2+n_3\right) \\ n_0' = \dfrac{1}{3}\left(n_1'+n_2'+n_3'\right) \end{cases}$$

where the suffices show the scale readings corresponding to the three azimuths respectively. We obtain, also, the following formulae :—
(i) from the first and the second settings,

$$(6)\qquad \begin{cases} \dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x^2} = \dfrac{2}{\sqrt{3}}\dfrac{1}{\left(a+a'\dfrac{b}{b'}\right)}\left\{\left[(n_1-n_0)+2(n_2-n_0)\right]\right. \\ \qquad\qquad\qquad \left.+\dfrac{b}{b'}\left[(n'_1-n'_0)+2(n'_2-n'_0)\right]\right\}, \\ \dfrac{\partial^2 U}{\partial x\partial y} = -\dfrac{1}{\left(a+a'\dfrac{b}{b'}\right)}\left\{(n_1-n_0)+\dfrac{b}{b'}(n_1'-n_0')\right\}, \\ \dfrac{\partial^2 U}{\partial x\partial z} = \dfrac{1}{\sqrt{3}}\dfrac{1}{\left(b+b'\dfrac{a}{a'}\right)}\left\{\left[(n_1-n_0)+2(n_2-n_0)\right]\right. \\ \qquad\qquad\qquad \left.-\dfrac{a}{a'}\left[(n'_1-n'_0)+2(n'_2-n'_0)\right]\right\}, \\ \dfrac{\partial^2 U}{\partial y\partial z} = -\dfrac{1}{\left(b+b'\dfrac{a}{a'}\right)}\left\{(n_1-n_0)-\dfrac{a}{a'}(n'_1-n'_0)\right\}, \end{cases}$$

(ii) from the second and the third settings,

$$(6)\qquad \begin{cases} \dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x^2} = \dfrac{2}{\sqrt{3}}\dfrac{1}{\left(a+a'\dfrac{b}{b'}\right)}\left\{\left[(n_2-n_0)-(n_3-n_0)\right]\right. \\ \qquad\qquad\qquad \left.+\dfrac{b}{b'}\left[(n'_2-n'_0)-(n'_3-n'_0)\right]\right\}, \\ \dfrac{\partial^2 U}{\partial x\partial y} = \dfrac{1}{\left(a+a'\dfrac{b'}{b}\right)}\left\{\left[(n_2-n_0)+(n_3-n_0)\right]\right. \\ \qquad\qquad\qquad \left.+\dfrac{b}{b'}\left[(n'_2-n'_0)+(n'_3-n'_0)\right]\right\}, \end{cases}$$

$$\frac{\partial^2 U}{\partial x \partial z} = \frac{1}{\sqrt{3}} \frac{1}{\left(b + b' \frac{a}{a'}\right)} \left\{ \left[(n_2 - n_0) - (n_3 - n_0) \right] - \frac{a}{a'} \left[(n'_2 - n'_0) - (n'_3 - n'_0) \right] \right\},$$

$$\frac{\partial^2 U}{\partial y \partial z} = \frac{1}{\left(b + b' \frac{a}{a'}\right)} \left\{ \left[(n_2 - n_0) + (n_3 - n_0) \right] - \frac{a}{a'} \left[(n'_2 - n'_0) + (n'_3 - n'_0) \right] \right\},$$

(iii) from the third and the first settings,

(8)

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = -\frac{2}{\sqrt{3}} \frac{1}{\left(a + a' \frac{b}{b'}\right)} \left\{ \left[2(n_3 - n_0) + (n_1 - n_0) \right] + \frac{b}{b'} \left[2(n'_3 - n'_0) + (n'_1 - n'_0) \right] \right\},$$

$$\frac{\partial^2 U}{\partial x \partial y} = -\frac{1}{\left(a + a' \frac{b}{b'}\right)} \left\{ (n_1 - n_0) + \frac{b}{b'} (n'_1 - n'_0) \right\},$$

$$\frac{\partial^2 U}{\partial x \partial z} = -\frac{1}{\sqrt{3}} \frac{1}{\left(b + b' \frac{a}{a'}\right)} \left\{ \left[2(n_3 - n_0) + (n_1 - n_0) \right] - \frac{a}{a'} \left[2(n_3' - n'_0) + (n'_1 - n'_0) \right] \right\},$$

$$\frac{\partial^2 U}{\partial y \partial z} = -\frac{1}{\left(b + b' \frac{a}{a'}\right)} \left\{ (n_1 - n_0) - \frac{a}{a'} (n'_1 - n'_0) \right\}.$$

Substituting the numerical values of the constants, we finally obtain the following numerical formulae :—

(i) from the first and the second settings

$$
(6')\left\{
\begin{aligned}
10^9\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right) &= 4{\cdot}770\Big\{\Big[(n_1-n_0)+2(n_2-n_0)\Big] \\
&\qquad +(1+0{\cdot}0192)\Big[(n'_1-n'_0)+2(n'_2-n'_0)\Big]\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial x\partial y} &= -4{\cdot}126\,\Big\{(n_1-n_0) \\
&\qquad +(1+0{\cdot}0192)(n'_1-n'_0)\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial x\partial z} &= 1{\cdot}450\Big\{\Big[(n_1-n_0)+2(n_2-n_0)\Big] \\
&\qquad -(1+0{\cdot}0181)\Big[(n'_1-n'_0)+2(n'_2-n'_0)\Big]\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial x\partial z} &= -2{\cdot}510\Big\{(n_1-n_0) \\
&\qquad -(1+0{\cdot}0181)(n'_1-n'_0)\Big\},
\end{aligned}
\right.
$$

(ii) from the second and the third settings,

$$
(7)\left\{
\begin{aligned}
10^9\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right) &= 4{\cdot}770\Big\{\Big[(n_2-n_0)-(n_3-n_0)\Big] \\
&\qquad +(1+0{\cdot}0192)\Big[(n'_2-n'_0)-(n'_3-n'_0)\Big]\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial x\partial y} &= 4{\cdot}126\Big\{\Big[(n_2-n_0)+(n_3-n_0)\Big] \\
&\qquad +(1+0{\cdot}0192)\Big[(n'_2-n'_0)+(n'_3-n'_0)\Big]\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial x\partial z} &= 1{\cdot}450\Big\{\Big[(n_2-n_0)-(n_3-n_0)\Big] \\
&\qquad -(1+0{\cdot}0181)\Big[(n'_2-n'_0)-(n'_3-n'_0)\Big]\Big\}, \\
10^9\,\frac{\partial^2 U}{\partial y\partial z} &= 2{\cdot}510\Big\{\Big[(n_2-n_0)+(n_3-n_0)\Big] \\
&\qquad -(1+0{\cdot}0181)\Big[(n_2'-n_0')+(n'_3-n'_0)\Big]\Big\},
\end{aligned}
\right.
$$

(iii) from the third and the first settings,

$$
\left\{
\begin{aligned}
10^9\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right) &= -4{\cdot}770\Big\{\Big[2(n_3-n_0)+(n_1-n_0)\Big] \\
&\qquad +(1+0{\cdot}0192)\Big[2(n'_3-n'_0)+(n'_1-n'_0)\Big]\Big\},
\end{aligned}
\right.
$$

$$(8')\left\{\begin{aligned}
10^9\frac{\partial^2 U}{\partial x\partial y} &= -4\cdot126\Big\{(n_1-n_0) \\
&\quad +(1+0\cdot0192)(n'_1-n'_0)\Big\}, \\
10^9\frac{\partial^2 U}{\partial x\partial z} &= -1\cdot450\Big\{\Big[2(n_3-n_0)+(n_1-n_0)\Big] \\
&\quad -(1+0\cdot0181)\Big]2(n_3'-n'_0)+n'_3-n'_0)\Big]\Big\}, \\
10^9\frac{\partial^2 U}{\partial y\partial z} &= -2\cdot510\Big\{(n_1-n_0) \\
&\quad -(1+0\cdot0181)(n'_1-n'_0)\Big\}.
\end{aligned}\right.$$

(*d*) *Corrections for the Error of Azimuth.*

The azimuth is measured by means of the horizontal circle, whose index is in rigid connection with the brass case in which the horizontal arms hang. But the arms change their position, relative to the side of the case, in different settings. The deviation goes sometimes so far as 20 scale-divisions, which corresponds to an angle of 27′. Hence we must add, if sufficiently large, some correctional terms arising from this small error of azimuth.

Differentiating equations (1) and (2) with respect to α[1], we get

$$\frac{d(n_0-n)}{d\alpha} = a\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)\cos 2\alpha - 2a\frac{\partial^2 U}{\partial x\partial y}\sin 2\alpha$$
$$- b\frac{\partial^2 U}{\partial x\partial z}\cos\alpha - b\frac{\partial^2 U}{\partial y\partial z}\sin\alpha,$$

$$\frac{d(n'_0-n')}{d\alpha'} = a\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)\cos 2\alpha - 2a\frac{\partial^2 U}{\partial x\partial y}\sin 2\alpha$$
$$+ b\frac{\partial^2 U}{\partial x\partial z}\cos\alpha + b\sin\alpha.$$

For $d\alpha$ and $d\alpha'$ we may use

$$\varDelta\alpha = \frac{n_0-n}{2D},$$

$$\varDelta\alpha' = \frac{n_0'-n'}{2D}.$$

Using the values of the correctional terms $d(n_0-n)$ and $d(n'_0-n')$, due

1 R. v. Eötvös, Bestimmung der Gradienten der Schwerkraft und ihrer Niveaufläche mit Hülfe der Drehwage. Verh. 15 allg, Conf. der Intn. Erdmessung, 337, 1906

to the errors of the azimuths, we can derive from equations (1) and (2), the following corrections :—

$$\Delta\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)=\frac{1}{D}\left[-\frac{1}{4}a\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)^2+a\left(\frac{\partial^2U}{\partial x\partial y}\right)^2\right.$$
$$\left.+\frac{1}{2}\frac{bb'}{a'}\left(\frac{\partial^2U}{\partial x\partial z}\right)^2-\frac{1}{2}\frac{bb'}{a'}\left(\frac{\partial^2U}{\partial y\partial z}\right)^2\right],$$

$$\Delta\frac{\partial^2U}{\partial x\partial y}=\frac{1}{D}\left[-\frac{1}{4}a\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial x\partial y}+\frac{3}{4}a\frac{\partial^2U}{\partial x\partial y}\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\right.$$
$$\left.+\frac{1}{4}\frac{bb'}{a'}\frac{\partial^2U}{\partial x\partial z}\frac{\partial^2U}{\partial y\partial z}-\frac{3}{4}\frac{bb'}{a'}\frac{\partial^2U}{\partial y\partial z}\frac{\partial^2U}{\partial x\partial z}\right],$$

$$\Delta\frac{\partial^2U}{\partial x\partial z}=\frac{1}{D}\left[-\frac{1}{4}\frac{aa'}{b'}\left(\frac{\partial^2U}{\partial y^2}-\frac{v^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial x\partial z}+\frac{1}{2}\frac{aa'}{b'}\frac{\partial^2U}{\partial x\partial y}\frac{\partial^2U}{\partial y\partial z}\right.$$
$$\left.+\frac{1}{8}b\frac{\partial^2U}{\partial x\partial z}\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)-\frac{1}{4}b\frac{\partial^2U}{\partial y\partial z}\frac{\partial^2U}{\partial x\partial y}\right],$$

$$\Delta\frac{\partial^2U}{\partial y\partial z}=\frac{1}{D}\left[-\frac{1}{4}\frac{aa'}{b'}\left(\frac{\partial_{*}U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial y\partial z}+\frac{3}{2}\frac{aa'}{b'}\frac{\partial^2U}{\partial x\partial y}\frac{\partial^2U}{\partial x\partial z}\right.$$
$$\left.+\frac{1}{4}b\frac{\partial^2U}{\partial x\partial z}\frac{\partial^2U}{\partial x\partial y}-\frac{3}{8}b\frac{\partial^2U}{\partial y\partial z}\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\right].$$

Substituting the numerical values of the instrumental constants, we obtain

$$(9')\left\{\begin{aligned}
19^9\Delta\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)&=-0{\cdot}000024\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)+0{\cdot}000097\left(\frac{\partial^4U}{\partial x\partial y}\right)^2\\
&\quad+0{\cdot}000154\left(\frac{\partial^2U}{\partial y\partial z}\right)^2-0{\cdot}000154\left(\frac{\partial^2U}{\partial y\partial z}\right)^2.\\
10^9\Delta\frac{\partial^2U}{\partial x\partial y}&=+0{\cdot}000048\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial x\partial y}-0{\cdot}000154\frac{\partial^2U}{\partial x\partial z}\frac{\partial^2U}{\partial y\partial z},\\
10^9\Delta\frac{\partial^2U}{\partial x\partial z}&=-0{\cdot}000001\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial x\partial z}+0{\cdot}000007\frac{\partial^2U}{\partial x\partial y}\frac{\partial^2U}{\partial y\partial z},\\
10^9\Delta\frac{\partial^2U}{\partial y\partial z}&=+0{\cdot}000011\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)\frac{\partial^2U}{\partial y\partial z}+0{\cdot}000040\frac{\partial^2U}{\partial x\partial y}\frac{\partial^2U}{\partial x\partial z}.
\end{aligned}\right.$$

For $\left(\frac{\partial^2U}{\partial y^2}-\frac{\partial^2U}{\partial x^2}\right)$ &c., in the righthand members, the values not corrected for the error of azimuth and expressed in 10^{-9} c.g.s. should be used. These correctional terms should be added to the uncorrected values.

As we shall see later, the values of the second derivatives of the gravitational potential are generally of the order of 100 in 10^{-9} c.g.s., or less. Since the coefficients in the above formulae are very small, the correctional terms are generally not very significant, though sometimes as great as 1×10^{-9} c.g.s.

III. Observations on the Jaluit Atoll.

The observations began on February 10, 1915 and ended on March 9, during which time 18 observations at 12 different places were made.

The atoll is of a discontinuous chain of islets having a mean breath of about 150 meters. Some of them are very long, for example Jaluit (17 km.) and Mejado (5 km.), but generally they are short. Except in a few places, such as Jabor and Imroj there is no road for vehicles, so that the instruments were always carried by a sailing boat.

To avoid the complicated influences of the surface irregularities, it is advisable to locate observing stations at places as free as possible from such irregularities. In this atoll, the surface of the land is generally very even, especially near the native cottages. Except in the native villages, which are distributed here and there all over the atoll, the land is covered with bushes and forests of cocoanut trees, or constructed of fragments of coral reef, so that it was not cónvenient for making observations.

The atmospheric temperature and pressure were observed during the sajourn by self-recording instruments and shew the mean daily variation given in Pl. II. The former was nearly constant after sunset during the night. Since the gravity-variometer is much affected by the temperature, the above circumstance was very convenient for the observations They could be begun soon after sunset and repeated three or even four times under the best conditions before morning.

The data observed with the gravity-variometer are given in Table 2. Under the heading " Mag. N " are given the readings of the azimuth-circle when the positive direction of the arm of the balance No. I, i.e., the direction from the center of the arm to the end from which the hanging weight was suspended, was directed to the magnetic north. The magnetic declination in the Jaluit Atoll is $+8° 2'$. The columns under n_r aad n'_r give the observed scale-readings, while those under n and n' show the values reduced to 25.°0 C and corrected for the elastic after-effect.

Table 2. Observed data on the Jaluit Atoll.

(1) Jabor (a) Feb. 10–11, 1915. Magn. N=355° 57′

Time		Setting	T_a	T	n_r	n	T′	n'_r	n'
h	m								
7	10 p.	set free							
10	10	180°	26·0°	26·1°	150·3	151·0	26·3°	445·8	447·2
11	10	300	25·8	25·9	163·9	164·7	26·1	433·0	434·7
0	10 a.	60	25·6	25·7	172·9	173·8	25·9	443·9	445·8
1	10	180	25·6	25·6	149·9	150·9	25·7	445·4	447·5
2	10	300	25·4	25·5	163·9	164·9	25·6	432·4	434·8
3	10	60	25·2	25·3	173·0	174·2	25·4	443·6	446·1

(2) Jabor (b) Feb. 11–12, 1915. Magn. N=355° 57′

Time		Setting	T_a	T	n_r	n	T′	n'_r	n'
9	10	set free							
10	10	180	26·0	26·1	150·8	150·9	26·2	446·9	447·1
11	10	300	25·8	25·9	163·9	164·3	26·0	433·6	434·5
0	10 a.	60	25·7	25·7	173·1	173·8	25·8	444·2	445·7

(3) English Town Feb. 12–13, 1915. Magn. N=274° 26′

Time		Setting	T_a	T	n_r	n	T′	n'_r	n'
7	10 p.	set free							
9	40	100	25·7	25·9	144·2	144·8	26·0	439·6	440·8
10	40	220	26·0	25·8	157·2	157·8	25·9	434·7	436·2
11	40	340	26·2	26·0	187·0	187·8	26·1	452·1	453·9
1	10 a.	100	26·4	26·3	143·2	144·2	26·3	43·89	440·9
2	10	220	26·4	26·3	157·1	158·1	26·4	434·0	436·1
4	5	340	26·2	26·3	187·1	188·2	26·3	450·8	453·1
5	10	100	26·1	26·1	143·2	144·4	26·2	439·9	442·2
6	55	220	26·2	26·2	157·8	159·0	26·2	434·1	436·5

(4) Lyllel (a) Feb. 13–14, 1915. Magn. N. = 315° 12′.

Time		Setting	T_a	T	n_r	n	T′	n'_r	n'
6	10 p.	set free							
10	10	100	26·2	26·3	143·9	144·7	26·3	433·9	435·6
11	20	220	26·0	26·0	156·1	157·0	26·1	446·7	448·6
0	20	340	26·1	25·9	181·0	181·9	26·0	446·7	448·8
1	30	100	25·7	25·7	143·8	144·8	25·8	433·0	435·2
2	55	220	25·4	25·6	155·2	156·4	25·7	447·1	449·4
3	58	340	25·6	25·4	18·08	182·0	25·4	446·1	448·4

Time	Setting	T	T	n_r	n	T′	n'_r	n'
h m	°	°	°			°		
5 50 a.	100	25·5	25·5	143·1	144·6	25·6	432·4	434·8
6 45	220	25·6	25·4	155·1	156·7	25·6	346·8	449·2

(5) Lyllel (b) Feb. 14–15, 1915. Magn. *N.*=315° 14′.

Continued.

9 10	180	26·7	26·8	166·1	167·3	27·0	463·9	466·3
10 4	300	26·5	26·5	171·8?	173·0	26·7	439·2	442.7
11 5	60	26·5	26·5	151·1	152·3	26·6	430·8	433·3
0 15 a.	180	26·5	26·5	166·3	167·5	26·6	463·9	466·4
1 10	300	26·5	36·4	160·2	161·4	26·4	430·2	432·8
2 10	60	26·4	26 4	152·1	153·4	26·4	429·9	432·5
3 10	180	26·4	26·4	165·9	167·2	26·4	464·1	466·7
4 10	300	26·2	26·2	160·1	161·4	26·3	430·6	433·3
5 5	60	26·2	26·0	152·6	151·9	26·1	430·4	433·1
6 5	180	26·0	26·0	166·1	167·4	26·0	463·9	466·7
7 10	300	25·5	26·0	160·0	161·3	26·1	430·0	432·8

(6) Majurirok Feb. 15–16, 1915. Magn. *N.*=330° 38′.

8 8 p.	set free							
9 13	180	27·3	27·2	171·7	171·7	27·3	450·5	450·6
10 20	300	27·0	27·1	152·3	152·7	27·1	456·1	456·9
11 23	60	26·7	26·8	166·2	166·9	26·9	426·9	428·3
0 23 a.	180	26·6	26·6	171·2	172·0	26·7	450·1	451·7
2 8	300	26·8	26·7	151·2	152·2	26·7	451·2	458·2
3 16	60	26·7	26·7	165·2	166.2	26·7	425·8	428·0
5 38	180	26·8	26·6	168·9	170·0	26·7	448·3	450·6
6 33	300	26·8	26·6	150·2	151·4	26·7	454·3	457·0

(7) Jaluij. Feb. 16–17. 1915. Magn. *N.*=22° 13.′

7 7	set free							
8 27	220	26·3	26 5	160·2	160·3	26·6	458·9	459·2
9 27	340	25·5	26·1	154·7	155·2	26·1	448·2	449·3
10 42	100	25·3	25·5	171·2	172·0	25·6	421·4	423·1
11 57	220	25·1	25·1	159·8	160·8	25·2	457·8	459·7
1 17	340	25·1	25·0	153·8	154·9	25·0	447·8	449·8

Time	Setting	T_a	T	n_1	n	T′	n_1'	n'
h m	°	°						
2 12 a.	100	24·7	24·7	171·1	172·2	25·0	420·1	422·2
3 37	220	25·0	24·9	159·2	160·2	25·0	358·0	460·3
4 37	340	25·1	24·8	153·0	154·3	24·9	447·6	450·0
3 37	100	25·0	24·9	151·1	172·4	24·9	419·5	432·0
6 37	220	25·0	24·9	179·2	160·5	25·0	458·0	460·5

(8) Jabor (c). Feb 17–18, 1915. Magn. *N.* = 357° 39′.

Time	Setting	T_a	T	n_1	n	T′	n_1'	n'
8 0 p.	set free							
9 5	180	25·8	25·8	151·0	151·0	26·2	449·2	449·3
10 0	300	25·8	25·8	165·0	165·4	26·0	437·0	437·9
11 0	60	25·7	25·7	174·0	174·6	25·8	446·9	448·3
0 0 a.	180	25·5	25·4	149·9	150·8	25·3	448·7	450·5
0 58	300	25·3	25·2	164·3	165·3	25·3	436·0	438·0
1 55	60	35·3	25·3	173·9	174·9	25·3	446·2	448·3

(9) Jabor (d). Feb. 18–19, 1915. Magn. *N.* = 357° 39′.

Time	Setting	T_a	T	n_1	n	T′	n_1'	n'
9 40 p.	set free							
10 40	180	26·0	26·0	150·4	150·3	26·2	444·8	448·7
11 35	300	25·8	25·8	164·6	165·0	25·8	436·6	437·5
0 30 a.	60	25·8	25·8	173·9	174·5	25·8	446·2	447·5

(10) Imroj Feb. 19–20, 1915. Magn. *N.* = 14° 3′

Time	Setting	T_a	T	n_1	n	T′	n_1'	n'
6 6 p.	set free							
9 36	180	27·3	27·3	165·2	165·8	27·4	460·5	461·9
10 36	300	27·2	27·2	163·7	164·4	27·2	429·9	431·6
11 36	60	27·0	27·0	150·9	151·8	27·1	437·0	438·8
0 36 a.	180	27·0	27:0	165·2	166·2	27·1	460·3	462·3
1 45	300	26·2	27·6	163·9	165·0	26·8	430·6	432·8
2 45	60	26·2	26·4	150·3	151·4	26·5	436·8	439·1
3 56	180	26·0	26·0	165·0	166·2	26·1	460·1	462·4
4 56	300	26·2	26·2	163·1	164·5	26·3	428·8	431·2
5 56	60	26·0	26·1	151·0	152·4	26·2	437·1	439·5
7 1	180	27·0	26·5	164·8	166·3	26·5	460·3	462·7

Time	Setting	T_a	T	n_r	n	T′	n'_r	n'
(11) Lebjer Feb. 23–24, 1915. Magn. N.=311° 11′.								
h m 7 1 p.	set free							
8 31	220°	26°·1	26°·0	166·0	166·2	25°·9	427·3	427·8
9 31	340	25·8	25·8	169·3	169·9	25·7	454·9	456·1
10 31	100	25·9	25·8	148·0	148·7	25·9	446·4	447·9
0 1 a.	220	25·7	25·7	165·5	166·4	25·7	425·1	426·9
1 1	340	25·6	25·6	169·1	170·1	25·7	454·3	456·3
2 1	100	25·8	25·5	147·8	148·9	25·7	456·1	448·3
3 1	220	25·8	25·7	165·1	166·2	25·8	424·3	426·5
4 1	340	25·6	25·6	169·0	170·2	25·7	454·1	456·2
5 1	100	25·7	25·4	147·7	149·0	25·6	446·0	448·3
6 21	220	25·6	25·6	165·0	166·3	25·6	424·7	427·1
7 21	340	25·2	25·4	169·0	170·3	25·5	453·9	456·4
(12) Namolar Feb. 24–25, 1915 Magn. N=322° 14′								
6 31 p.	set free							
9 31	220	25·8	26·1	165·8	166·0	26·1	430·0	431·4
10 31	340	26·3	26·2	147·0	147·8	26·2	459·0	460·6
11 31	100	26·4	26·4	171·2	172·1	26·4	439·5	441·3
0 31 a.	220	26·3	26·2	165·9	166·8	26·2	430·0	432·0
1 31	340	26·5	26·2	147·0	148·0	26·2	458·8	460·9
2 31	100	26·6	26·5	171·5	172·6	26·5	439·6	441·8
3 41	220	26·8	26·6	165·7	166·8	26·6	429·3	431·6
5 1	340	26·8	26·8	147·0	148·1	26·8	458·0	460·3
6 1	100	26·8	26·8	171·9	173·1	26·9	439·8	442·2
(13) Mejado Feb. 25–26, 1915 Magn. N=359° 27′								
7 15 p.	set free							
8 40	220	27·6	27·6	161·1	161·2	27·8	446·1	446·5
9 42	340	27·2	27·2	178·9	179·4	27·3	442·0	443·1
10 45	100	26·8	26·9	147·5	148·2	27·0	447·3	448·7
11 45	220	26·6	26·7	160·3	161·2	26·8	445·0	446·7
0 45 a.	340	26·6	26·6	178·8	179·7	26·6	441·0	443·0
1 45	100	26·1	26·2	147·0	148·0	26·2	447·0	449·1
2 45	220	26·0	26·0	160·1	161·1	26·1	444·8	447·7
3 45	340	26·1	26·0	178·3	179·4	26·1	440·9	443·2

Time	Setting	T_a	T	n_r	n	T′	n'_r	n'
h m		°	°					
4 40 a.	100°	25·8	25·9	147·0	148·1	26·1	446·7	449·0
5 30	220	25·9	25·8	159·9	161·1	25·9	444·3	446·7
6 25	340	25·3	25·8	178·3	179·5	25·9	440·9	443·3
7 30	100	24·9	25·3	146·9	147·6	25·3	446·3	448·8
(14) Bokalokouj (a) Feb. 27–28, 1915 Magn. N = 306° 20′								
6 3 p.	set free							
8 3	220	26·8	26·7	140·2	140·6	26·7	439·2	440·1
9 3	340	26·7	26·8	168·1	168·7	26·8	444·1	445·5
10 3	100	26·9	26·9	190·9?	191·7	26·9	444·9	446·5
0 23 a.	220	26·7	26·7	140·2	141·1	26·7	438·7	440·8
1 53	340	26·5	26·5	185·2?	169·2	26·5	443·9	446·1
2 48	100	26·6	26·7	169·9?	171·0	26·7	444·1	446·4
4 29	220	26·7	26·8	140·2	141·3	26·9	438·7	441·0
5 23	340	26·9	26·9	167·9	169·1	26·9	443·5	445·9
7 18	100	27·3	27·1	178·2	179·4	27·1	444·3	446·6
(15) Bokalokouj (b) Feb. 28–March 1, 1915 Magn. N = 306° 20′								
6 53 p.	set free							
10 3	340	27·2	27·0	168·1	168·8	27·2	444·4	445·7
11 3	100	27·0	27·0	178·2	179·0	27·1	445·0	446·6
0 5 a	220	27·0	27·0	140·2	141·1	27·0	439·0	441·8
1 3	340	27·0	27·0	167.8	168·7	27·0	444·0	445·9
2 3	100	27·0	27·0	178·1	179·1	27·0	444·3	446·4
3 3	220	26·8	27·0	140·2	141·3	27·0	438·8	440·9
4 18	340	27·0	26·9	167·7	168·8	26·9	444·0	446·3
5 13	100	26·9	26·9	177·9	179·1	27·0	444·1	446·3
6 13	220	26·9	26·9	140·0	141·2	26·9	438·7	441·1
7 18	340	27·2	27·0	167·7	168 9	27·0	443·5	445·9
(16) Jabor (e) March 4–5, 1915 Magn. N = 356° 17′								
7 50 p.	set free							
8 55	180	26·7	27·6	152·0	151·8	27·6	447·4	448·5
9 45	300	26·5	27·2	164·9	165·2	27·2	435·0	435·8
10 40	60	26·4	26·6	173·5	174·0	26·6	445·7	448·1

Time	Setting	T_a	T	n_r	n	T′	n'_r	n'
h m								
11 30 p.	180°	26·3°	26·3°	150·7	151·5	26·3°	447·3	448·9
0 20 a.	300	26·0	26·0	164·6	165·5	26·0	434·2	435·9
1 10	60	26·0	26·0	173·7	174·7	26·0	445·1	448·0
2 0	180	26·0	26·0	150·3	151·3	26·0	447·0	469·0
2 50	300	26·0	26·0	164·3	165·4	26·0	434·0	436·6
3 40	60	26·0	26·0	173·6	174·7	26·0	445·1	447·4

(17) Namnum March 5–6, 1915 Magn. N = 345° 38′

Time	Setting	T_a	T	n_r	n	T′	n'_r	n'
4 58 p.	set free							
7 58	220	27·0	27·2	157·9	158·5	27·2	448·8	450·1
8 58	340	27·0	27·0	166·3	167·1	27·1	460·1	461·6
9 58	100	26·9	26·9	163·1	164·0	27·0	421·3	423·0
10 58	220	26·6	26·7	157·4	158·4	26·8	447·9	449·9
11 58	340	26·4	26·4	166·2	167·3	26·4	460·0	462·1
0 58 a.	100	26·2	26·3	162·9	164·0	26·3	420·9	422·1
1 58	220	26·2	26·2	157·1	158·2	26·2	447·4	449·7
2 58	340	26·2	26·1	166·0	167·2	26·1	460·0	462·4
3 58	100	26·2	26·1	163·0	164·2	26·2	460·2	422·6
4 58	220	26·2	26·2	157·0	158·2	26·2	448·0	450·3
5 58	340	26·2	26·2	166·2	167·5	26·2	459·3	461·7
6 58	100	26·7	26·4	162·8	164·1	26·3	419·7	422·1

(18) Imej March 8–9, 1915 Magn. N = 18° 7′

Time	Setting	T_a	T	n_r	n	T′	n'_r	n'
7 15 p.	set free							
9 50	220	25·7	25·7	150·8	151·4	25·9	452·0	453·2
10 55	340	25·8	25·8	177·3	178·0	25·9	430·6	432·1
11 55	100	25·2	25·4	163·2	164·2	25·7	449·0	450.7
0 55 a.	220	25·0	25·0	150·8	151·8	25·1	451·5	453·5
1 55	340	25·2	25·1	177·0	178·1	25·2	430·1	432·3
2 55	100	25·0	25·0	163·1	164·3	25·0	448·8	451·1
3 55	220	25·1	25·0	149·8	151·0	25·1	451·2	453·5
4 55	340	25·2	25·0	177·1	178·4	25·0	429·9	432·3
5 55	100	25·2	25·0	162·9	164·3	25·0	448·5	450·9

IV. Discussion of the Observed Data.

(*a*) *Corrections for tne Tidal Effect.*

As we have already seen, we need at least three observations of equilibrium-positions in order to determine the second derivatives of the gravitational potential. These observations take some three hours. The surface-irregularities of the land in the neighbourhood of the station are, of course, constant during this short interval, and their effect may afterwards be reduced if the necessary data be had. But the effect due to tidal disturbances is variable even during this short interval and hence requires careful investigation.

The tide-generating potential may, as usual, be written in the form

$$V = \frac{3}{4}\frac{GM}{\Delta^3}a^2\Big[\cos^2\varphi\cos^2\delta\cos 2t + \sin 2\varphi\sin 2\delta\cos t + \left(1-3\sin^2\delta\right)\left(\frac{1}{3}-\sin^2\varphi\right)\Big].$$

By differentiations, we may easily derive the expressions for the second derivatives of this potential with respect to the coordinates of a point on the earth's surface. They will all have the form

$$\frac{3}{2}\frac{GM}{\Delta^3}\times\begin{pmatrix}\text{trigonometrical sines and}\\ \text{cosines of the coordinates}\end{pmatrix}.$$

The coefficient will have values of the order of 10^{-13} c. g. s. in the case of the moon. Thus the direct effects of the tide-generating potential are far less than the limitting accuracy of the instrument.

The Jaluit Atoll is a group of small islets in the open Pacific. The surface of the sea-water around it, under the influence of the tidal force, may be considered to take the form expressed by the equation

$$\zeta = A\left(\cos^2\theta - \frac{1}{3}\right),$$

where θ is the arcual distance between the highest point and the point under consideration. This effect may be considered in two ways. First, the surface at a point will change its inclination, and secondly, it will change its height above the mean sea-level. If ε be the inclination of the real surface to the level-surface, we have

$$\varepsilon = \frac{A}{a}\sin 2\theta\left\{1+\frac{A}{a}\left(\cos^2\theta-\frac{1}{3}\right)\right\}^{-1}$$

In the Jaluit Atoll the amplitude of the spring tide is nearly 1 meter. Assuming that this value holds in the neighbouring ocean, the inclination of the surface of the sea-water will not exceed 0.″03 which again we need not to consider.

We will, therefore, proceed to consider the effect of the rise and fall of the water-surface around the station. Take a system of cylindrical coordinates ρ, α, ζ where α starts from the x-axis in the direction north to east and the ζ-axis is the normal through the origin, taken positive upward from the mean sea-level. Then, neglecting small quantities, we can derive the expressions for the effects of a sector bounded by radial lines of azimuths α_1 and α_2 and concentric circular arcs of radii ρ_1 and ρ_2, as follows:—

$$
(10)\quad \left\{
\begin{aligned}
\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2} &= \frac{1}{2}\, G\sigma\zeta \left\{\sin 2\alpha_2 - \sin 2\alpha_1\right\}\left\{\frac{3\rho_2^{\,2}+2h^2}{(\rho_2^{\,2}+h^2)^{\frac{3}{2}}}-\frac{3\rho_1^{\,2}+2h}{(\rho_1^{\,2}+h^2)^{\frac{3}{2}}}\right\},\\
\frac{\partial^2 U}{\partial x\partial y} &= \frac{1}{4}\, G\sigma\zeta \left\{\cos 2\alpha_2 - \cos 2\alpha_1\right\}\left\{\frac{3\rho_2^{\,2}+2h^2}{(\rho_2^{\,2}+h^2)^{\frac{3}{2}}}-\frac{3\rho_1^{\,2}\theta+2h^2}{(\rho_1^{\,2}+h^2)^{\frac{3}{2}}}\right\},\\
\frac{\partial^2 U}{\partial x\partial z} &= G\sigma \frac{\zeta}{h}\left(1-\frac{\zeta}{2h}\right)\left\{\sin \alpha_2 - \sin \alpha_1\right\}\left\{\frac{\rho_2^{\,3}}{(\rho_2^{\,2}+h^2)^{\frac{3}{2}}}\right.\\
&\qquad\qquad \left. -\frac{\rho_1^{\,3}}{(\rho_1^{\,2}+h^2)^{\frac{3}{2}}}\right\},\\
\frac{\partial^2 U}{\partial y\partial z} &= -G\sigma \frac{\zeta}{h}\left(1-\frac{\zeta}{2h}\right)\left\{\cos \alpha_2 - \cos \alpha_1\right\}\left\{\frac{\rho_2^{\,3}}{(\rho_2^{\,2}+h^2)^{\frac{3}{2}}}\right.\\
&\qquad\qquad \left. -\frac{\rho_1^{\,3}}{(\rho_1^{\,2}+h^2)^{\frac{3}{2}}}\right\}.
\end{aligned}
\right.
$$

where h is the height of the origin of the xyz-system above the sea-level. From these expressions, it may easily be seen that when the water-surface has circular symmetry around the station these effects will all vanish.

For numerical culculations, the surface around each station has been divided into systems of sectors by radial lines of azimuths differing by 30° successively, and concentric circles of radii 1×2^n meters starting from $n=0$. For sectors whose inner radii, as in the present case, are greater than 16 meters, the above formulae are applicable. Table 3 gives the numerical values of the effects of each sectors of height 1 cm. The two values of $\dfrac{\partial^2 U}{\partial x\partial z}$ and $\dfrac{\partial^2 U}{\partial y\partial z}$ for each sector are the coefficients of ζ and ζ^2 respectively. The effect of a sector will

Radius in meters	0° 180 – 30° 210	30° 210 – 60° 240	60° 240 – 90° 270	90° 270 – 120° 300	120° 300 – 150° 330	150° 330 – 180° 360

$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}$ in 10^{-15} c.g.s.

Radius	0°–30°	30°–60°	60°–90°	90°–120°	120°–150°	150°–180°
16						
32	−709	0	+709	+709	0	−709
64	−401	0	+401	+401	0	−401
128	−203	0	+203	+203	0	−203
256	−101	0	+101	+101	0	−101
512	− 51	0	+ 51	+ 51	0	− 51
1024	− 25	0	+ 25	+ 25	0	− 25

$\frac{\partial^2 U}{\partial x \partial y}$ in 19^{-15} c.g.s.

Radius	0°–30°	30°–60°	60°–90°	90°–120°	120°–150°	150°–180°
16						
32	+450	+899	+450	−450	−899	−450
64	+232	+463	+232	−232	−463	−232
128	+117	+234	+117	−117	−234	−117
256	+ 59	+117	+ 59	− 59	−117	− 59
512	+ 29	+ 59	+ 29	− 29	− 59	− 29
1024	+ 15	+ 29	+ 15	− 15	− 29	− 15

$\frac{\partial^2 U}{\partial x \partial z}$ in 10^{-15} and in 10^{-18} c.g.s.

Radius	0°–30°		30°–60°		60°–90°		90°–120°		120°–150°		150°–180°	
16												
32	±58	±102	± 42	±75	±16	±27	∓16	∓27	∓42	∓75	∓58	∓102
64	±16	± 30	± 12	±22	± 4	± 8	∓ 4	∓ 1	∓12	∓22	∓16	∓ 30
128	± 4	± 4	± 3	± 3	± 1	± 1	∓ 1	∓ 8	∓ 3	∓ 3	∓ 4	∓ 4
256	± 1	± 2	0	± 1	0	0	0	0	0	0	∓ 1	∓ 2
512	0	± 1	0	0	0	0	0	0	0	0	0	∓ 1
1024	0	0	0	0	0	0	0	0	0	0	0	0

$\frac{\partial^2 U}{\partial y \partial z}$ in 10^{-15} and in 10^{-18} c.g.s.

Radius	0°–30°		30°–60°		60°–90°		90°–120°		120°–150°		150°–180°	
16												
32	∓16	∓27	∓42	∓75	∓58	∓102	∓58	∓102	∓42	∓75	∓ 6	∓2
64	∓ 4	∓ 8	∓12	∓22	∓16	∓ 30	∓16	∓ 30	∓12	±22	∓ 4	∓ 8
128	± 1	± 1	∓ 3	∓ 3	∓ 4	∓ 4	∓ 4	∓ 4	∓ 3	∓ 3	∓ 1	∓ 1
256	0	0	0	0	∓ 1	∓ 2	∓ 1	∓ 2	0	0	0	0
512	0	0	0	0	0	± 1	0	0	0	0	0	0
1024	0	0	0	0	0	0	0	0	0	0	0	0

be obtained by multiplying these values by the height ζ of the water-surface in centimeters.

As it may be seen from the table, the coefficients of ζ^2 are very small and we may omit them without making appreciable error. Since our discussion refers to the change of height of water-surface common to all sectors, we may, for each station, write

$$
(11)\quad \left\{
\begin{aligned}
\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)_s &= \left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)+A_1\zeta_s\,,\\
\left(\frac{\partial^2 U}{\partial x\partial y}\right)_s &= \frac{\partial^2 U}{\partial x\partial y}+A_2\zeta_s\,,\\
\left(\frac{\partial^2 U}{\partial x\partial z}\right)_s &= \frac{\partial^2 U}{\partial x\partial z}+B_1\zeta_s\,,\\
\left(\frac{\partial^2 U}{\partial y\partial z}\right)_s &= \frac{\partial^2 U}{\partial y\partial z}+B_2\zeta_s\,.
\end{aligned}
\right.
$$

where the suffix s in the left hand members denotes the values when the height of the water-surface is ζ_s. The coefficients A and B depend on the distribution of the water-area around a station. Their values for each station are given in Table 4 in 10^{-9} c.g.s.

Table 4. Coefficients of tidal effects.

Station	A_1	A_2	B_1	B_2
Jabor	−0·00011	+0·00081	0·00000	0·00000
Eng. Town	+ 105	− 5	− 1	− 4
Lyllel	− 178	− 71	+ 19	− 3
Majurirök	− 10	+ 14	− 3	+ 1
Jaluij	− 55	− 105	− 4	− 1
Imroj	− 28	+ 61	+ 1	0
Lebjer	− 34	− 121	+ 11	+ 6
Namolar	+ 66	− 117	− 2	− 4
Mejado	+ 91	+ 51	0	+ 1
Bokalokouj	− 24	− 49	+ 4	− 28
Namnum	+ 59	+ 75	+ 7	− 8
Imej	+ 55	+ 173	+ 15	− 18

Now substituting the relations (11) in the equations of equilibrium (1) and (2) we get the following expressions for the torsionless positions instead of (5):—

$$(12)\quad \begin{aligned} n_0 &= \frac{1}{3}\left(n_1+n_2+n_3\right)+\left(\frac{1}{3}\,aA_2+\frac{1}{3}\,bB_2\right)\left(\zeta_1-\zeta_2\right)\\ &\quad+\left(-\frac{1}{4\sqrt{3}}\,aA_1+\frac{1}{6}\,aA_2-\frac{1}{2\sqrt{3}}\,bB_1+\frac{1}{6}\,bB_1\right)\left(\zeta_2-\zeta_3\right),\\ n'_0 &= \frac{1}{3}\left(n'_1+n'_2+n'_3\right)+\left(\frac{1}{3}\,a'A_2-\frac{1}{3}\,b'B_2\right)\left(\zeta_1-\zeta_2\right)\\ &\quad+\left(-\frac{1}{4\sqrt{3}}\,a'A_1+\frac{1}{6}\,a'A_2+\frac{1}{2\sqrt{3}}\,b'B_1-\frac{1}{6}\,b'B_2\right)\left(\zeta_2-\zeta_3\right), \end{aligned}$$

or, substituting the numerical values of a and b,

$$(12')\quad \begin{aligned} n_0 &= \frac{1}{3}\,(n_1+n_2+n_3)+(0{\cdot}040\,A_2+0{\cdot}065\,B_2)(\zeta_1-\zeta_2)\\ &\quad+(-0{\cdot}017\,A_1+0{\cdot}020\,A_2+0{\cdot}058\,B_1-0{\cdot}033\,B_2)(\zeta_2-\zeta_3),\\ n'_0 &= \frac{1}{3}\,(n'_1+n'_2+n'_3)+(0{\cdot}040\,A_2-0{\cdot}066\,B_2)(\zeta_1-\zeta_2)\\ &\quad+(-0{\cdot}017\,A_1+0{\cdot}018\,A_2+0{\cdot}056\,B_1-0{\cdot}033\,B_2)(\zeta_2-\zeta_3) \end{aligned}$$

where A and B should be expressed in 10^{-9} c.g.s. Since A and B are of the order of 10^{-3} or less and ζ less than 100 cm., we see that the tidal change of sea-level does not affect the values of n_0 and n_0' in the order of 10^{-2} scale division.

Denote the values of the second derivatives calculated by the formulae (10), (11) or (12) by the suffices p, q when they are obtained from n_p and n_q. In order to obtain the normal values not affected by the tidal influence, we must substitute the values given by (11) in the equations of eqailibrium (1) and (2) and solve the resulting ones. In this way we get the following relations:—

(i) from the first and the second settings,

$$(13)\quad \left\{ \begin{aligned} \frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2} &= \left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)_{1,2}-A_1\zeta_2+\frac{2}{\sqrt{3}}\,A_2\left(\zeta_1-\zeta_2\right),\\ \frac{\partial^2 U}{\partial x\partial y} &= \left(\frac{\partial^2 U}{\partial x\partial y}\right)_{1,2}-A_2\zeta_1,\\ \frac{\partial^2 U}{\partial x\partial z} &= \left(\frac{\partial^2 U}{\partial x\partial z}\right)_{1,2}-B_1\zeta_2+\frac{1}{\sqrt{3}}\,B_2\left(\zeta_1-\zeta_2\right),\\ \frac{\partial^2 U}{\partial y\partial z} &= \left(\frac{\partial^2 U}{\partial y\partial z}\right)_{1,2}-B_2\zeta_1, \end{aligned} \right.$$

(ii) from the second and the third settings,

$$(14)\quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x^2}=\left(\dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x^2}\right)_{2,3}-\dfrac{1}{2}A_1\left(\zeta_2+\zeta_3\right)-\dfrac{1}{\sqrt{3}}A_2\left(\zeta_2-\zeta_3\right), \\ \dfrac{\partial^2 U}{\partial \partial xy}=\left(\dfrac{\partial^2 U}{\partial x\partial y}\right)_{2,3}-\dfrac{\sqrt{3}}{4}A_1\left(\zeta_2-\zeta_3\right)-\dfrac{1}{2}A_2\left(\zeta_2+\zeta_3\right), \\ \dfrac{\partial^2 U}{\partial x\partial z}=\left(\dfrac{\partial^2 U}{\partial x\partial z}\right)_{2,3}-\dfrac{1}{2}B_1\left(\zeta_2-\zeta_3\right)-\dfrac{1}{2\sqrt{3}}B_2\left(\zeta_2-\xi_3\right), \\ \dfrac{\partial^2 U}{\partial y\partial z}=\left(\dfrac{\partial^2 U}{\partial y\partial z}\right)_{2,3}-\dfrac{\sqrt{3}}{2}B_1\left(\zeta_2-\zeta_3\right)-\dfrac{1}{2}B_2\left(\zeta_2+\zeta_3\right), \end{cases}$$

(iii) from the third and the first settings,

$$(15)\quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x_2}=\left(\dfrac{\partial^2 U}{\partial y^2}-\dfrac{\partial^2 U}{\partial x^2}\right)_{3,1}-A_1\zeta_3+\dfrac{2}{\sqrt{3}}A_2\left(\zeta_3-\zeta_1\right), \\ \dfrac{\partial^2 U}{\partial x\partial y}=\left(\dfrac{\partial^2 U}{\partial x\partial y}\right)_{3,1}-A_2\zeta_1, \\ \dfrac{\partial^2 U}{\partial x\partial z}=\left(\dfrac{\partial^2 U}{\partial x\partial z}\right)_{3,1}-B_1\zeta_3+\dfrac{1}{\sqrt{3}}B_2\left(\zeta_3-\zeta_1\right), \\ \dfrac{\partial^2 U}{\partial y\partial z}=\left(\dfrac{\partial^3 U}{\partial y\partial z}\right)_{3,1}-B_2\zeta_1. \end{cases}$$

The tidal observations at Jaluit Atoll showed the amplitude of the spring and the neap tides to be 98 and 60 cm. respectively and the phase lag $4^h\ 12^m$. The author prepared two tide-gauges of Prof. Honda's type[1]; and their records, though incomplete, were consistent with the tides calculated from the above data and the positions of the sun and the moon. On the other hand the observations with the gravity-variometer were generally repeated two or three times at each station, and the results showed that the tidal effects were not very large. Hence we need not any accurate value of ζ.

The calculated values of the second derivatives, corrected for the tidal effects according to the above method are given in Table 5. The resulting values show some disagreements for one and the same station and they sometimes go so far as 17×10^{-9} c. g. s. Some part of these disagreements may have been due to the unevenness of the sea-bottom near the shore, but more probably to some unknown disturbance of the instrument. As to this disturbance the author desires to make further investigation, here satisfying himself with the mean values.

---

1 K. Honda, Phil. Mag. **10**, 1905. Zeitschr. f. Instr.-Kunde **26**, 1906.

Table 5. Values corrected for the tidal effect.

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
			(1) Jabor (a). $A_x = 192°5'$				
1	−15	...	...	−97·8	+30·8	+3·4	+42·4
2	+24	163·2	442·6	−96·9	+30·2	+3·3	+42·8
3	+57	163·1	442·7	−96·0	+30·8	+3·6	+43·2
1	+76	163·2	442·7	−97·2	+30·8	+3·3	+43·2
2	+76	163·3	442·8	−98·8	+32·1	+3·2	+43·3
3	+57	...	...	...	...	...	...
Mean	...	163·20	442·70	−97·34	+30·94	+3·36	+42·98
			(2) Jabor (b) $A_x = 192° 6'$				
1	−50	...	...	−99·7	+30·3	+2·7	+42·3
2	− 2	163·0	442·4	−99·7	+30·3	+2·8	+42·3
3	+72	...	...	...	...	...	...
Mean	...	163·0	442·4	−99·7	+30·3	+2·75	+42·3
			(3) English Town. $A_x = 193° 36'$				
1	−88	...	...	−228·4	+88·9	−17·1	+39·6
2	−63	163·5	443·6	−229·3	+89·7	−17·4	+40·1
3	−23	163·3	443·7	−228·4	+90·4	−16·7	+41·3
1	+45	163·4	443·6	−228·6	+90·4	−17·3	+41·3
2	+77	163·5	443·4	−225·0	+91·7	−18·5	+40·9
3	+89	163·5	443·8	−225·2	+91·4	−17·9	+44·5
1	+51	163·8	443·9	−219·0	+91·4	−17·7	+44·5
2	−28	...	..	...	...	...	...
Mean	...	163·50	443·67	−222·67	+90·56	−17·51	+41·74
			(4). Lyllel (a). $A_x = 152° 55'$				
1	−95	...	...	−119·2	+104·8	−36·1	+19·1
1	−69	161·2	444·3	−119·7	+105·3	−36·0	+18·7
3	−27	161·2	444·2	−118·9	+105·9	−36·7	+17·8
1	+32	161·0	444·5	−117·4	+105·9	−38·6	+17·8
2	+89	161·1	444·3	−117·6	+106·2	−38·4	+17·3

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
3	+97	161·0	444·2	−116·4	+107·3	−38·3	+17·3
1	+57	161·1	444·1	−116·1	+107·3	−37·9	+17·3
2	+15	...	...	...	...	...	...
Mean	...	161·60	444·27	−117·90	+106·10	−37·43	+17·90

(5) Lyllel (6) $A_x = 230° \ 48'$

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	−54	...	...	...	...	...	...
2	−96	164·2?	447·4?	...	...	...	...
3	−94	164·3?	447·5?	+41·2	−122·7	+14·4	+39·8
1	−62	160·4	444·2	+40·2	−122·7	+12·4	+39·8
2	−21	160·8	443·9	+40·0	−122·4	+11·3	+41·8
3	+27	160·7	444·0	+42·1	−122·4	+11·3	+40·7
1	+71	160·7	444·2	+44·4	−122·4	+12·0	+ 0·7
2	+94	160·2	444·4	+45·6	−123·4	+13·2	+38 5
3	+95	160·2	444·4	+44·9	−124·0	+13·7	+44·3
1	+74	160·2	444·2	+43·0	−124·0	+14·2	+44·3
2	+30	...	...	...	...	—	...
Mean	...	160·46	444·19	+42·68	−123·00	+12·81	+41·14

(6) Majorirök $A_x = 217° \ 24'$

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	−54	...	...	+69·0	−55·2	−62·4	−6·3
2	−89	163·8	445·3	+72·2	−57·9	−63·2	−5·0
3	−96	163·9	445·6	+71·2	−58·9	−63·8	−6·3
1	−78	163·7	446·1	+77·9	−58·9	−66·2	−6·3
2	− 4	163·5	446·0	+73·7	−54·6	−66·0	−6·2
3	+49	162·8	445·6	+76·5	−52·2	−65·4	−5·1
1	+94	162·5	445·2	+68·9	−52·2	−64·2	−5·1
2	+77	...	...	...	...	...	...
Mean	...	163·37	445·63	+72·91	−55·70	−64·46	−5·76

(7) Jaluij $A_x = 205° \ 49'$

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	−11	...	...	+45·6	−55·4	−63·1	+44·7
2	−35	162·5	443·9	+47·5	−57·2	−63·2	+44·9
3	−79	162·7	444·0	+46·2	−58·4	−63·4	+44·5

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	−92	162·6	444·2	+50·3	−58·4	−64·8	−44·5
2	−69	162·6	443·9	+51·1	−59·1	−66·0	−46·5
3	−21	162·4	444·1	+50·8	−59·3	−65·9	−46·6
1	+21	162·2	444·2	+49·1	−59·3	−67·4	−46·6
2	+65	162·3	444·1	+49·9	−60·1	−67·5	−46·7
3	+87	162·4	444·2	+49·6	−60·8	−67·7	−46·5
1	+97	...	...	...	...	...	...
Mean	...	162·46	444·08	+48·90	−58·67	−65·44	−45·72
			(8) Jabor (c) $A_x = 190° \ 23'$				
1	+15	...	...	−96·2	+34·9	+2·9	+42·3
2	−28	163·7	445·2	−94·2	+33·2	+1·9	+43·8
3	−65	163·6	445·6	−95·3	+28·9	+2·4	+44·7
1	−85	163·6	445·6	−95·2	+28·9	+1·5	+44·6
2	−84	163·7	445·6	−95·7	+32·6	+1·4	+44·9
3	−63	...	...	...	...	...	...
Mean	...	163·65	445·50	−95·32	+31·70	+2·02	+44·06
			(9) Jabor (d) $A_x = 190° \ 23'$				
1	−24	...	...	−94·1	+35·7	+1·0	+43·0
2	−55	163·3	444·6	−93·9	+36·0	+1·0	+43·1
3	−75	...	...	...	...	...	...
Mean	...	163·3	444·6	−94·0	+35·85	+1·0	+43·1
			(10) Imroj $A_x = 173° \ 59'$				
1	+43	...	...	+23·8	−96·0	+28·9	+32·6
2	+10	160·7	444·1	+26·3	−98·2	+28·8	+32·8
3	−27	160·8	444·2	+28·0	−97·5	+28·5	+32·4
1	−55	161·0	444·6	+34·9	−97·5	+28·5	+32·4
2	−70	160·9	444·7	+33·1	−95·2	+29·1	+31·1
3	−67	160·9	444·8	+30·3	−97·5	+29·9	+32·5
1	−46	160·7	444·2	+18·7	−97·5	+30·5	+32·5
2	−6	161·0	444·4	+18·4	−97·2	+29·8	+33·5
3	+31	161·1	444·5	+17·1	−98·3	+30·1	+33·0
1	+58	...	...	...	...	...	...
Mean	...	160·89	444·44	+25·62	−97·21	+29·34	+32·33

Setting	ζ in cm.	n_0	n_0'	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
			(11) Lebjer $A_x=276^\circ\ 51'$				
1	0	...	...	+142·2	+48·8	+18·1	−52·7
2	+30	161·6	443·9	+140·9	+50·1	+18·8	−55·0
3	+53	161·7	443·6	+142·4	+51·5	+18·4	−54·6
1	+62	161·7	443·7	+142·3	+51·5	+18·7	−54·6
2	+49	161·8	443·8	+140·0	+53·4	+19·0	−55·1
3	+24	161.7	443·7	+140·7	+54·0	+19·2	−55·0
1	− 8	161·8	443·7	+139·0	+54·0	+19·4	−55·0
2	−37	161·8	443·7	+140·0	+53·0	+19·0	−55·4
3	−59	161·8	443·9	+139·3	+52·3	+19·2	−54·0
1	−63	161·9	443·9	+141·3	+52·3	+18·9	−54·0
2	−49	...	...	...	...	...	...
Mean	...	161·76	443·77	+140.81	+53·09	+18·87	−54·44
			(12) Namolar $A_x=265^\circ\ 48'$				
1	+ 1	...	...	−23·3	+36·5	−63·6	−44·4
2	+34	162·2	444·4	−21·6	+35·0	−64·2	−44·0
3	+59	162·2	444·6	−21·4	+32·9	−63·8	−43·8
1	+69	162·3	444·7	−21·6	+32·9	−64·0	−43·8
2	+62	162·5	444·9	−24·4	+37·6	−63·7	−44·3
3	+40	162·5	444·8	−24·6	+37·4	−63·6	−44·1
1	+ 2	162·5	444·6	−29·2	+37·4	−62·8	−44·1
2	−42	162·7	444·7	−30·5	+38·6	−63·0	−43·8
3	−65	...	...	...	...	...	...
Mean	...	162·41	444·67	−24·58	+36·04	−63·59	−44·04
			(13) Mejado $A_x=228^\circ\ 35'$				
1	−57	...	...	+121·4	+5·5	+53·7	+5·4
2	−25	162·9	446·1	+121·7	+5·2	+53·6	+5·6
3	+15	162·9	446·2	+121·9	+5·5	+53·8	+5·9
1	+50	163·0	446·1	+120·9	+5·5	+54·9	+5·9
2	+65	162·9	446·3	+122·3	+4·3	+54·6	+6·5
3	+76	163·0	446·6	+120·2	+2·6	+54·9	+7·0

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	+62	162·9	446·7	+122·3	+2·6	+53·7	+7·0
2	+31	162·9	446·6	+120·6	+4·0	+54·1	+6·3
3	− 4	162·9	446·3	+122·2	+5·3	+53·7	+5·4
1	−36	162·9	446·3	+123·8	+5·3	+53·1	+5·4
2	−64	162·7	446·3	+120·1	+4·5	+54 3	+5·1
3	−79	...	...	...	...	...	...
Mean	...	162·90	446·35	+121·58	+4·57	+54·13	+5·95
			(14) Bokarokouj (a) $A_x = 281° 42'$				
1	−84	...	...	...	...	...	...
2	−89	167·0?	444 0	...	...	...	...
3	−70	167·2?	444·3	...	...	...	...
1	−24	173·0?	444·5	...	...	...	...
2	+78	166·1?	444·4	...	...	...	...
3	+92	166·2?	444·5	...	...	...	...
1	+65	160·5?	444·4	−52·6	+105·4	−13·9	+46·2
2	+40	163·3	444·5	−52·5	+105·3	−13·6	+46·2
3	+21	...	...	...	...	...	...
Mean	...	163·3	444·5	−52·55	+105·35	−13·9	+46·2
			(15) Bokalokouj (b) $A_x = 41° 42'$				
1	−90	...	...	+203 9	−28·0	+48·0	−12·1
2	−64	163·0	444·7	+204·2	−28·4	+47·9	−11·9
3	−21	162·9	444·8	+203·6	−29·0	+48·4	−11·1
1	+26	163·0	444·7	+204·7	−29·0	+47·4	−11·1
2	+68	163·0	444·4	+206·9	−29·9	+46·9	− 9·9
3	+92	163·1	444·5	+206·3	−31·1	+46 8	− 9·9
1	+90	163·1	444·5	+206·5	−31·1	+46·9	− 9·9
2	+65	163·0	444·6	+206·1	−30·7	+47·3	−10·5
3	+22	163·1	444·4	+206·6	−30·3	+47·1	−10·9
1	−22	...	...	...	...	...	...
Mean	...	163·25	444·58	+205·42	−29·76	+47 41	−10·81
			(16) Jabor (e) $A_x = 191° 45'$				
1	+18	...	...	−101·9	+ 0·6	+5·7	+41·1
2	−22	163·7	444·1	−101·4	+30·2	+5·4	+41·5
3	−58	163·6	444·3	−101·1	+31·0	+5·9	+42·4

Setting	ζ in cm.	n_0	n'_0	$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 U}{\partial x \partial z}$	$\frac{\partial^2 U}{\partial y \partial z}$
1	−80	163·7	444·3	−101·1	+31·0	+5·2	+42·4
2	−87	163·9	444·3	−102·2	+31·9	+4·5	+43·7
3	−75	163·8	444·3	−101·7	+32·2	+4·2	+43·2
1	−58	163·8	444·5	− 98·2	+32·2	+2·8	+43·2
2	−26	163·8	444·3	− 97·5	+31·6	+2·6	+43·5
3	+10	...	...	...	...	...	...
Mean	...	163·76	444·30	−100·64	+31·34	+4·54	+42·63

(17) Namnum $A_x = 242^\circ\ 24'$

1	+71	...	...	+202·9	−2·5	−52·6	+25·1
2	+43	163·2	444·9	+202·7	−2·3	−52·6	+25·1
3	+ 5	163·2	444·8	+203·7	−1·4	−52·6	+25·0
1	−34	163·2	445·0	+209·2	−1·4	−53·8	+25·0
2	−65	163·2	444·7	+209·6	−1·9	−54·2	+25·8
3	−78	163·2	444·6	+211·4	−0·3	−54·6	+25·1
1	−84	163·1	444·7	+208·8	−0·3	−54·5	+25·1
2	−49	163·2	444·9	+208·2	−0·2	−54·4	+25·0
3	−13	163·2	445·1	+205·9	−0·2	−53·5	+26·5
1	+26	163·3	444·9	+206·8	−0·2	−53·1	+26·5
2	+58	163·3	444·7	+208·3	−0·3	−53·5	+27·2
3	+75	...	...	...	...	...	...
Mean	...	163·21	444·83	+207·05	−1·00	−53·58	+25·58

(18) Imej $A_x = 209^\circ\ 55'$

1	+60	...	...	−26·2	+21·1	+47·6	+53·1
2	+56	164·5	445·3	−24·3	+19·9	+47·5	+53·0
3	+38	164·7	445·4	−24·4	+20·2	+47·4	+52·7
1	+11	164·7	445·5	−23·6	+20·2	+47·8	+52·7
2	−19	164·7	445·6	−25·6	+21·5	+47·6	+53·1
3	−43	164·5	445·6	−24·0	+22·7	+48·1	+54·6
1	−58	164·6	445·6	−23·6	+22·7	+48·1	+54·6
2	−58	164·6	445·6	−24·4	+22·5	+48·0	+54·5
3	−34	...	...	...	...	...	...
Mean	...	164·61	445·51	−24·51	+21·35	+47·76	+53·54

(*b*) *Reduction to a Uniform Set of Coordinate-axes.*

The direction of the x-axis was not the same for each station, as is shown by the values of A_x in the table. As a new system of coordinates, use the meriodinal, the prime vertical and the plumb-line directions as the axes. The formulae of transformation are

$$\frac{\partial^2 U}{\partial\eta^2}-\frac{\partial^2 U}{\partial\xi^2}=\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)\cos 2A_x+2\frac{\partial^2 U}{\partial x\partial y}\sin 2A_x,$$

$$\frac{\partial^2 U}{\partial\xi\partial\eta}=-\frac{1}{2}\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)\sin 2A_x+\frac{\partial^2 U}{\partial x\partial y}\cos 2A_x,$$

$$\frac{\partial^2 U}{\partial\xi\partial\zeta}=\frac{\partial^2 U}{\partial x\partial z}\cos A_x-\frac{\partial^2 U}{\partial y\partial z}\sin A_x,$$

$$\frac{\partial^2 U}{\partial\eta\partial\zeta}=\frac{\partial^2 U}{\partial x\partial z}\sin A_x+\frac{\partial^2 U}{\partial y\partial z}\cos A_x.$$

(*c*) *Correction for the Effect of the Neighboring Terrain.*

The resulting values are under the influence of the terrain around each station and also of the ellipticity of the geoid. The effect of the former origin was corrected by means of the Eötvös' method[1], with $h=1{\cdot}32$ m instead of $1{\cdot}00$ m and $\sigma=2{\cdot}0$[2] instead of $\sigma=1{\cdot}8$. In this method the field is divided into sectors by radial lines of azimuth differing by 45° successively and concentric circles of radii 5 m, 20 m, 50 m, 100 m and 1000 m. If ε and $\varkappa$ be the inclinations of the earth's surface in the innermost circle in the meridional and prime vertical directions and ζ the heights at the intersections of the radial lines and the circles, then the effects are given by the following numerical formulae :—

$$\begin{aligned}10^9\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right)=&-0{\cdot}53623\,(\zeta_1-\zeta_3+\zeta_5-\zeta_7)_{\rho=5\,m}\\&-0{\cdot}09104\,(\eta_1-\eta_3+\eta_5-\zeta_7)_{\rho=20\,m}\\&-0{\cdot}03534\,(\zeta_1-\zeta_3+\zeta_5-\zeta_7)_{\rho=50\,m}\\&-0{\cdot}03308\,(\zeta_1-\zeta_3+\zeta_5-\zeta_7)_{\rho=100\,m}\\&-0{\cdot}00474\,(\zeta_1-\zeta_3+\zeta_5-\zeta_7)_{\rho=1000\,m}\end{aligned}$$

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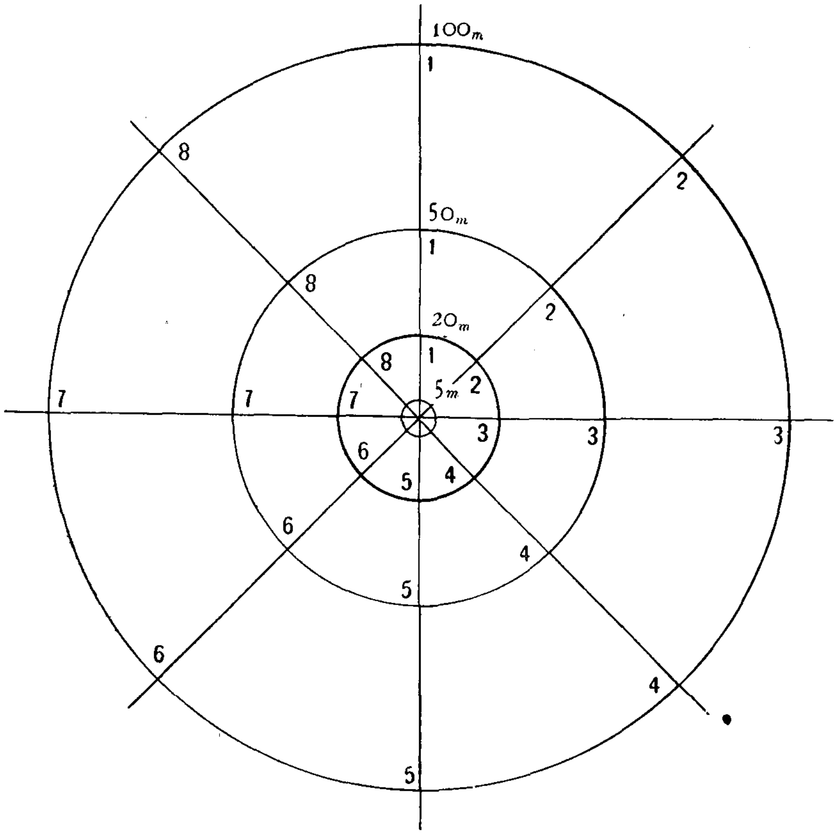
1 R. v. Eötvös, Bestimmung der Gradienten der Schwerkraft und ihre Niveauflächen mit Hülfe der Drehwage. Verh. 15 Allg. Conf. d. Intn. Erdmessung, **337**, 1906.

2 F. H. Helmert, Die Schwerkraft und die Massenverteilung der Erde. Encyk. d. math. Wiss. VI 1B, 131, 1910.



$$10^9 \frac{\partial^2 U}{\partial x \partial y} = +0{\cdot}25701\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)_{\rho=5\,\mathrm{m}}$$
$$+0{\cdot}04552\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)_{\rho=20\,\mathrm{m}}$$
$$+0{\cdot}01768\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)_{\rho=50\,\mathrm{m}}$$
$$+0{\cdot}01710\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)_{\rho=100\,\mathrm{m}}$$
$$+0{\cdot}00238\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)_{\rho=10000\,\mathrm{m}}$$

$$10^9 \frac{\partial^2 U}{\partial x \partial z} = +8{\cdot}538\,\varepsilon$$
$$+\{0{\cdot}0421\,\varepsilon\,(\zeta_1+\zeta_5)+0{\cdot}0068\,\varepsilon\,(\zeta_3+\zeta_7)$$
$$+0{\cdot}0246\,\varepsilon\,(\zeta_2+\zeta_4+\zeta_6+\zeta_8)$$
$$+0{\cdot}0178\,\varkappa\,(\zeta_2-\zeta_4+\zeta_6-\zeta_8)\}_{\rho=5\mathrm{m}}$$
$$+\{0{\cdot}14496\,(\zeta_1-\zeta_5)+0{\cdot}10250\,(\zeta_2-\zeta_4-\zeta_6+\zeta_8)\}_{\rho=5\,\mathrm{m}}$$
$$+\{0{\cdot}01303\,(\zeta_1-\zeta_5)+0{\cdot}00922\,(\zeta\;-\zeta_4-\zeta_6+\zeta_8)\}_{\rho=20\,\mathrm{m}}$$
$$+\{0{\cdot}00120\,(\zeta_1-\zeta_5)+0{\cdot}00086\,(\zeta_2-\zeta_4-\zeta_6+\zeta_8)\}_{\rho=50\,\mathrm{m}}$$

$$+\{0{\cdot}00031\,(\zeta_1-\zeta_5)+0{\cdot}00022\,(\zeta_2-\zeta_4-\zeta_6+\zeta_8)\}_{\rho=100\text{ m}}$$
$$+\{-0{\cdot}000017\,(\zeta_1-\zeta_5)-0{\cdot}000010\,(\zeta_2-\zeta_4-\zeta_6+\zeta_8)\}_{\rho=1000\text{ m}}$$
$$+\{-0{\cdot}091941\,(\zeta_1^2-\zeta_5^2)-0{\cdot}001372\,(\zeta_2^2-\zeta_4^2-\eta_6^2+\eta_8^2)$$
$$-0{\cdot}001444\,(\zeta_1\zeta_2-\zeta_3\zeta_4-\zeta_5\zeta_6+\zeta_8\zeta_1)$$
$$-0{\cdot}000598\,(\zeta_2\zeta_3-\zeta_3\zeta_4-\zeta_6\zeta_7+\zeta_7\zeta_8)\}_{\rho=5\text{ m}}$$

$$10^9\,\frac{\partial^2 U}{\partial y\partial z}=+8{\cdot}538\,\varkappa$$
$$+\{0{\cdot}0421\,\varkappa(\zeta_3+\zeta_7)+0{\cdot}0068\,\varkappa(\zeta_1+\zeta_5)+0{\cdot}0246\,\varkappa(\zeta_4+\zeta_6+\zeta_8+\zeta_2)$$
$$+0{\cdot}0178\,\varepsilon\,(\zeta_4-\zeta_6-\zeta_8+\zeta_2)\}_{\rho=5\text{ m}}$$
$$+\{0{\cdot}14499\,(\zeta_3-\zeta_7)+0{\cdot}10250\,(\zeta_4-\zeta_6-\zeta_8+\zeta_2)\}_{\rho=5\text{ m}}$$
$$+\{0{\cdot}01303\,(\zeta_3-\zeta_7)+0{\cdot}00922(\zeta_4-\zeta_6-\zeta_8+\zeta_2)\}_{\rho=20\text{ m}}$$
$$+\{0{\cdot}00120\,(\zeta_3-\zeta_7)+0{\cdot}00086\,(\zeta_4-\zeta_6-\zeta_8+\zeta_2)\}_{\rho=50\text{ m}}$$
$$+\{0{\cdot}00031\,(\zeta_3-\zeta_7)+0{\cdot}00022\,(\zeta_4-\zeta_6-\zeta_8+\zeta_2)_{\rho=100\text{ m}}$$
$$+\{0{\cdot}000017\,(\zeta_3-\zeta_7)-0{\cdot}000010\,(\zeta_4-\zeta_6-\zeta_8+\zeta_2)\}_{\rho=1000\text{ m}}$$
$$+\{-0{\cdot}001941\,(\zeta_3^2-\zeta_7^2)-0{\cdot}001372\,(\zeta_4^2-\zeta_6^2-\zeta_8^2+\zeta_4^2)$$
$$-0{\cdot}001444\,(\zeta_3\zeta_4-\zeta_6\zeta_7-\zeta_7\zeta_8+\zeta_2\zeta_3)$$
$$-0{\cdot}000598\,(\zeta_4\zeta_5-\zeta_5\zeta_6-\zeta_8\zeta_1+\zeta_1\zeta_2)\}_{\rho=5\text{ m}}$$

As we have noted before, the land-surface of the Jaluit Atoll is rather flat and not very high above the mean sea level. The correctional terms due to such a land-form at each station were naturally very small.

(*d*) *Correction for the Ellipsoidal Form of the Geoid.*

At a point on the surface of the geoid, we have

$$\frac{\partial^2 U}{\partial\eta^2}-\frac{\partial^2 U}{\partial\xi^2}=g\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right),$$
$$\frac{\partial^2 U}{\partial\xi\partial\eta}=0,$$

where ρ_1 and ρ_2 are the principal radii of curvature in the meridional and prime vertical sections. The equation of the geoid, center origin,

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1$$

becomes, when transformed into our coordinate system,

$$\xi^2\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)+\frac{\eta^2}{a^2}+\zeta^2\left(\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2}\right)+2\xi\zeta\sin\theta\cos\theta\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$$
$$-2\xi\left(\frac{p\cos\theta}{a^2}-\frac{q\sin\theta}{b^2}\right)-2\zeta\left(\frac{p\sin\theta}{a^2}+\frac{q\cos\theta}{b^2}\right)=0$$

where θ is the inclination of the tangential plane at the origin to the equatoreal, and p and q the co-ordinates of the origin referred to the initial coordinate axes. We have

$$\tan\theta=\frac{p}{q}\frac{b^2}{a^2},$$
$$p=r\cos\varphi,$$
$$q=r\sin\varphi,$$
$$r^2=\frac{b^2}{1-e^2\cos^2\varphi},$$

φ being the geocentric latitude of the origin. From the above equation we obtain the principal radii of culvature as follows:

$$\rho_1=\left(\frac{p\sin\theta}{a^2}+\frac{q\cos\theta}{b^2}\right)\Big/\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right),$$
$$\rho_2=\left(\frac{p\sin\theta}{a^2}+\frac{q\cos\theta}{b^2}\right)\Big/\left(\frac{\sin^2\theta}{a^2}+\frac{\cos^2\theta}{b^2}\right).$$

In our case $\varphi=+6^0$ so that

$$\rho_1=6323\text{ km.},$$
$$\rho_2=6364\text{ km.},$$

and from these values we get

$$\frac{\partial^2 U}{\partial\eta^2}-\frac{\partial^2 U}{\partial\xi^2}=10{\cdot}04\times10^{-9}\text{ c. g. s.}$$

Again, the gravitational acceleration at a point on the geoid at latitude φ is given by[1]

$$\gamma_0=978{\cdot}030\,(1+0{\cdot}005302\,\sin^2\varphi-0{\cdot}000007\,\sin^2 2\varphi)\,\frac{\text{cm.}}{\text{sec}^2}.$$

Differentiating this with respect to φ, we obtain

$$\frac{d\gamma_0}{d\varphi}=978{\cdot}030\,(0{\cdot}005302\sin2\varphi-0{\cdot}000014\sin4\varphi).$$

Since

1 F, R. Helmert, die Schwerkraft und die Massenverteilung der Erde. Encyk. d. math. Wiss. VI 1B, 95, 1910.

$$d\varphi = \frac{d\zeta}{r}$$

and also in the present case $\varphi = +6^0$, we finally obtain

$$\frac{\partial^2 U}{\partial\xi\partial\zeta} = +1{\cdot}68 \times 10^{-9} \text{ c. g. s.}$$

Also we have

$$\frac{\partial^2 U}{\partial\eta\partial\zeta} = 0.$$

These corrections due to the ellipsoidal form of the geoid should be subtracted from the observed values of the second derivatives of the gravitational potential.

(*e*) *The Final Values.*

The resulting values after the application of the above corrections are given in Table 6.

Table 6. The Final Values.

Station	$\frac{\partial^2 U}{\partial\eta^2} - \frac{\partial^2 U}{\partial\xi^2}$	$\frac{\partial^2 U}{\partial\xi\partial\eta}$	$\frac{\partial^2 U}{\partial\xi\partial\zeta}$	$\frac{\partial^2 U}{\partial\eta\partial\zeta}$
Jabor (a)	− 73·11	+48·78	+ 4·05	−42·71
„ (b)	− 76·22	+48·08	+ 4·50	−41·92
„ (c)	− 76·68	+46·48	+ 6·73	−43·30
„ (d)	− 72·52	+50·13	+ 5·10	−42·56
„ (e)	− 77·05	+48·66	+ 1·54	−42·73
„ mean	− 75·12	+48·43	+ 4·38	−42·64
Eng. Town	−128·72	+132·33	+23·88	−36·31
Lyllel (a)	−252·56	+14·45	+23·49	−32·98
„ (b)	−255·91	+12·48	+23·31	−35·08
„ mean	−254·24	+13·47	+23·40	−34·03
Majurirök	− 98·50	−46·71	+46·14	+43·76
Jaluij	− 71·58	−55·37	+85·79	−12·68
Imroj	+ 55·74	−68·88	−33·95	−29·30
Lebjer	−171·95	−34·87	−53·49	−25·24
Namolar	+ 24·78	−33·72	−20·96	+66·64
Mejado	− 16·16	−60·78	−33·04	−44·53
Bokalokouj (a)	− 45·60	−107·13	+40·71	+22·98
„ (b)	− 45·46	−105·46	+40·93	+23·47
„ mean	− 45·53	−106·30	+40·82	+23·23
Namnum	−129·88	−84·58	+45·80	+35·63
Imej	+ 14·26	+21·80	−16·35	−70·21

(*f*) *Graphical Representation.*

For graphical representation of the resulting values, we proceed as follows. Let A_0 be the azimuth of the principal section ξ_0 of the larger curvature of the equipotential surface at a point. Then

$$\frac{\partial^2 U}{\partial \eta_0^2}-\frac{\partial^2 U}{\partial \xi_0^2}=\sqrt{\left(\frac{\partial^2 U}{\partial \eta^2}-\frac{\partial^2 U}{\partial \xi^2}\right)^2+\left(-2\frac{\partial^2 U}{\partial \xi \partial \eta}\right)^2},$$

$$\tan 2A_0=-2\frac{\partial^2 U}{\partial \xi \partial \eta}\Big/\left(\frac{\partial^2 U}{\partial \eta^2}-\frac{\partial^2 U}{\partial \xi^2}\right).$$

In obtaining A_0, we must take the minimum angle of $2A_0$ when $\frac{\partial^2 U}{\partial \eta^2}-\frac{\partial^2 U}{\partial \xi^2}$ is positive and add π when $\frac{\partial^2 U}{\partial \eta^2}-\frac{\partial^2 U}{\partial \xi^2}$ is negative. In other words, if A_0 be the minimum value, the azimuth of ξ_0 is A_0 or $A_0+\frac{\pi}{2}$ according as $\frac{\partial^2 U}{\partial \eta^2}-\frac{\partial^2 U}{\partial \xi^2}$ is positive or negative. In either case the addition of π to A_0 makes no difference.

The gradient of gravity at each station is obtained by

$$\frac{\partial g}{\partial s}=\sqrt{\left(\frac{\partial^2 U}{\partial \xi \partial \zeta}\right)^2+\left(\frac{\partial^2 U}{\partial \eta \partial \zeta}\right)^2},$$

$$\tan A_s=\frac{\partial^2 U}{\partial \eta \partial \zeta}\Big/\frac{\partial^2 U}{\partial \xi \partial \zeta}.$$

For A_s we must take the minimum angle or add π to it according as $\frac{\partial^2 U}{\partial \xi \partial \zeta}$ is positive or negative.

These curvature and gradient terms are shown in Pl. III. and Pl. IV. On the whole, the results agree with those which we may expect in the present case.

V. On the Depth of Coral Reef.

Among the geodetic meanings, which the second derivatives of the gravitational potential contain, suggestions concerning the distribution of density under the ground is most interesting in the present case. Coral atolls are naturally built upon foundations probably of volcanic origin. The difference in density of the coral reef and the material of the foundation gives a means whereby to estimate the depth of the former. This is important whatever theory, subsision or solution, we assume concerning the formation of the atolls.

For this we first calculate the values of the second derivatives of the gravitational potential upon various assumptions as to the depth of the coral reef. The Jaluit Atoll has an irregular rhombic form difficult of mathematical treatment. Hence a way for mechanical integration must be found.

Consider a system of cylindrical coordinates with ζ-axis in the plumb-line direction. Then the gravitational potential at a point ρ, α, ζ due to an elementary mass $\sigma r\,dr\,d\theta\,dz$ at another point r, θ, z is given by

$$\Delta U = \frac{G\,\sigma r\,dr\,d\theta\,dz}{\sqrt{\rho^2+r^2-2\rho r\cos(\alpha-\theta)+(\zeta-z)^2}}\,.$$

Differentiate this expression with respect to the coordinates of the former point and put $\rho=0, \zeta=0$. Then we find the second derivatives of the potential at the origin due to tne elementary mass as follows:—

$$\Delta\left(\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2}\right) = -3G\sigma\frac{r^3\cos 2\theta}{(r^2+z^2)^{\frac{5}{2}}}\,dr\,d\theta\,dz,$$

$$\Delta\frac{\partial^2 U}{\partial x\partial y} = \frac{3}{2}\,G\sigma\frac{r^3\sin 2\theta}{(r^2+z^2)^{\frac{5}{2}}}\,dr\,d\theta\,dz,$$

$$\Delta\frac{\partial^2 U}{\partial x\partial z} = -\,G\sigma\frac{r^2\zeta\cos\theta}{(r^2+z^2)^{\frac{5}{2}}}\,dr\,d\theta\,dz,$$

$$\Delta\frac{\partial^2 U}{\partial y\partial z} = -G\sigma\frac{r^2\zeta\sin\theta}{(r^2+z^2)^{\frac{5}{2}}}\,dr\,d\theta\,dz.$$

Integrating these expressions, we obtain the effects due to a sectoreal column bounded by the limits r_1 and r_2, θ_1 and θ_2 and z_1 and z_2. The results are as follows:—

$$\frac{\partial^2 U}{\partial y^2}-\frac{\partial^2 U}{\partial x^2} = \sigma P(r,z)(\sin 2\theta_2-\sin 2\theta_1),$$

$$\frac{\partial^2 U}{\partial x\partial y} = \sigma P(r,z)\,\frac{1}{2}\,(\cos 2\theta_2-\cos 2\theta_1),$$

$$\frac{\partial^2 U}{\partial x\partial z} = \sigma Q(r,z)(\sin\theta_2-\sin\theta_1),$$

$$\frac{\partial^2 U}{\partial y\partial z} = -\sigma Q(r,z)(\cos\theta_2-\cos\theta_1),$$

where

$$P(r,z) = \frac{1}{2}\,G\left|\frac{z}{(r^2+z^2)^{\frac{1}{2}}}-2\log\frac{z}{z+(r^2+z^2)^{\frac{1}{2}}}\right|_{r_1,z_1}^{r_2,z_2},$$

$$Q(r,z) = G\left|\frac{r}{(r^2+x^2)^{\frac{1}{2}}}+\log\frac{r}{r+(r^2+z^2)^{\frac{1}{2}}}\right|_{r_1,z_1}^{r_2,x_2}.$$

It is to be noted that, in these expressions, r and z occur in ratio, so that the calculation becomes very simple if we choose successive values of r and z by

$$r_1 = z_1 \text{ finite,}$$
$$r_m = c^{m-1} r_1,$$
$$z_n = c^{n-1} z_1.$$

when the smallest limits of r and z are zero at the same time, the values of P and Q become infinite. In Table 8, the values of these functions for different valucs of r and z are given. The values of the trigonometrical factors in the above expressions may also be easily calculated from trigonometrical tables as shown in the annexed table.

As to the depth of the sea around the Jaluit Atoll the author could obtain no exact data, except a small map[1] kindly shown by Prof. Ogawa in the Geographical Institute, which was magnified and used in the calculation.

Any finite values of the second derivatives of the gravitational potential, corrected for various influences before described, must be due to the difference in density of the coral reef and the material of the foundation on the one hand and of the sea-water on the other. We know nothing about the material of the foundation, though it is probably of submarine volcanic origin. The density of volcanic rocks varies over a somewhat large range as follows[2] :—

syenite	2.54
granite	2.68
trachyte	2.63
tonschiefer	2.81
basalt	2.99
mean	2.73

The composition of volcanic rocks, moreover, is not the same for different portion of the earth's surface. Volcanoes in the Pacifiic Ocean are generally composed of rocks of greater mean density,[3] such as 3.12 for the Hawaiian rocks. That the foundations of the atolls of the Marshall Group, if volcanic, can not be of recent formation, must be taken into account; and their density is not likely to reach 3.12. Thus

1 Andrees, Hand atlas.

2 M. P. Rudzki, Physik der Erde, 99, 1911.

3 J. P. Iddings, The Problem of Volcanism, 123, 1914.

Table 7. Values of $P(r, z)$

ζ \ r	0m–1	1–3	3–9	9–27	27–81
0m–1		−49·682 +62·726	−21·180 + 4·561	−7·342 +0·404	−3·552 +0·060
1–3	− 2·702 +10·807	−20·454 +35·298	−28·960 +22.211	−13·850 + 2·539	−4·880 +0·477
3–9	− 0·069 + 0·716	− 2·679 +10·555	−20·454 +35·298	−28·960 −22·211	−13·850 + 2·539
9–27	− 0·001 + 0·033	−0·066 +0·683	− 2·679 +10·555	−20·454 +35·298	−28·960 +22·211
27–81	0 + 0·001	−0·001 +0·033	− 0·066 + 0 683	− 2·679 +10·555	−20·454 +25·298
81–243		0 +0·001	− 0·001 + 0·033	−0·066 +0·683	− 2·679 +10·555
243–729			0 + 0·001	−0·001 +0·033	−0·006 +0·683
729–2187				0 +0·001	−0·001 +0·033
2187–6561					0 +0·001

Table 8. Values of the

	0°–30°	30°–60°	60°–70°
$\sin 2\theta_2 - \sin 2\theta_1$	+0·8660	0	−0·8660
$\frac{1}{2}(\cos 2\theta_2 - \cos 2\theta_1)$	−0·2500	−0·5000	−0·2500
$\sin \theta_2 - \sin \theta_1$	±0·5000	±0·3660	±0 1340
$\cos \theta_2 - \cos \theta_1$	∓0·1340	∓0·3660	∓0·5000
	180°–210°	210°–240°	240°–270°

the writer has assumed cases where the density of the foundation is 2.6, 2.8 and 3.0

and $Q(r, z)$ in 10^{-9} c.g.s.

81	243	729	2187	6561	19683	59047
−0·819 +0·007	−0·272 +0·001	−0·090 0	−0·030 0	−0·010 0	−0·003 0	
−1·634 +0·053	−0·547 +0·007	−0·182 +0·001	−0·060 0	−0·020 0	−0·007 0	
−4·880 +0·477	−1·634 +0·053	−0·547 +0·007	−0·182 +0·001	−0·060 0	−0·020 0	
−13·850 +2·539	−4·880 +0·477	−16·34 +0·053	−0·547 +0·007	−0·182 +0·001	−0·060 0	
−28 960 +22·211	−13·850 + 2·539	−4·880 +0·477	−1·634 +0·053	−0·547 +0·007	−0·182 +0·001	
−20·454 +35·298	−28·960 +22·211	−13·850 + 2·539	−4·880 +0·477	−1·634 +0·053	−0·547 +0·007	
− 2·679 +10·555	−20·454 +35·298	−28·960 +22·211	−13·850 + 2·539	−4·880 −0·477	−1·634 +0·053	
−0·066 +0·683	− 2·679 +10·555	−20·454 +35·298	−28·960 +22·211	−13·850 + 2·539	−4·880 +0·477	
−0·001 +0·033	−0·066 +0.683	− 2·679 +10·555	−20·454 +35·298	−28·960 +22·211	−13 850 + 2·539	

trigonometrical factors.

90°	120°	150°	180°
−0·8660	0	+0·8660	
+0·2500	+0·5000	+0·2500	
∓0·1340	∓0·3660	∓0·5000	
∓0·5000	∓0·3660	∓0·1340	
270°	300°	330°	360°

The coral reef is exceedingly porous. The density of the material is 2·4 according to the author's determination. The effective den-

Table 9. (1). Calculated values of

Station	Jabor	Eng. Town	Lyllel	Major	Jaluij	Imroj
Obs. values	−75·12	−128·72	−254·24	−98·50	−71·58	+55·74
			for $\sigma^{f} = 2{\cdot}6$			
Depth in meters						
3	−69·43	−162·12	−253·59	−110·17	− 78·66	+85·39
9	−69·67	−158·97	−247.89	−109·92	− 91·55	+79·55
27	−70·06	−155·19	−248·83	−111·13	−101·07	+75·81
81	−69·50	−146·57	−241·92	−108·85	− 99·96	+73·10
243	−68·24	−135·49	−229·35	−102·32	− 88·00	+63·73
729	−62·85	−129·49	−225·66	− 92·73	− 76·16	+52·86
2187	−55·59	−119·79	−194·62	− 80·99	− 69·81	+38·58
4000	−45·33	−107·72	−179·67	− 83·91	− 61·12	+29·52
			for $\sigma_f = 2{\cdot}8$			
3	−76·69	−174·45	−270·02	−116·01	− 82·57	+97·82
9	−76·97	−170·38	−263·06	−115·70	− 98·33	+90·67
27	−77·38	−165·76	−264·21	−107·19	−109·94	+86·09
81	−75·21	−155·22	−255·77	−113·51	−108·61	+82·80
243	−73·09	−141·67	−240·40	−106·42	− 93·98	+71·35
729	−66·55	−134·33	−218·58	− 94·69	− 79·50	+58·06
2187	−57·66	−122·46	−197·95	− 87·47	− 71·74	+40·59
4000	−45·33	−107·72	−179·67	− 83·91	− 61·12	+29·52
			for $\sigma_f = 3{\cdot}0$			
3	−83·91	−186·78	−286·45	−121·85	− 86·48	+110·25
9	−84·27	−181·79	−278·23	−121.48	−105·13	+101·79
27	−84·70	−176·33	−285·45	−123·25	−118·83	+ 96·39
81	−80·92	−163·87	−269·62	−113·51	−117·26	+ 92·50
243	−77·96	−147·85	−251·45	−110·89	− 99·96	+ 78·97
729	−70·25	−139·17	−211·50	− 96·65	− 82·84	+ 63·26
2187	−59·73	−125·13	−202·55	− 93·95	− 73·67	+ 42·60
4000	−45·33	−107·72	−197·69	− 83·91	− 61·12	+ 29·52

$$10^9 \left(\frac{\partial^2 U}{\partial \eta^2} - \frac{\partial^2 U}{\partial \xi^2} \right)$$

Lebjer	Namolar	Mejado	Bokal.	Namnum	Imej
−171·95	+24.78	−16·16	−45·53	−129·88	+14·26
		for $\sigma_f = 2{\cdot}6$			
−187·36	+28·55	−27·41	−47·81	−190·88	+25·99
−187·11	+27·80	−26·78	−47·77	−179·13	+25·79
−185·45	+27·17	−24·99	−49·17	−169·24	+21·51
−182·20	+25·28	−23·05	−48·82	−151·71	+15·34
−176·41	+23·46	−20·42	−47·58	−132·52	+12·51
−168·08	+21·77	−18·44	−44·70	−115·17	+10·69
−162·77	+19·76	−16·71	−41·75	− 97·60	+ 9·47
−154·05	+17·12	−15·16	−37·26	− 80·27	+ 5·40
		for $\sigma_f = 2{\cdot}8$			
−194·77	+31·09	−30.13	−50·16	−215·48	+30·56
−194·45	+30·17	−29·36	−50·11	−201·11	+30·33
−192·41	+29·40	−27·21	−51·82	−188·99	+25·10
−188·47	+27 09	−24·80	−51·39	−167·59	+27·55
−181·38	+24·87	−21·58	−49·87	−144·15	+14·09
−172 79	+22·80	−19·17	−46·36	−122·93	+11·86
−164·71	+20·35	−17·05	−42·75	−101·45	+10·37
−154·05	+17·12	−15·16	−37·26	− 80·27	+ 5·40
		for $\sigma_f = 3{\cdot}0$			
−202·18	+33·64	−32·85	−52·51	−240·08	+35·19
−211·79	+32·54	−31·94	−52·45	−223·09	+34·87
−199·41	+31·63	−29·40	−54·48	−208·74	+28·69
−194·74	+28·90	−26·54	−53·96	−183·47	+19·76
−186·34	+26·28	−22·74	−52·16	−155·78	+16·67
−174·90	+23·83	−19·90	−48·02	−130·69	+13·03
−166·65	+20·94	−17·39	−43·74	−105·30	+11·27
−154·05	+17·12	−15·16	−37·26	− 80·27	+ 5·40

Table 9. (2). Calculated values of

Station	Jabor	Eng. Town	Lyllel	Majur	Jaluij	Imroj
Obs. values	+48·43	+132·33	+13·47	−46·71	−55·37	−68·88
			for $\sigma_f = 2{\cdot}6$			
Depth in meters						
3	+37·75	+169·57	+13·86	−58·34	−71·77	−101·37
9	+42·61	+167·29	+13·73	−59·73	−71·78	− 96·16
27	+56·14	+157·92	+12·72	−55·98	−66·86	− 88·58
81	+51·38	+145·76	+12·58	−55·37	−59·38	− 79·31
243	+42·94	+140·04	+12·09	−51·78	−51·70	− 76·13
729	+29.68	+134·53	+12·24	−43·62	−41·51	− 67·64
2187	+19·61	+123·11	+11·66	−31·57	−36·57	− 52·20
4000	+13·05	+117·41	+10·22	−24·94	−29·81	− 43·27
			for $\sigma_f = 2{\cdot}8$			
3	+43·25	+181·13	+14·67	−65·82	−81·10	−114·27
9	+49·01	+178·39	+14·51	−65·04	−81·12	−107·71
27	+65·73	+167·54	+13·27	−62·88	−75·10	− 98·64
81	+59·91	+152·18	+13·10	−62·13	−65·96	− 87·33
243	+49·59	+145·41	+12·50	−57·74	−56·79	− 83·44
729	+33·39	+138·59	+12·71	−47 77	−44·12	− 70·84
2187	+21·07	+124·37	+11·98	−33·04	−38·55	− 54·18
4000	+13·05	+117·41	+10·22	−24·94	−29·81	− 4327
			for $\sigma_f = 3{\cdot}0$			
3	+48·75	+192·59	+15·48	−73·24	−90·43	−127·17
9	+55·37	+189·49	+15·29	−70·35	−90·46	−119·26
27	+75·32	+177·16	+13·82	−69·78	−83·34	−108·70
81	+68·44	+158·66	+13·62	−68·89	−72·54	− 95·35
243	+56·24	+150·78	+12·91	−63·70	−61·88	− 90·75
729	+37·10	+142·65	+13·18	−51.92	−46·73	− 74·04
2187	+22·53	+125·63	+12·30	−34·51	−40·14	− 56·16
4000	+13·05	+117·41	+10·22	−24·94	−29·81	− 43·27

$$10^9 \frac{\partial^2 U}{\partial\xi\partial\eta}$$

Lebjer	Namolar	Mejado	Bokal.	Namnum	Imej
−34·87	−33·72	−60·78	−106·30	−84·58	+21·80
		for $\sigma_f = 2{\cdot}6$			
−45·38	−47·43	−82·51	−114·32	−88·30	+39·73
−43·92	−45·87	−74·92	−112·40	−84·76	+32·51
−41·39	−44·77	−69·39	−110·58	−84·40	+29·68
−38·28	−38·46	−64·62	−109·08	−80·52	+22·09
−36·08	−32·77	−59·01	−104·33	−79·70	+18·34
−33·07	−28·51	−50·48	−100·43	−45·71	+16·24
−29·16	−24·88	−41·58	− 96·05	−67·73	+15·07
−24·43	−21·32	−41·11	− 91·12	−55·32	+10·33
		for $\sigma_f = 2{\cdot}8$			
−50·03	−53·24	−91·72	−119·48	−95·63	+46·27
−48·25	−51·33	−82·44	−117·13	−91·30	+43·55
−45·61	−49·98	−75·68	−114·91	−90·87	+33·98
−41·35	−42·27	−69·73	−111·76	−86·12	+24·71
−38·67	−35·31	−62·99	−107·27	−85·14	+20·12
−34·99	−30·11	−57·02	−102·38	−79·75	+17·57
−30·21	−25·67	−48·36	− 97·15	−70·49	+16·12
−24·43	−21·32	−41·11	− 91·12	−55·32	+10·33
		for $\sigma_f = 3{\cdot}0$			
−54·68	−59·05	−100·93	−124·64	−102·96	+52·81
−52·58	−56·79	− 89·96	−121·86	− 97 84	+49 59
−49·83	−55·19	− 81·97	−119·24	− 97·34	+38·28
−41·74	−46·08	− 75·04	−114·44	− 91·72	+27·33
−41·26	−37·85	− 66·97	−110·21	− 90 58	+21·90
−36·91	−31·71	− 63·56	−104·43	− 84·19	+18·89
−31·26	−26·46	− 55·14	− 98·25	− 73·25	+17·17
−24·43	−21·32	− 41·11	− 91·12	− 55·32	+10·33

Table 9. (3). Calculated values of

Station	Jobor	Eng. Town	Lyllel	Majur.	Jaluij	Imroj
Obs. values	+4·38	+23·88	+23·40	+46·14	+85·79	−33·95
			for $\sigma_f = 2{\cdot}6$			
Depth in meters						
3	+10·19	+28·05	+25·60	+29·09	+42·15	−28·83
9	+ 9·92	+27·85	+25·09	+29·14	+43·82	−27·96
27	+ 8·52	+27·34	+24·57	+31·01	+50·13	−28.66
81	+ 9·69	+27·43	+23·67	+35·11	+53·32	−31·15
243	+ 8·77	+25·34	+22·40	+34·99	+49·59	−29·34
729	+ 7·36	+22·55	+20·90	+33·41	+46·59	−26 82
2187	+ 5·65	+20·00	+18·59	+25·81	+28·71	−18·02
4000	+ 3·33	+17·68	+15·47	+16·93	+16·35	−12·34
			for $\sigma_f = 2{\cdot}8$			
3	+11·71	+30·36	+27·85	+31·79	+47·89	−31·28
9	+11·39	+30·11	+27·23	+31·86	+49·93	−31·43
27	+ 9·69	+29·49	+26·59	+34·14	+57·65	−32·29
81	+11·10	+29.37	+25·49	+39·15	+61·54	−35·11
243	+ 9 98	+27·04	+23·94	+39·01	+56·98	−33·12
728	+ 8·25	+23·63	+22·11	+37·08	+51·10	−30 04
2187	+ 6·16	+20·51	+19·28	+27·78	+31·46	−19·38
4000	+ 3·33	+17·68	+15·47	+16·93	+16·35	−12·34
			for $\sigma_f = 3{\cdot}0$			
3	+13·23	+32·67	+30·10	+34·49	+53·63	−34·73
9	+12·86	+32·37	+28·37	+34·58	+56·04	−34·90
27	+10·84	+31 64	+28·61	+37·27	+65·17	−35·92
81	+12·73	+30·31	+27·31	+43·19	+69·76	−39·07
243	+11·19	+28·74	+25·48	+42·83	+64·37	−36·90
729	+ 9·14	+24·71	+23·32	+40·75	+55·61	−33·26
2187	+ 6·67	+21·02	+19·97	+29·75	+34·21	−20·74
4000	+ 3·33	+17·68	+15·47	+16·93	+16·35	−12·34

$$10^9 \frac{\partial^2 U}{\partial \xi \partial \zeta}$$

Lebjer	Namolar	Mejado	Bokal.	Namnum	Imej
−53·49	−20·96	−33·04	+40·82	+45·80	−16·35
		for $\sigma_f = 2{\cdot}6$			
−60·74	−24.19	−29·34	+46·35	+34·90	−19·65
−61·35	−23·67	−29·63	+46·82	+36·65	−19·54
−61·42	−23·55	−29·82	+47·04	+41·11	−19·48
−60·85	−22·15	−30·27	+46·03	+42·04	−16·90
−55·09	−18·95	−39·37	+43·39	+39·14	−15·40
−50·63	−15·34	−28·32	+39·46	+33·13	−13·28
−43·71	−10·70	−21·83	+34·68	+20·42	−11·72
−38·11	− 5·91	−16·44	+31·07	+23·38	− 9·09
		for $\sigma_f = 2{\cdot}8$			
−65·79	−27·93	−32·21	+40·75	+37·47	−22·00
−66·32	−27·62	−32·57	+50·32	+35·61	−21·86
−69·60	−27·47	−32·80	+50·59	+45·05	−20·54
−65·91	−25·99	−32·35	+49·85	+46·19	−14·63
−58·87	−21·85	−32·25	+47·13	+42·64	−16·80
−53·41	−17·44	−30·96	+41·33	+35·30	−14·21
−44·95	−11·76	−23·03	+45·48	+27·46	−12·31
−38·11	− 5·91	−16·44	+31·07	+23·38	− 9·69
		for $\sigma_f = 3{\cdot}0$			
−60·83	−32·21	−35·08	+53·15	+40·04	−24·35
−71·69	−31·57	−35·51	+53·82	+22·57	−24·18
−71·78	−31·39	−35·78	+54·14	+48·99	−22·60
−70·97	−29·83	−36·43	+53·27	+50·34	−20·36
−62·65	−24·75	−35·13	+48 87	+46·14	−18·20
−56·19	−19·54	−33·60	+43·20	+37·47	−15·14
−46·19	−12·82	−24·23	+36·28	+29·50	−12·90
−38·11	− 5·91	−16·44	+36·07	+23·38	− 9·09

Table 9. (4). Calculated values of

Station	Jabor	Eng. Town	Lyllel	Majur.	Jaluij	Imroj
Obs. values	−42·64	−36·31	−34·03	+43·76	−12·68	−29·30
			for $\sigma_f = 2\cdot6$			
Depth in meter						
3	−35·98	−22·61	−23·30	+34·01	−36·04	−19·84
9	−36·42	−21·90	−23·92	+34·45	−35·48	−19·63
27	−39·12	−22·06	−24·90	+36·91	−33·52	−19·46
81	−38·20	−23·01	−29·01	+37·14	−36·54	−19·24
243	−36·88	−26·61	−31·51	+36·49	−35·61	−18·92
729	−34·21	−31·50	−32·00	+34·71	−33·34	−21·55
2187	−25·30	−40·55	−33·62	+26·43	−16·43	−30·26
4000	−14·16	−52·16	−39·59	+25·65	−12·41	−36·78
			for $\sigma_f = 2\cdot8$			
3	−40·84	−27·16	−25·81	+35·87	−38·22	−22·45
9	−41·38	−26·56	−26·33	+36·41	−38·39	−22·27
27	−44·66	−26·69	−27·16	+39·41	−28·44	−22·13
81	−43·54	−27·49	−30·64	+39·70	−39·68	−21·94
243	−41·93	−30·54	−32·75	+38·90	−38·55	−21·67
729	−36·44	−34·68	−33·17	+36·73	−35·77	−23·89
2187	−27·78	−42·34	−34·54	+28·83	−17·33	−21·26
4000	−14·16	−52·16	−39·59	+25·65	−12·41	−36·78
			for $\sigma_f = 3\cdot0$			
3	−45·70	−31·71	−28·32	+37·73	−40·40	−25·06
9	−46·33	−31·22	−28·74	+38·37	−41·30	−24·91
27	−50·20	−31·32	−29·42	+41·91	−41·36	−24·80
81	−48·88	−31·97	−32·27	+42·26	−42·82	−24·64
243	−46·98	−34·47	−33·99	+41·31	−41·49	−24·42
729	−38·69	−47·86	−34·34	+38·75	−38·20	−26·23
2187	−30·26	−44·13	−35·46	+31·23	−18·23	−32·26
4500	−14·16	−52·16	−30·59	+25·65	−12·41	−36·78

$$10^9 \frac{\partial^2 U}{\partial\eta\partial\zeta}$$

Lebjer	Namolar	Mejado	Bokal.	Namnum	Imej
−25·24	+66·64	−44·53	+23·23	+35·63	−70·21
		for $\sigma_f = 2{\cdot}6$			
−28·81	+71·86	−38·45	+25·06	+28·85	−48·08
−29·80	+71·13	−40·28	+25·31	+33·10	−49·56
−30·03	+70·34	−40·13	+25·83	+36·05	−55·02
−30·06	+69·32	−40·17	+25·25	+33·73	−57·09
−27·64	−63·87	−37·93	+22·66	+33·06	−57·33
−23·45	+60·17	−40·55	+18·78	+21·06	−54·62
−19·67	+56·50	−45·63	−15·73	+22·57	−50·37
−14·63	+51·16	−49·57	+13·09	+18·61	−33·31
		for $\sigma_f = 2{\cdot}8$			
−31·96	+76·46	−40·16	+27·78	+31·13	−51·37
−33·17	+75·12	−41·71	+28·03	+36·32	−53·18
−33·46	+74·61	−41·58	+28·66	+39·93	−57·62
−33·27	+73·35	−41·62	+27·95	+37·09	−60·16
−30·53	+68·92	−39·72	+24·79	+36·27	−60·23
−25·41	+62·18	−41·94	+20·05	+31·74	−57·14
−20·79	+57·69	−46·23	+16·32	+23·45	−51·94
−14·63	+51·16	−49·57	+13·09	+18·61	−33·31
		for $\sigma_f = 3{\cdot}0$			
−35·11	+81·06	−41·87	+30·50	+33·41	−54·66
−36·54	+79·11	−43·14	+30·75	−39·54	−56·80
−36·89	+78·88	−43·03	+31·49	+43·81	−60·22
−36·48	+77·38	−43·07	+30·65	+40·45	−63·23
−33·42	+73·97	−41·51	+26·92	+39·48	−63·13
−27·37	+64·19	−43·33	+21·32	+25·84	−59·66
−21·91	+58·88	−46·83	+16·91	+24·33	−53·51
−14·63	+51·16	−49·57	+13·09	+18·61	−33·31

sity which causes the gravitational irregularities on the surface is but the mean density when all the cavities are filled up with sea-water. This mean density was determined to be 1·69.

With these data were calculated the values of the second derivatives of the gravitational potential for various depths of the coral reef. The resulting values are shown in Table 9.

Comparing these values with the observed, the probable residuals, corresponding to each assumed depth of the coral reef, was calculated for the curvature and the gradient terms, with the following results:—

TABLE 10.

Depth in meter	$\sigma_f = 2{\cdot}6$	2·8	3·0
	for the curvature term		
3	±8·90	±12·38	±15·55
9	7·60	10·94	14·36
27	6·03	9·31	12·41
81	3·86	6·37	8·55
243	2·64	3·54	5·27
729	4·00	3·45	3·59
2187	7·54	6·68	5·35
4000	10·39	10·39	10·39
	for the gradient term.		
3	±3·60	±3·58	±3·50
9	3·50	3·40	3·40
27	3·25	2·90	3·10
81	2·98	2·71	2·90
243	2·96	2·58	2·56
729	3.19	2·85	2·83
2187	4·61	4·27	3·94
4000	6·23	6·23	6·23

The minimum residual for the curvature term occurs at the depth of 243 m., if we take $\sigma_f = 2{\cdot}6$, and of 729 m., if $\sigma_f = 2{\cdot}8$ or 3·0. For the gradient term, it is 243 m. for all assumed densities of the foundation. Considering the mode of variation of the residual with depth, we notice that, in the latter case, the probable depth will lie between 243 m, and 729 m. when we assume $\sigma_f = 2{\cdot}8$ or $3{\cdot}0$

In Pl. V the abscissae give the depth in logarithmic scale i.e.,

$$n = \log z \,/\, \log 3$$

and the ordinates give the residuals. The minimum residuals seem to occur at the depths.

	$\sigma_f = 2{\cdot}6$		$\sigma_f = 2{\cdot}8$		$\sigma_f = 3{\cdot}0$	
	n	z	n	z	n	z
Curvature term	5·0	m 243	5·7	m 520	6·3	m 1000
Gradient term	5·0	243	5·2	301	5·6	466
Mean n	5·0		5·45		5·95	
$3^{(\text{mean n})}$		243		396		683

It must be noticed that the mean residual is the least for the curvature term, if we assume $\sigma_f = 2{\cdot}6$ and $z = 243$ m. while in the case of the gradient term when $\sigma_f = 3{\cdot}0$ and $z = 466$ m. These do not harmonize, and we can not conclude which assumption is the more probable.

Boring[1] at the Funafuti Atoll reached the depth of $1114\frac{1}{2}$ feet or about 337 meters. Throughout this depth, reef-building corals were the main material. The lower third, however, was hard enough to yield the core. Thus the lower part of the reef is very compact and has greater density than the upper part. Our calculation was founded on the assumption that the reef had a uniform density. The depth of the coral reef calculated upon this assumption must give some value smaller than the actual.

VI. Conclusion.

The interest of the writer in the subject of this study was specially aroused by the fact that the Jaluit Atoll is an isolated island of simple construction in an open sea. The values of the second derivatives of the gravitational potential at 12 stations on the atoll have been determined by the Eötvös' gravity-variometer although the value of the gravitational acceleration in that island is not yet known.

[1] The Atoll of Funafuti, published by R. S. London, 1904.

Necessary corrections due to the elastic after-effect of the torsion-wire, the deviation of the azimuth of the arm, the tidal effect, the attraction of the surrounding terrain and the effect of oblateness of the geoid, have been considered in full and applied to the observed data. The resulting values have been given in numerical tables and represented graphically.

Generally speakiug, the gradients of the gravitational force on that atoll are directed inwards, perpendicular to the line of the reef. The curvature of the equipotential surface is larger in the direction perpendicular to the line of the reef than in the direction parallel to it. The stations are all situated near the inner shore of the atoll and the value of the mean gradient at those stations is about 60×10^{-9} c. g. s. The values of the differences of curvatures multiplied by the gravitational acceleration varied from 60×10^{-9} to 280×10^{-9} c. g. s., the mean value being about 150×10^{-9} c. g. s.

These observed values have been compared with the values calculated for each station under several assumptions as to the formation of the atoll. As the density of the coral reef, the value 1.69 has been taken, being the mean value obtained for all specimens when the pores were filled up with sea-water. The density of the foundation has been assumed to be 2·6, 2·8 or 3·.0 The mean residuals, thus obtained, shew that the effective depth of the coral reef should be from 243 to 1000 meters, according to the assumptions of the density of the foundation.

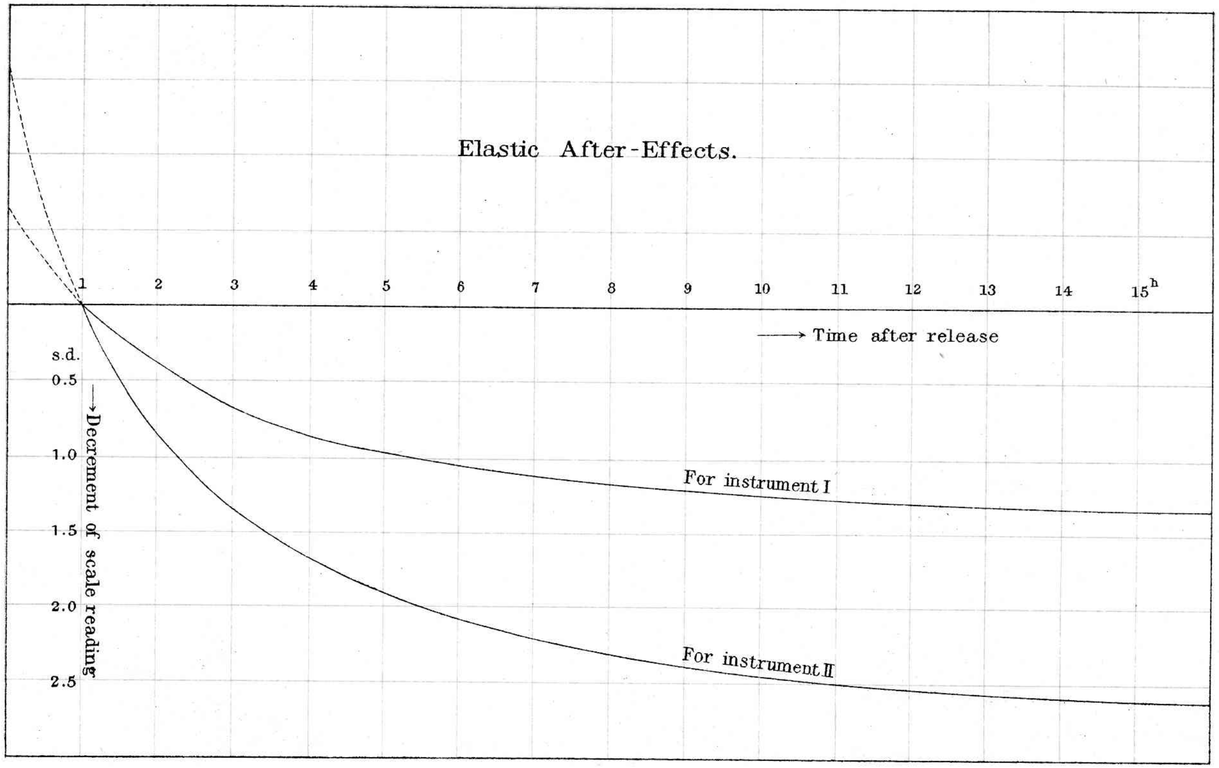
This estimation of the depth may be subject to several criticisms. The author has taken 1·69 as the density of the coral reef throughout its material; though the boring at the Funafuti Atoll suggests that it is compressed and has greater density at the lower part. Hence the depth obtained as above must be too small.

Again in the calculations, it has been assumed that the reef was built up on a flat foundation. More probably it was built up on a peak or peaks of a submarine volcano.

In order to consider these points more fully it would be necessary to introduce more assumptions of density distribution in the coral reef and of the form of the peaks; and for the present, we content ourselves with having obtained certain estimations of the effective depth of the coral.

In conclusion, the author owes hearty thanks to Prof. Dr. Shinjō, by whose intercession he has started on his excursion and under whose suggestion and care most of this paper has been completed.

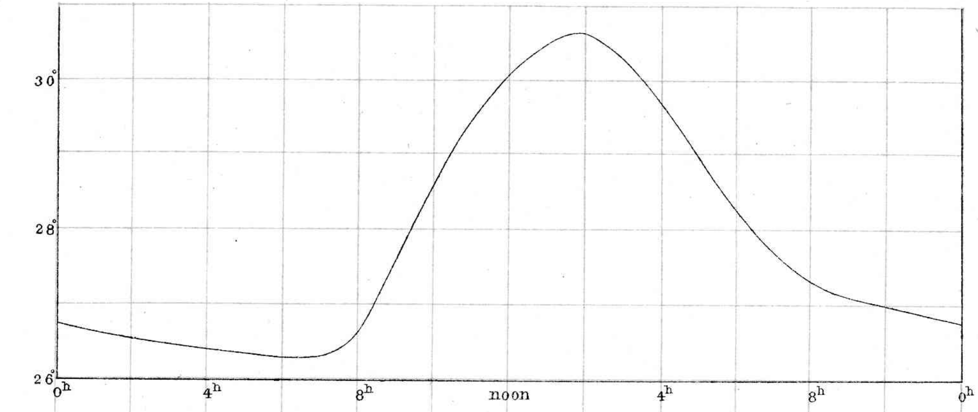
Plate I.



Elastic After-Effects.
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15^h
Time after release
s.d.
0.5
1.0
1.5
2.0
2.5
Decrement of scale reading
For instrument I
For instrument II

Plate II.

Daily Variation of Temperature.



Daily Variation of Pressure.

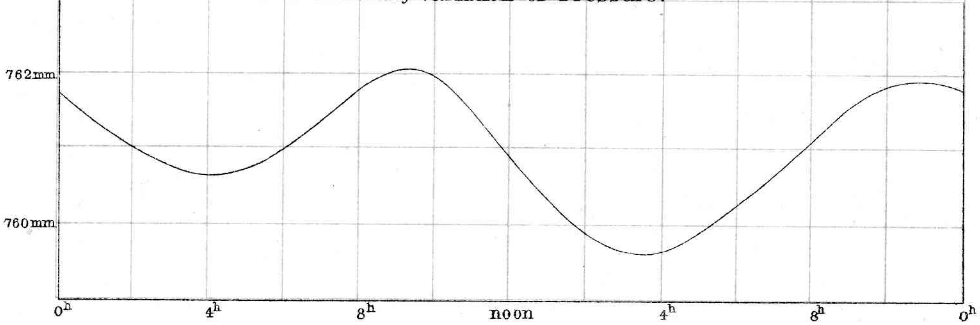
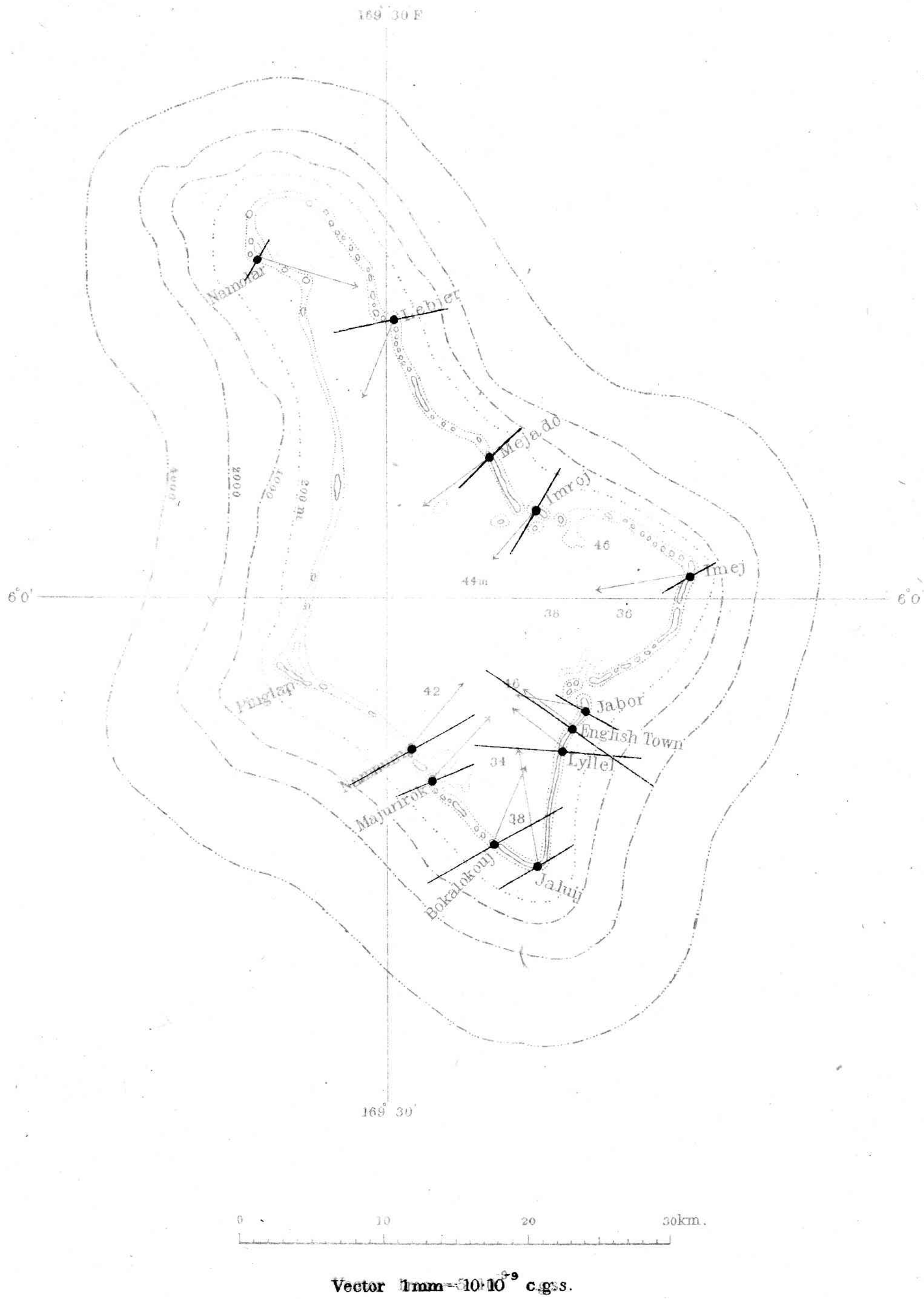


Plate III.

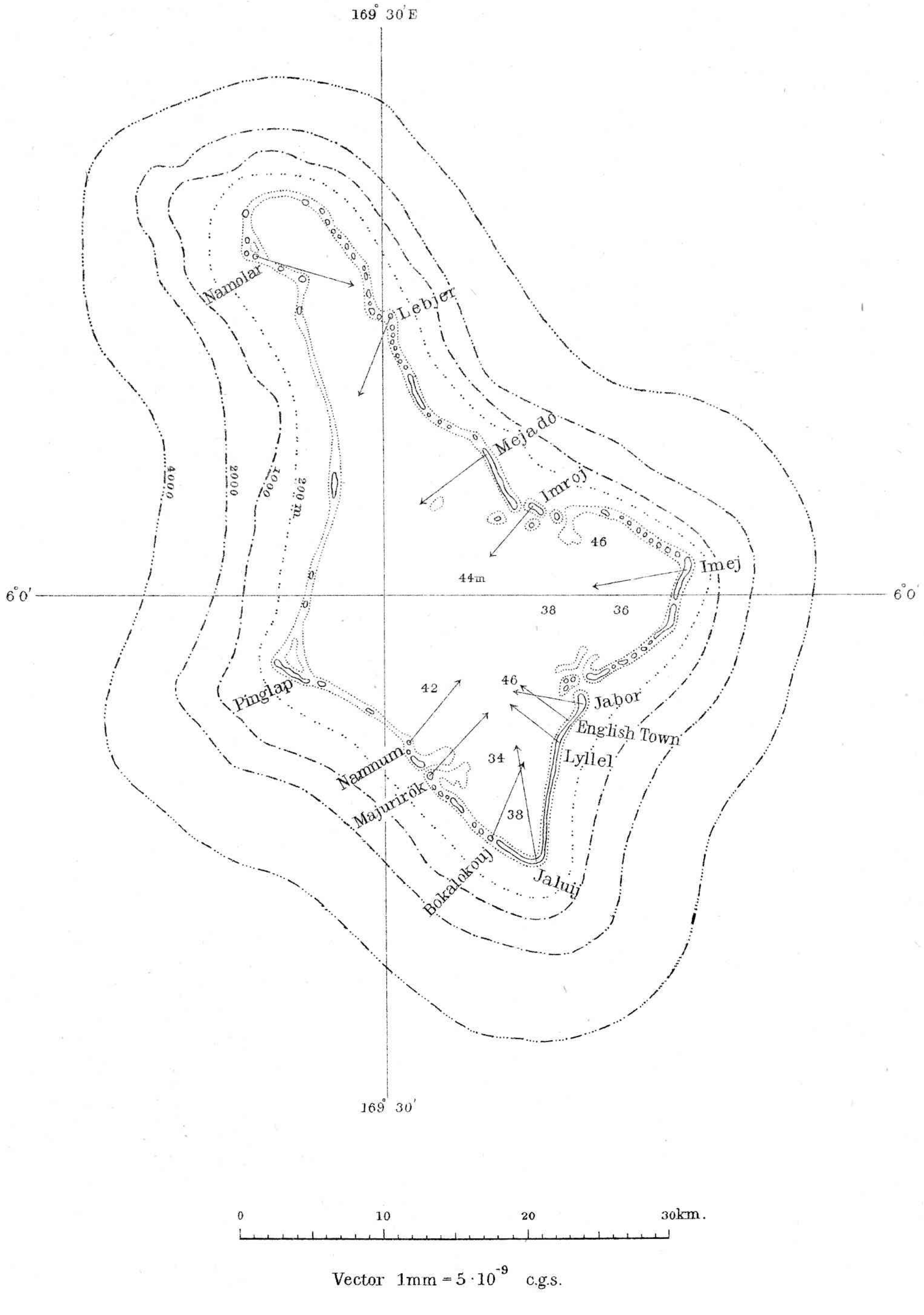
The Curvature-vectors in the Jaluit Atoll



Vector 1mm = 10·10⁻⁹ c.g.s.

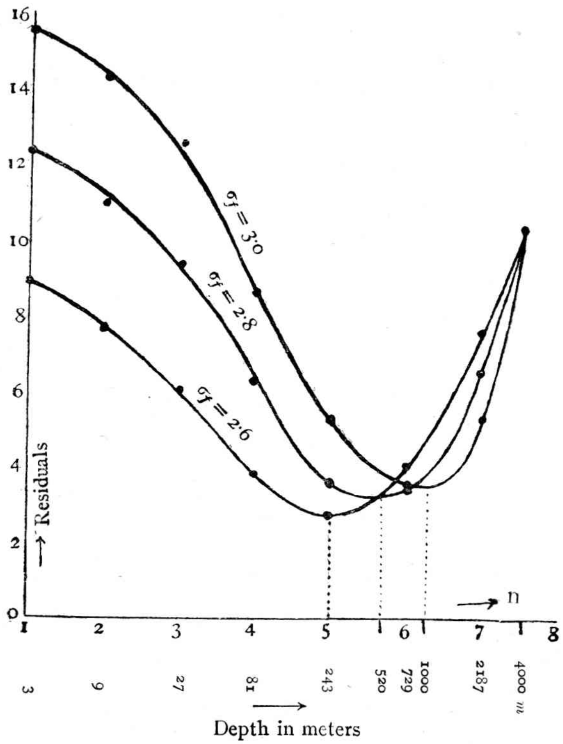
Plate IV.

The Gradients of Gravity in the
Jaluit Atoll



Vector 1mm = $5 \cdot 10^{-9}$ c.g.s.

Residuals
of
Curvature Terms



Residuals
of
Gradient Terms.