On the Rotation of Celestial Bodies.

By

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Although the rotation of celestial bodies or of celestial systems has hitherto very often been discussed, the amount of their angular momenta, strange to say, has not so far been made the subject of thorough investigation, except in the case of our solar system. The main cause of this may of course lie in the fact that good reliable parallaxes have always been rare. Since, as is well known, the angular momentum of an isolated body or of an isolated system must remain forever constant, any knowledge of its amount will always prove to be a valuable and important data for the discussion of the evolution of such a body or system.

Three years ago we calculated the masses and angular momenta of certain binary systems—11 visual binaries with known parallaxes, and 8 eclipsing spectroscopic binaries—and found the remarkable fact that not only the masses but also the angular momenta of these binary systems, were of about the same order of magnitude, the latter being indeed several hundred times greater than that of our solar system. Upon this fact we built our theory of the cause of the rotation of celestial bodies; and showed that the observed fact could be well accounted for, if we assumed the binary systems to have been built up of a large multitude of meteorites, each about the size of the asteroids in our solar system. So far was the result which we presented before the annual meeting of the Tôkyô Mathematico-Physical Society in 1915[1]. This was not, however, printed in full.

Later the number of binary systems with known parallaxes largely increased so that we could test our theory by a far greater multitude

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1 Proc. Tokyo Math.-Phys. Soc., **8**, 116 (1915).

of cases, and build it up on a broader and consequently firmer basis. Especially the recent publication by Adams of five hundred parallaxes determined spectroscopically provided us with a solid stand-point and so gave us an opportunity to take up the subject afresh.

The material at hand may be classified as follows:—

(A) Visual systems with known parallaxes.
  (a) Those for which the orbits are known.
  (b) Those for which sensible orbital motions have been observed.
  (c) Triple systems.
  (d) So-called wide pairs, of which the components are widely separated.

(B) Eclipsing spectroscopic binaries, for which both lines are observed.

These will be considered in turn.

(A) Visual Systems with Known Parallaxes.

In order to find out visual systems with known parallaxes, the first thing is to compare the catalogues of double stars with the list of known parallaxes.

For the former, we took

Burnham's General Catalogue of Double Stars, 1906,

Aitken's Catalogue of the Orbits of Visual Binary Stars, in Lick Obs. Bull. No. 84, 1905,

as our main sources, and supplemented them, when necessary, from

See—Evolution of Stellar Systems, vol. I, 1896,

Lohse—Doppelsterne, Potsdamer Publ. Nr. 58, 1908,

Aitken—Measures of Double Stars, Lick Obs. Publ. XII, 1914, and other sources.

For the latter, we consulted principally

Adams—The Luminosities and Parallaxes of Five Hundred Stars. First List. Ap. J. **46**, 1917,

as a reliable basis, and supplemented it, when necessary, from

Comstock—The Luminosity of the Fixed Stars, A.J. No. 597, 1907,

Flint—Results for Stellar Parallax, Washburn Obs., A.J. No. 631, 1912,

v. Maanen—Stellar Parallaxes, Mt. Wilson Obs., A.J. 723, 1917,

Miller—Stellar Parallaxes, Sproul Obs., Pop. Astr. **24**, 670, 1916, Mitchell—Stellar Parallaxes, Leander McCormick Obs., Pop. Astr., **25**, 23, 1917;

Elkins and others—Stellar Parallaxes, Yale Obs. Transactions.

We regret that we had not Kapteyn's compilation of measured parallaxes at hand, and so could not avail ourselves of it.

As may be easily seen from the formulae of calculation, the uncertainties of orbital elements have, generally speaking, only minor influence, in comparison with those arising from the uncertainties attached to the measured values of parallaxes. The significance of the result of calculation depends mainly on the selection of good parallaxes.

As to reliability of the values of parallaxes, measured by several observers, by several methods often totally different in principles of measurements, it is naturally very difficult to find appropriate criterion for assigning relative weights. Not entering into any detailed discussion, we, as a preliminary trial, arranged the parallaxes somewhat as follows :—

(i) Large parallaxes, for instance, those of α Centauri, Sirius, etc.,

(ii) Parallaxes derived from radial velocity observations, for instance, those of ε Hydrae, $\varkappa$ Pegasi, etc.,

(iii) Parallaxes determined spectroscopically by Adams's procedure,

(iv) Parallaxes determined relatively either by photographic, or by heliometric method,

(v) Parallaxes determined by meridian observations.

In case of (i) or (ii), we adopted the values as final ; for (iii) and (iv), we gave equal weights; and when (v) are the only values so far determined , we rejected them at all, as of inferior quality.

As the result of comparison of the catalogues of double stars with those of known parallaxes, we find that there are over 200 systems common to both. Only 56 of them have their orbits known, determined with more or less degree of certainty; and for these we are able to calculate their masses and angular momenta with ease, if we only assume the ratios of the masses of their components. These form of course the principal material of our present investigation, being the sub-class (a) above mentioned.

For the remainder, we have then, first of all, to discriminate those which form real physical systems, leaving aside those which are only optical pairs. As our preliminary criterion, we selected only those, for which the relative movements of the components were surely observable and yet were largely surpassed by the common proper motions of the systems. Evidently, these are very probably physical systems and should give at the same time some information concerning their masses and angular momenta; these belong to our sub-class (b).

The visual systems with known parallaxes, so far found out and taken into consideration, are shown, together with necessary data for calculation, in the following tables :—

TABLE I.

Visual Systems, for which the orbits are known.

No.	βGC	Name	α	δ	Orbital Elements P	Orbital Elements a	Orbital Elements e	Parallax Spk.	Parallax Trig.	Parallax Adopted	Remark
			h m		y	″					
1	12755	Σ 3062	0 1.0	+57°53′	104.61	1.371	0.450	0.038	0.036	0.037	
2	104	OΣ 4	0 11.5	+35 56	135.2	.53	.506		.115	—	
3	335	β 395	0 32.2	−25 19	25.0	.66	.171	.066		.066	
4	426	η Cassiop.	0 43.1	+57 17	345.6 507.6	10.10 12.21	.376 .522	.190 .166	.191	.190	
5	1070	γ Androm.BC	1 57.9	+41 51	55.0	.346	.82		.007	—	
6	1144	Σ 228	2 7.6	+47 1	123.1	.899	.309		.069	.069	
7	1471	20 Persei AB	2 47.4	+37 9	33.33	.16	.60	.040	.013	.027	triple
8	2109	o Eridani BC	4 10.7	− 7 49	180.0	4.791	.134	.200	.182	.191	triple
9	2134	55 Tauri	4 14.2	+16 17	88.9	.57	.625		.025	.025	
10	2381	β 883	4 45.7	+10 54	16.61	.19	.445	.033	.007	.020	C, optical companion
11	2383	β 552	4 46.2	+13 29	56.0	.528		.027	.007	.027	
12	3474	OΣ 149	6 30.2	+27 22	85.9	.55	.460		.048	.048	
13	3596	Sirius	6 40.8	−16 35	48.84	7.594	.588		.376	.376	
14	4122	α Gemin.	7 28.2	+32 6	346.82	5.756	.441	.100	.080	.090	
15	4187	Procyon	7 34.1	+ 5 29	39	4.05	.324	.331	.309	.320	
16	4310	9 Argus	7 47.2	−13 38	23.34	.69	.75		.035	—	
17	4414	β 581	7 58.4	+12 35	46.5	.53	.40		.082	.082	triple
18	4477	ζ Cancri AB	8 6.5	+17 57	59.11	.858	.381	.043	.033	.038	quadruple
19	4771	ε Hydrae AB	8 41.5	+ 6 47	15.3	.23	.65	.025*	.004	.025	quadruple
20	5005	Σ 3121	9 12.0	+29 0	34.00	.669	.33	.083	.067	.075	
21	5223	φ Urs. Maj.	9 45.3	+54 32	99.70	.32	.44		−.038	—	
22	5734	ξ Urs. Maj.	11 12.9	+32 6	60.0	2.508	.397	.158 .138	.158	.158	
23	5811	OΣ 235	11 26.7	+61 38	71.9	.78			.051	.051	
24	6243	γ Virginis	12 36.6	− 0 54	194.0	3.989	.897	.078	.068	.073	
25	6406	42 Comae	13 5.1	+18 3	25.56	.642	.461		.062	.062	
26	6578	β 612	13 34.7	+11 15	23.05	.225	.52		.26	—	
27	—	α Centauri	14 32.8	−60 25	81.185	17.71	.529		.759	.759	triple
28	7034	ξ Bootis	14 46.8	+19 31	148.46	4.988	.545	.152	.230	.191	
29	7251	η Coron. B.	15 19.1	+30 39	41.51	.891	.278	.069	.078	.074	
30	7259	μ_2 Bootis	15 20.7	+37 42	275.73	1.482	.601	.052	.055	.054	triple
31	7332	OΣ 298	15 32.5	+40 8	52.0	.799	.581		.046	.046	
32	7368	γ Coron. B.	15 38.5	+26 37	73.0	.736			.031	.031	
33	7487	ξ Scorpii	15 58.9	−11 6	44.70	.72	.75		.053	.053	triple
34	7563	σ Coron. B.	16 10.9	+34 7	370.0	3.82		.049	.031	.049	
35	7649	λ Ophiuchi	16 25.9	+ 2 12	134	1.0			.021	.021	
36	7717	ζ Herculis	16 37.5	+31 47	34.53	1.355	.457	.066	.118	.092	
37	7783	Σ 2107	16 47.9	+28 50	186.21	1.0	.387		.006	—	
38	7929	β 416	17 12.2	−34 53	41.47	1.86			.170	.170	
39	8099	26 Draconis	17 34.0	+61 57	197.3	1.905	.522	.076	.087	.081	
40	8162	μ Herculis BC	17 42.6	+27 47	43.23	1.30	.20	.091	.095	.093	triple
41	8340	70 Ophiuchi	18 0.4	+ 2 31	88.395	4.548	.500	.205	.187	.196	
42	8372	99 Herculis	18 3.2	+30 33	63.0 53.51	1.00 1.11	.76 .763	.105	.074	.105	
43	8679	A 88	18 33.2	− 3 17	12.12	.176	.273		.033	.033	
44	8933	β 648	18 53.3	+32 46	45.85	1.04	.305	.079	.116	.098	
45	8965	ζ Sagittarii	18 56.3	−30 1	21.17	.565	.185		.115	—	
46	9114	Secchi 2, BC	19 7.8	+38 37	58.0	.40	.50		.024	.024	triple
47	9605	δ Cygni	19 41.9	+44 53	376.66	2.39	.327		.049	.049	
48	10363	β Delphini	20 32.9	+14 15	26.79	.480	.350	.038	.017	.028	
49	10643	ε Equulei	20 54.1	+ 3 55	97.4	.61		.038	.048	.043	
50	10829	δ Equulei	21 9.6	+ 9 36	5.70	.27	.39	.070* .063	.067	.070	
51	10846	τ Cygni	21 10.8	+37 37	47.0 55.11	.91 1.08	.22 .34	.042	.031	.037	C, optical companion
52	11222	χ Pegasi	21 40.1	+25 11	11.37	.29	.40	.025* .066	.040	.025	
53	11761	Krüger 60	22 24.5	+57 12	46.0 54.9	2.49 2.86		.251	.256	.254	
54	12290	β 80	23 13.8	+ 4 52	95.2 156.0	.72 .955		.048	.016	.032	
55	12701	85 Pegasi AB	23 57.0	+26 33	25.70	.78	.43	.095	.099	.098	

* derived from radial velocity observations.

TABLE II.

Visual Systems, for which orbital motions have been observed.

No.	βGC	Name	α	δ	Orbital Motion: Date	Orbital Motion: r	Orbital Motion: θ	Parallax: Spk.	Parallax: Trig.	Parallax: Adopted	Remark
			h m			″	°		″	″	
56	—	Gr. 34	0 12.7	+43°27′	1864.35 1908.85	39.86 38.65	53.0 57.0		0.270	0.270	
57	1393	θ Persei	2 37.4	+48 48	1850.57 1896.43	15,94 17.18	296.2 299,5	.100	.088	.094	
58	1854	Σ 443	3 40 2	+41 9	1849.74 1892.66	8.93 8.53	45.2 48.2		.055	.055	
59	3239	η Gemini	6 8.8	+22 32	1884.71 1899.44	0.93 1.09	295.6 291.4	.041	.005	.023	
60	4815	Fed. 1384	8 46.0	+71 11	1870.57 1903.96	6.29 4.87	39.0 44.6	.113	.086	.099	
61	4866	ι Urs. Maj. A—(BC) B—C	8 52.4	+48 26	1848.48 1881.35 1904.30 1903.38 1905.32	10.62 9.59 8.31 0.93 0.68	351.8 357.0 359.4 203.3 195.4		.09	.09	triple
62	4972	Lal. 18115	9 7.6	+53 7	1840.62 1898.14	20.1 19.2	50.4 63.9	.160	.152	.156	
63	5331	α Leonis BC A—(BC)	10 3.0	+12 27	1878.68 1904.21	3.27 2.46 176.7	87.5 82.7	.048		.048	triple
64	5652	α Urs. Maj. A—B	10 57.6	+62 17	1889.99 1895.78 1898.77	0.86 0.85 0.57	321.0 303.5 284.0		.05	.05	
65	5706	Fed. 1831 AC	11 8.7	+74 1	1862.23 1902.27	7,99 7.33	296,1 301.5	.072	.088	.080	B, optical companion
66	5812	88 Leonis	11 26.6	+14 55	1857.81 1894.82	15,27 15.49	322.1 324.2	.055		.055	
67	5858	Urs. Maj. 290	11 33.5	+45 40	1844.52 1900.24	10.43 9.97	265.5 259.7	.030	.038	.034	
68	6442	Lal. 24652	13 11.9	+17 35	1899.78 1906.84	2.66 3.14	114.9 111.4	.087		.087	
69	6630	τ Bootis	13 42.5	+17 57	1873.10 1898.45	8,88 8.51	350.5 355.6	.052	.040	.046	
70	6993	ε Bootis	14 40.6	+27 30	1843.61 1900.48	2.73 2.90	322.3 329.3	.021	.050	.021	
71	7060	Pi 14.212	14 51.6	−20 58	1878.05 1904.28	15.11 17.39	290.4 295.0	.174	.174	.174	
72	7318	δ Serpentis	15 30.0	+10 52	1836.81 1870.33 1904.47	2,73 3.18 3.57	196.6 191.3 185.9		.018	.018	
73	7631	α Scorpii	16 23.3	−26 13	1850.50 1874.93 1895.58	3.03 3.05 3.06	273.2 271.4 271.9	.014	.030	.022	
74	7905	36 A Oph. 30 Scorpii	17 9.2 17 10.1	−26 27 −26 24	1841.00 1904.52	4.80 4.29 732	219.6 190.0	.166	.152	.166	triple
75	7914	α Herculis	17 10.1	+14 30	1839.23 1901.47	4.68 4.69	118.6 114.6	.015		.015	
76	8798	Pos. Med. 2164	18 41.7	+59 29	1838.54 1886.62 1905.05	12.70 16.86 17.04	135.9 147.4 151.1	.229	.2 0	.260	
77	9137	Σ 2486	19 9.5	+49 40	1876.79 1902 37	9.86 9.42	221.0 217.6	.057	.045	.051	
78	9485	θ Cygni	19 33.8	+49 59	1890.88 1898.55	3,71 3.54	45.5 48.1	.066	.073	.069	
79	9560	16 Cygni	19 39.2	+50 17	1844.07 1895.96	37.48 38.16	136.0 135.0	.047		.047	
80	9602	Σ 2576	19 41.0	+33 20	1841.81 1882.19 1904.36	3.50 3.07 2.45	315.1 302.1 291.5		.042	.042	
81	9617	χ Cygni	19 42.6	+33 30	1856.46 18 8.92	25.71 25.57	72.7 71.1		.068	.068	
82	9602 9617					786				.055	quadruple?
83	9724	β Aquilae	19 50.4	+ 6 9	1874.74 1902.04	11.91 12.30	17.0 14.0	.072	.071	.072	
84	10509	γ Delphini	20 42.0	+15 46	1841.37 1894.94	11.83 11.16	273.8 2,0.6	.022	.076	.049	
85	10732	61 Cygni	21 2.4	+38 15	1831.20 1865.20 1904.00	15.62 18.50 22.41	90.8 110.8 126.7	.295	.311	.303	
86	11125	24 Aquarii	21 34.4	− 0 30	1892.49 1898.33 1903.51	0.50 0.60 0.53	259.0 267.2 275.1	.048	.017	.033	

No.	βGC	Name	α	δ	Orbital Motion			Parallax			Remark
					Date	r	θ	Spk.	Trig.	Adopted	
87	11214	μ Cygni	21 39.7	+28 17	1836.86 1904.20	5.51 2.32	114.4 124.3		.046	.046	
88	11514	Boss 5682	22 1.8	+82 23	1859 99 1905.21	13.70 13.67	75.9 73.1	.022	.007	.015	
89	11690	Boss 5772	22 18.8	+20 21	Probably (P=420,	a=2.5,	i=90)	.048	.060	054	
90	11716	Boss 5786	22 21.5	+ 3 53	1877.13 1898.67	2.62 2.96	221.1 218.1	.046	.067	.056	
91	11957	ξ Pegasi	22 41.7	+11 40	1873.09 1898.05	12.05 12.31	115.2 110.1		.09	.09	
92	12608	Boss 6129	23 47.5	+74 59	1884.69 1904.70	5.48 5.71	65.9 71.9	.132		.132	
93	12740	OΣ 547 AB (AB)—C	23 59.2	+45 9	1877.39 1002.11	4.36 4.59 328	112.0 131.2		.098	.098	triple

TABLE III.

Visual Multiple Systems.

No.	Name	Magn.	Orbital Motion			Adopted Parallax
			Date	r	θ	
				″	°	″
17	β 581	8.7, 8.7, 10.5	1878·18	4.76	184.8	0·082
	(AB)—C		1905·11	4·69	196·5	
18	ζ Cancri	5.6, 6.3, 6.0, —				·038
	(AB)—(CD)		(P=677·3, a=5″.49)			
	C—D		(P=17.43, a=0·158)			
19	ε Hydrae	3.7, 5.2, 7.5, 12.5				·025
	(AB)—C		(P=650, a=3·22)			
	(AB)—D			19·76		
33	ξ Scorpii		1832·41	6·93	76·4	
	(AB)—C		1871·92	7·09	69·5	·053
			1908·00	7·42	62·5	
40	μ Herculis	3·5, 10.0, 10.5	1850·69	30·74	242·8	·093
	A—(BC)		1895·53	32·02	244·2	
46	Secchi 2	8·0, 8·7, 8·7	1860·34	3·98	229·3	·024
	A—(BC)		1902·98	4·13	219·9	
7	20 Persei, (AB)—C	5.6, 6.7, 10.0		14·00		·027
8	o Eridani A—(BC)	4.5, 9.4, 10.8		82·26		·191
27	α Centauri	0.0, 1.5, 11.0		2° 12′		·759
				″		
30	μ Bootis, A—(BC)	4·5, 7.2, 7.8		108·29		·054
49	ε Equulei, (AB)—C	5.8, 6.3, 7·1		10·74		·038
61	ι Urs. Maj.	3.1, 9.5, 9.8	given in Table II			·09
63	α Leonis	1.5, 8.4, —				·048
74	36 A Ophiuchi 30 Scorpii	5·4, 5.4, 7.0		732″		·166
82	Σ 2576 χ Cygni	7.8, 7.8, 5 1, 8.1		786		·055
93	oΣ 547, (AB)—C	8.3, 8.3, 9.5		328		·098

Formulae of Calculation for Visual Systems.

Sub-class (a).

If we take the distance of the sun from the earth, the mass of the sun and the year as our units of length, mass and time, and put

$$\left.\begin{array}{ll} P & = \text{period of revolution, in years} \\ a & = \text{semi-major axis, in arc} \end{array}\right\} \text{of the relative orbit,}$$

e = eccentricity,
p = parallax,
m_1, m_2 = masses of the components,
M = $m_1 + m_2$ = mass of the system,
α = $\frac{m_2}{m_1}$ = ratio of the component masses,
H = angular momentum of the system,

then we have, as can be easily verified, for the mass and angular momentum of a double star system

$$M = \frac{\left(\frac{a}{p}\right)^3}{P^2}, \tag{1}$$

$$H = 2\pi \frac{\left(\frac{a}{p}\right)^5}{P^3} \sqrt{1-e^2} \cdot \frac{\alpha}{(1+\alpha)^2} + \text{terms due to the rotation of both components} \tag{2}$$

Since the second part in the expression of H is usually insensible compared with the first part, we have not taken it into account in our calculation.

The values of α, the ratio of the masses of two components, are determined in some cases, as for instance, in the table IV.

The values of α, however, are not always well determined, since they result from absolute measurements, which are usually not so accurate as relative measurements. As may be seen from the table, the values range usually between 1 and 0.5, and such a slight variation has only minor influences on the values of H. The variation of $\frac{\alpha}{(1+\alpha)^2}$ according to the variation of α is shown in the annexed diagram. For these two reasons, we have put $\alpha = 1$ throughout, for all systems in our calculation.

Sub-class (b).

For the visual systems of the sub-class (b), where the orbits are

TABLE IV.

Visual Systems for which α is determined.

No.	Star	α
4	η Cassiop.	0·76
13	Sirius	·393
14	α Gemin.	{6 (Curtiss) / 1 (Boss)
15	Procyon	·33
22	ξ Urs. Maj.	1·09
24	γ Virginis	1·1
27	α Centauri	·85
28	ξ Bootis	·87
34	σ Coron. B.	·47
36	ζ Herculis	·43
41	70 Ophiuchi	·82
55	85 Pegasi	1·8

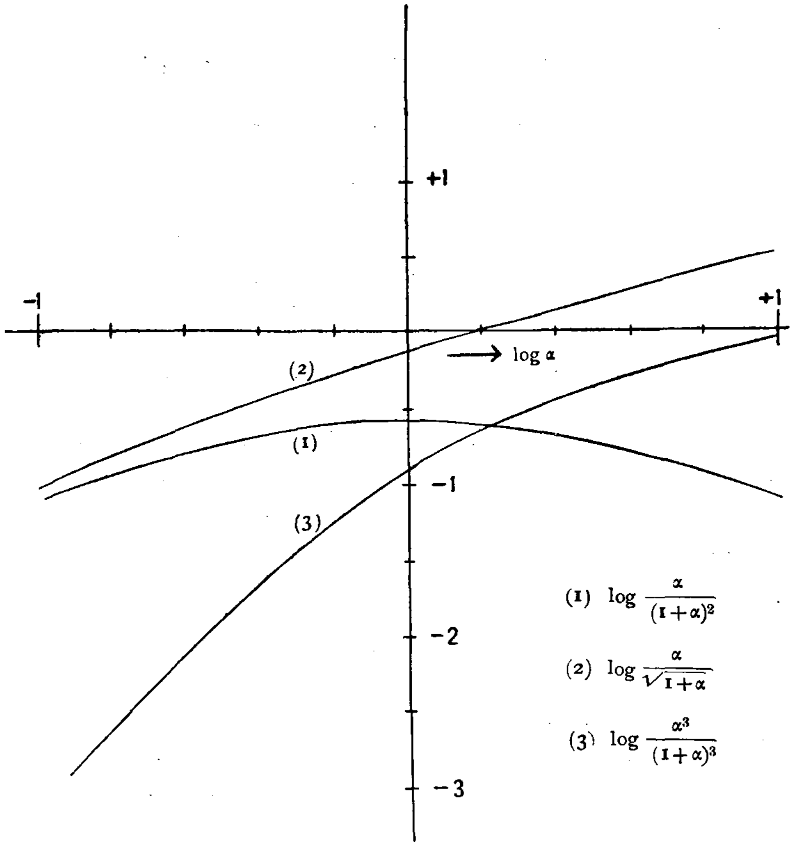
not known, and hence the masses and angular momenta of the individual systems are not to be calculated by the ordinary methods, we proceeded to estimate the probable values by the following considerations.

From the relation

$$h^2 = GMa\sqrt{1-e^2},$$

where h denotes twice the areal velocity of the relative orbital motion and G stands for the constant of gravitation, we obtain easily

$$M = \frac{1}{4\pi^2}\frac{\left(r_1 r_2 \frac{d\theta}{dt}\right)^2}{ap^3} \times \frac{1}{\sqrt{1-e^2}} \times \frac{1}{(\cos i)^2},$$



$$H=\sqrt{GM^3a(1-e^2)}\times\frac{a}{(1+a)^2}$$

$$=\frac{1}{4\pi^2}\cdot\frac{\left(r_1r_2\frac{d\theta}{dt}\right)^3}{ap^5}\cdot\frac{1}{1-e^2}\cdot\frac{a}{(1+a)^2}\cdot\frac{1}{(\cos i)^3},$$

where r denotes the distance, and θ the position angle of the components, and i the inclination of the orbital plane.

Since the values of a, e and i are not known, we replaced them by probable values, using for a the relation between a and the mean value of r,

$$r=r'\cos j,$$

$$\text{time mean of } r'=\frac{1}{T}\int r'\frac{dt}{d\theta}d\theta=\frac{1}{hT}\int r'^3d\theta$$

$$=\frac{2}{hT}a^3(1-e^2)^3\int_0^\pi\frac{d\theta}{(1+e\cos\theta)^3}=\left(1+\frac{e^2}{2}\right)a,$$

and putting mean values for e, i and j, under the supposition e to vary from zero to one, i and j to vary from 0° to 90°, we have

$$M=\frac{3}{4\pi^2}\frac{\left(r_1r_2\frac{d\theta}{dt}\right)^2}{rp^3}\times c_1, \tag{3}$$

$$H=\frac{9}{32\pi^2}\frac{\left(r_1r_2\frac{d\theta}{dt}\right)^3}{rp^5}\times c_2. \tag{4}$$

We added the factors c_1 and c_2 as reserve, since the probable ranges of e, i and j might not be such as we assumed; and determined them by applying these formulae to the case of sub-class (*a*), where the apparent orbits also are known, and comparing the values so obtained with those calculated by the formulae (1) and (2). As the result of comparison in 25 cases, we obtained

$$\left.\begin{aligned}c_1&=1{\cdot}5\overset{\times}{\div}1{\cdot}5,\\ c_2&=1{\cdot}9\overset{\times}{\div}1{\cdot}9\end{aligned}\right. \tag{5}$$

These formulae we used for sub-class (*b*), and also for the case of multiple systems, sub-class (*c*), in combination with (1) and (2).

The uncertainties thus introduced into the results in cases (*b*) and (*c*) are not so large, as is to be expected at the first sight, compared with those arising from other sources. The uncertainties arising from inferior quality of parallaxes are always pretty large, and affect all the sub-classes without exception.

Sub-class (d).

In case where the relative motion of the components is either not known or very small, and the relative position of the components is the only available data, it is evidently too bold to attempt any estimate of the amount of the angular momentum. The following procedure, however, may serve to give some idea of the magnitude even in such cases.

We assume first the value of α, which may in most cases be reasonably derived from a consideration of their magnitudes and colors, and put

$$M = m_1(1+\alpha). \tag{6}$$

Then, since

$$H = \sqrt{GM^3 a(1-e^2)} \times \frac{\alpha}{(1+\alpha)^2},$$

we have, proceeding just as in the previous case,

$$H = 2\pi \frac{\alpha}{\sqrt{1+\alpha}} \cdot \frac{\sqrt{1-e^2}}{\sqrt{1+\frac{e^2}{2}}} \times m_1^{\frac{3}{2}} r^{\frac{1}{2}},$$

and then, putting

$$e = 0{\cdot}5,$$

as seems to be appropriate in the case of wide pairs, we obtain finally

$$H = [1{\cdot}0113] \times m_1^{\frac{3}{2}} r^{\frac{1}{2}} \times \frac{\alpha}{\sqrt{1+\alpha}}. \tag{7}$$

The values of $\frac{\alpha}{\sqrt{1+\alpha}}$ are shown graphically in the diagram.

The formulae, (6) and (7), we used for the case of wide pairs, sub-class (*d*).

(B) Eclipsing Spectroscopic Binaries.

For spectroscopic binaries with known elements, we have

$$M = \frac{(a \sin i)^3}{P^2} \div \left\{\sin^3 i \times \frac{\alpha^3}{(1+\alpha)^3}\right\}, \tag{8}$$

$$H = 2\pi M^{\frac{5}{3}} P^{\frac{1}{3}} \sqrt{1-e^2} \cdot \frac{\alpha}{(1+\alpha)^2}, \tag{9}$$

where ($a \sin i$) and P are known from observations. Hence, if α and i also be known, we would be able to calculate M and H. Now, if a spectroscopic binary be an eclipsing variable at the same time, we could find i, and, if both the lines of a spectroscopic binary be observable, we could find α.

Comparing

Campbell—Second Catalogue of Spectroscopic Binaries, Lick Obs. Bull., **181,** 1910,

with

Shapley—A Study of the Orbits of Eclipsing Variables, Contrib. Princeton Univ. Obs. No. 3, 1915,

we find 19 systems common to both; and, if we confine ourselves to those for which both lines are observable, we obtain at last ten systems left for our purpose. Since the expression $\frac{\alpha^3}{(1+\alpha)^3}$ varies largely with α, we dare not assume the value of α when it is unknown. The values of $\frac{\alpha^3}{(1+\alpha)^3}$ are shown in the diagram.

The material with necessary data for calculation is given in the following table.

TABLE V.

Eclipsing Spectroscopic Binaries.

No.	Shapley	H.R.	Name	α	δ	P day	$a \sin i$ 10^6.km	e	i	α
94	8	815	RZ Cassiop.	h m 2 39·9	+69°13′	1·195	1·170			·55
95	11	936	β Persei	3 1·7	+40 34	2·867	1·641	0·039		·5
96	—	1458	*d* Tauri	4 30·2	+ 9 57	3·571	3·748	0·000		·47
97	22	2088	β Aurigae	5 52·2	+44 56	3·960	11·899*	0·005	77°	·99
98	28	3129	V Puppis	7 55·3	−48 58	1·455	12·200*			1
99	—	5056	α Virgo	13 19·9	−10 38	4·014	6·930	0·10		·51
100	49	5586	δ Librae	14 55·6	− 8 7	2·328	2·450	0·054	81·5	.70
101	53	6431	u Herculis	17 13·6	+33 12	2·051	2.800	0·053		·39
102	61	—	RX Herculis	18 26·0	+12 32	1·779	2·590	0·00		1
103	66	7106	β Lyrae	18 46·4	+33 15	12·91	34·339	0·11		2·2

* $(a_1+a_2) \sin i$.

The Result of Calculation.

The results obtained according to the foregoing formulae are put together in the following tables.

Table VI.

Masses and Angular Momenta of Visual Double Stars. (*a*)

No.	Name	Magn.	Spk.	Adopted Parallax	M	H
1	Σ 3062	6·5, 7·5	G2, G8	0″.037	4·7	85·6
3	β 395	7 8, 8·5	G6	·066	1·6	10·0
4	η Cassiop.	3·6, 7·9	G0, K0	·190	{ 1·3 1·0	{ 16·1 13·2
6	Σ 228	6·7, 7·6		·069	·15	·30
9	55 Tauri	7·5, 9·3		·025	1·5	10·7
10	β 883	7·9, 7·9	F8	·020	3·3	23·8
11	β 552	7·0, 10·0	F6	·027	2·4	25·6
12	OΣ 149	6·9, 9.4		·048	·20	·44
13	Sirius	−1·6, 9·0	A	·376	3·5	36·6
14	α Gemin.	2·0, 2·8	A	·090	2·2	36·2
15	Procyon	0·5, 13·5	F4	·320	1·3	8·2
20	Σ 3121	7·6, 7·9	K4	·075	·61	2·1
22	ξ Urs. Maj.	4·4, 4·9	F9, G1	·158	1·1	6.8
23	OΣ 235	5·9, 7·2	F	·051	·69	2·9
24	γ Virginis	3·6, 3·7	A8	·073	4·3	46·3
25	42 Comae	5·2, 5·2		·062	1·7	10·0
28	ξ Bootis	4·8, 6·7	G6, K3	·191	·8	4·9
29	η Coron. B.	5·6, 6·1	G0	·074	1·0	5·3
31	OΣ 298	7·4, 7·7		·046	1·9	14·3
32	γ Coron. B.	7·1, 7·6	A	·031	2·5	24·8
34	σ Coron. B.	5·8, 6·8	G1, F9	·049	3·5	74·4
35	λ Ophiuchi	4·0, 6·1	A	·021	6·0	133·1
36	ζ Herculis	3·0, 6·5	G1	·092	2·7	23·5
38	β 416	6·0, 8·0		·170	·8	2·8
39	26 Draconis	5·3, 10·0	G0	·081	·33	1·3
41	70 Ophiuchi	4·1, 6·1	G9, K7	·196	1·6	13·3
42	99 Herculis	5·2, 10·5	F6, —	·105	{ ·22 ·41	{ ·32 ·9
43	A 88	7·2, 7·2		·033	1·0	3·7
44	β 648	5·2, 8·7	F9	·098	·57	2·1
47	δ Cygni	3·0, 7.9	A	·049	·8	7·7
48	β Delphini	4·0, 5·0	F3	.028	7·0	113
50	δ Equulei	5·3, 5·4	F5	·070	1·6	6·7
51	τ Cygni	3·8, 8·0	F1	·037	{ 6·7 8·2	{ 131 191
53	Krüger 60	9·6, 11·3	Mb	.254	·46	1·3
54	β 80	8·3, 9·3	G9	·032	1.2	8·5
55	85 Pegasi	5·8, 11·0	G1	·098	.8	2·7

TABLE VII.

Masses and Angular Momenta of Visual Double Stars. (*b*)

No.	Name	Magn.	Spk.	Adopted Parallax	M	H
		m m				
56	Gr. 34	7·7, 10·7	Ma	0″·270	0 9	13·3
57	θ Persei	4·2, 10·0	F7	·094	1·0	17·0
58	Σ 443	8·2, 8·8		·055	·64	9·0
59	η Gemin.	3, 8·8	G6	·023	·23	1·0
60	Fed. 1384	7·5, 7·6	K6, K5	·099	·18	·8
62	Lal 18115	7·4, 7·4	K7, K5	·156	4·0	12·1
64	α Urs. Maj.	2·0, 11·1	K	·05	2·2	18·6
65	Fed. 1831 AC	7·0, 10·2	K5	·080	·55	5·4
66	88 Leonis	6·4, 8·4	F9	·055	2·5	88·3
67	Urs. Maj. 290*	5·9, 8·0	F9	.034	10·2	774
68	Lal. 24652	7·1, 10·2	K1	·087	·29	1·2
69	τ Bootis*	4·8, 11·4	F6	·046	8·1	446
70	ε Bootis	3·0, 6·3	G8	·021	1·2	22·0
71	Pi. $14^h.212$	7, 8	K6	·174	2·0	34·5
72	δ Serpentis	3·0, 4·0		·018	4·7	184
73	α Scorpii	1, 7·1	G2	·022	·56	6·8
75	α Herculis	3·0, 6·1	G5	·015	4·7	254
76	Pos. Med. 2164	8·2, 8·7	Mb	·260	·35	2·3
77	Σ 2486	6·0, 6·5	G3, G2	·051	4·2	160
78	θ Cygni	5·0, 14·3	F4	·069	·58	4·4
79	16 Cygni*	5·1, 5·3	G1, G3	·047	6·7	682
83	β Aquilae	3·4, 11·3	G7	·072	1·9	47·9
84	γ Delphini*	4·0, 5·0	K1	·049	16·0	427
85	61 Cygni	5·3, 5·9	K7, K7	·303	2·0	29·3
86	24 Aquarii	6·5, 6·9	F6	·033	·34	1·1
87	μ Cygni	4·0, 5·0		·046	·35	2·5
88	Boss 5682*	6·2, 7·0	F7	·015	98	$4·0 \times 10^4$
89	Boss 5772	6·0, 9·2	F5	·054	·56	3·8
90	Boss 5786*	6·0, 12·5	F6	·056	·08	·23
91	ξ Pegasi	5, 18		·09	3·6	109
92	Boss 6129	6·8, 11·7	K3	·132	.24	1·0

* Probably the adopted parallaxes are largely in error. The corresponding values of M and H are consequently rejected in forming the statistical table X.

Table VIII.

Masses and Angular Momenta of Visual Multiple Systems. (*c*)

No.	Name	Magn.	Spk.	Adopted Parallax	Assumed α	M		H
7	20 Persei	5·6, 6·7, 10·0	F2	0″·027		AB	0.19	0·25
					0·5	ABC	·29	7·7
8	O Eridani	4·5, 9·4, 10.8	Ko	·191		BC	·49	2·7
					2	ABC	1·5	84·9
17	β 581	8·7, 8·7, 10·5		·082		AB	·13	·16
						ABC	1·2	14·5
18	ζ Cancri	5·6, 6·3, 6·0, —	F9, Go	·038		AB	3·3	41·3
						CD	·24	·37
						ABCD	6·6	318
19	ε Hydrae	3·7, 5·2, 7·5, 12·5	Go, F6	·025		AB	3·3	21·6
						ABC	5·1	203
					·2	ABD	4·0	320
27	α Centauri	0·0, 1·5, 11·0	Go	·759		AB	1·9	17·2
					·1	ABC	2·1	267
30	μ Bootis	4·5, 7·2, 7·8	Fo, Go	·054		BC	·27	·94
					2	ABC	·81	74·5
33	ξ Scorpii	4·8, 5·1, 7·2		·053		AB	1·3	5·4
						ABC	2·9	77·2
40	μ Herculis	3·5, 10·0, 10·5	G4, Mb	·093		BC	1·5	10.2
						ABC	1·3	37·6
46	Secchi 2	8·0, 8·7, 8·7		·024		BC	1·4	9·0
						ABC	6·5	417
61	ι Urs. Maj.	3·1, 9·5, 9·8	A5	·09		BC	·40	1·2
						ABC	·67	9·3
63	α Leonis	1·5, 8·4, —	B8, Ko	·048		BC	·26	1·4
					5	ABC	1·5	20·9
74	36 A Ophiuchi / 30 Scorpii	5·4, 5·4, 7·0	K1, —	·166		AB	·17	.50
					·5	ABC	·26	1·95
82	Σ 2576 / χ Cygni	7·8, 7·8, 5·1, 8·1		·055		AB	·9	8·4
						CD	4·2	364
						ABCD	5·1	$2·0 \times 10^3$
93	OΣ 547	8·3, 8·3, 9·5		·098		AB	1·9	25·9
					·5	ABC	2·9	645

Table IX.

Eclipsing Spectroscopic Binaries.

No.	Name		Magn.	Spk.	Density ⊙=1	M	H
94	Rz	Cassiop.	6·4—7·6	A	0·21	1·00	0·21
95	β	Persei	2·2—3·4	B8	·076	.66	.14
96	d	Tauri	4.4	A2	—	5·0	4·24
97	β	Aurigae	2·1—2·2	Ap	·12	4·8	4·75
100	δ	Librae	4·8—5·7	A	·035	1·50	·55
102	Rx	Herculis	7·0—7.5	A	·30	1·76	·68
98	V	Puppis	4·1—4·8	B1p	.056	34·3	90 0
99	α	Virgo	1·2	B2	—	15·4	31·3
101	u	Herculis	4·6—5·3	B3	·058	9·4	9·5
103	β	Lyrae	3·4—4·3	B2p	{ ·0043 ·0002	30·6	132

Taking geometrical means of each kind separately, we obtain the mean values and the probable dispersion ranges as follows:—

Table X.

The Mean Values and the Probable Dispersion Ranges of the Masses and Angular Momenta.

	No.	M	H
Visual doubles with known orbits (a)	36	$1{\cdot}4 \overset{\times}{\div} 1{\cdot}6$	$9 \overset{\times}{\div} 2{\cdot}2$
Visual doubles with known orbital motion (b)	25	$1{\cdot}0 \overset{\times}{\div} 1{\cdot}6$	$11 \overset{\times}{\div} 2{\cdot}2$
Visual multiple systems (c)	15	$1{\cdot}7 \overset{\times}{\div} 1{\cdot}7$	$83 \overset{\times}{\div} 2{\cdot}2$
Eclipsing Spectroscopic binaries			
type A	6	$1{\cdot}9 \overset{\times}{\div} 1{\cdot}4$	$0{\cdot}8 \overset{\times}{\div} 1{\cdot}8$
type B	4	$20 \overset{\times}{\div} 1{\cdot}2$	$43 \overset{\times}{\div} 2$
Our solar system	—	1.0	0·022

From these values we may conclude that

(i) The masses and angular momenta of star-systems are, on the whole, of the same order of magnitude, thus confirming the result obtained by us three years ago,

(ii) The multiple systems have somewhat greater angular momenta, the masses remaining about the same,

(iii) Our solar system has an angular momentum, over hundred times less,

(iv) For spectroscopic binaries, the angular momenta are comparatively less than for those of visual systems, the masses, however, being considerably greater.

We may here add, for sake of reference, a result obtained elsewhere, regarding the direction of the orbital planes of visual systems. The distribution of the orbital planes of visual systems has been investigated by See[1], Bohlin[2], Poor[3] and others. Although the conclusions arrived at by these investigators are diversified, yet we would not be much in error if we summarized them as follows :—

(v) There is no positive evidence that the distribution of the orbital planes has any regularity.

That the masses of celestial bodies are, on the whole, of about the same order of magnitude, has been noticed by many investigators. This fact has also been theoretically accounted for by Jeans[4] and Eddington[5], the former finding the cause in the phenomenon of rotation, the latter, on the contrary, in the phenomenon of radiation pressure. Which is the proper explanation of the fact, is an interesting question, requiring further consideration.

Cosmogonical Considerations concerning the Origin of Celestial Rotation.

The observed facts so far stated in the foregoing could surely not be a product of mere accidence. That almost all star-systems so far investigated have, broadly speaking, about the same amount of angular momentum, requires a sufficient reason to account for it; and indeed, it seems to us, the search for the appropriate cause leads directly to

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1 Evolution of Stellar Systems. **1.** 1896.
2 A. N. **176**, 196, 1907.
3 A. J. **23**, 145, 1914.
4 Jeans, Monthly Notices R. A. S., **77**, 186 (1917).
5 Eddington, do. **77**, 16 (1916).

the very question of cosmogony,—How was our stellar universe created, and how has it evolved to its present state?

The theories hitherto proposed to account for the origin of celestial rotation seem always to have been merely qualitative. Chamberlin and Moulton[1] attribute it to near approaches of celestial bodies, which might eventually take place during their translational motion through space. Jeans[2] attributes it to the tidal action between the celestial bodies, which might have been large enough at an early stage of evolution of these bodies. These two theories have therefore one thing in common, that is, they look at the rotation of celestial bodies as transformed from their translational motion through space. We do not know whether these theories are able to account for the observed facts quantitatively.

See[3] assumes the primordial forms of celestial bodies to have been large swarms of meteorites, immense in number. According to him, it must have been rather rare that the condensation of these swarms took place in just such a manner as to cause no rotation; on the contrary, rotation, in one sense or other, would enevitably follow as a consequence of the condensation of these meteoric swarms. The idea seems to us quite right in principle; he has not, however, given any quantitative account of it.

Theoretical Calculation.

Let us begin by considering the following problem:— A meteoric swarm of immense multitude is assumed to have a spherical symmetry, the density of meteoric distribution and the "mean square" velocity of individual meteorites being functions of the distance from the center. The size of all meteorites is assumed to be the same, and the velocity distribution at any point to follow Maxwell's law of velocity distribution for gaseous molecules. Let us call such a swarm for later reference a primordial swarm. It is required to find the probable amount of angular momentum of such a primordial swarm.

Let the probability that a meteorite taken at random should lie just in an elementary volume $dxdydz$ at (x, y, z) be expressed by

$$\rho(x, y, z)\, dxdydz,$$

1 Moulton, Ap. J. **22.** 165–181, 1905.

2 Jeans, Ap. J. **22,** 102, 1905.

3 See, Evolution of Stellar Systems, **2,** 1910.

so that
$$\int \rho\,(x, y, z)\,dxdydz = 1, \qquad (10)$$

in which the integration is to be extended over all the space within the swarm. Further, let the probability that the velocity-components of a meteorite at (x, y, z) should lie between u and $u+du$, v and $v+dv$, w and $w+dw$, be expressed by

$$p\,(u, v, w, x, y, z)\,dudvdw,$$

so that
$$\int p\,(u, v, w, x, y, z)\,dudvdw = 1, \qquad (11)$$

the integration being taken to extend over all the values of u, v, w which are possible at (x, y, z).

The functions ρ and p should satisfy, besides the conditions (10) and (11), the so-called equation of continuity, which may be written in the form

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \bar{u}}{\partial x} + \frac{\partial \rho \bar{v}}{\partial y} + \frac{\partial \rho \bar{w}}{\partial z} = 0, \qquad (12)$$

where we put

$$\int u\,p\,dudvdw = \bar{u},$$
$$\int v\,p\,dudvdw = \bar{v},$$
$$\int w\,p\,dudvdw = \bar{w}.$$

Since we assume the swarm to be in a stationary state, we have

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{and} \quad \frac{\partial p}{\partial t} = 0,$$

and hence the equation (12) will be satisfied, if

$$\bar{u} = \bar{v} = \bar{w} = 0,$$

which is the case when the swarm has a spherical symmetry, and accordingly p is an even function with respect to u, v and w.

Let now

m = the mass of a single meteorite,

n = the number of meteorites in the swarm,

$M = nm$ = the total mass of the swarm,

H = the resultant angular momentum of the swarm;

and put further

$$\left.\begin{aligned}&\int(vz-wy)\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\nu_1,\\&\int(wx-uz)\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\nu_2,\\&\int(uy-vx)\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\nu_3,\end{aligned}\right\}\quad(13)$$

$$\left.\begin{aligned}&\int(vz-wy)^2\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\mu_1^2,\\&\int(wx-uz)^2\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\mu_2^2,\\&\int(uy-vx)^2\cdot p(u,v,w,x,y,z)\cdot\rho(x,y,z)\,dudvdwdxdydz=\mu_3^2,\end{aligned}\right\}\quad(14)$$

In case of spherical symmetry, we shall have obviously

$$\nu_1=\nu_2=\nu_3=0,$$

and

$$\mu_1^2=\mu_2^2=\mu_3^2=\frac{2}{3}\int\lambda^2(r)\,.\,r^2.\,\rho(r)\,dxdydz=c^2k^2,$$

where

λ = the "mean square" velocity at (x, y, z),
c = a kind of "mean square" velocity taken throughout the swarm, as specified by the equation,
k = the radius of gyration of the swarm about an axis through the center of mass of the swarm.

We have then for the square of the resultant angular momentum of the swarm

$$H^2=m^2\left[\left\{\sum_{i=1}^{n}(v_iz_i-w_iy_i)\right\}^2+\left\{\sum_{i=1}^{n}(w_ix_i-u_iz_i)\right\}^2+\left\{\sum_{i=1}^{n}(u_iy_i-v_ix_i)\right\}^2\right].$$

Further, the compound probability that a meteorite m_1 lying in an elementary volume at (x_1, y_1, z_1) has its velocity-components between u_1 and u_1+du_1, v_1 and v_1+dv_1, w_1 and w_1+dw_1, and a second meteorite m_2 lying in an elementary volume at (x_2, y_2, z_2) has its velocity-components between u_2 and u_2+du_2, v_2 and v_2+dv_2, w_2 and w_2+dw_2, and so on, will be

$$\begin{aligned}&\rho(x_1y_1z_1)\cdot p(u_1v_1w_1x_1y_1z_1)\,du_1dv_1dw_1dx_1dy_1dz_1\\&\times\rho(x_2y_2z_2)\cdot p(u_2v_2w_2x_2y_2z_2)\,du_2dv_2dw_2dx_2dy_2dz_2\\&\cdots\cdots\cdots\\&\times\rho(x_ny_nz_n)\cdot p(u_nv_nw_nx_ny_nz_n)\,du_ndv_ndw_ndx_ndy_ndz_n.\end{aligned}$$

Consequently we obtain as the mean square value of the resultant angular momentum of such a swarm,

$$M_n(H^2) = m^2\iint\Big[\Big\{\sum_{i=1}^{n}(v_iz_i - w_iy_i)\Big\}^2 + \Big\{\sum_{i=1}^{n}(w_ix_i - u_iz_i)\Big\}^2 + \Big\{\sum_{i=1}^{n}(u_iy_i - v_ix_i)\Big\}^2\Big] \times$$

$$p(u_1v_1w_1x_1y_1z_1)\cdot\rho(x_1y_1z_1)\ \ldots\ldots\ p(u_nv_nw_nx_ny_nz_n)\cdot\rho(x_ny_nz_n) \times$$

$$du_1dv_1dw_1dx_1dy_1dz_1\ \ldots\ldots\ du_ndv_ndw_ndx_ndy_ndz_n.$$

$$= m^2\iint\Big[\sum_{i=1}^{n}(v_iz_i - w_iy_i)^2 + + + 2\sum_{\substack{h,k=1\\h\neq k}}^{n}(v_hz_h - w_hy_h)(v_kz_k - w_ky_k) + +\Big] \times$$

$$p\ldots\rho\ldots \times du_1dv_1dw_1dx_1dy_1dz_1\ldots,$$

in which the integration is to be extended over all the possible values of $u_1v_1w_1x_1y_1z_1$, $u_2v_2w_2x_2y_2z_2$ and so on.

Remembering now that

$$\int p\cdot\rho\, du_idv_idw_idx_idy_idz_i = \int\rho\, dx_idy_idz_i\int p\, du_idv_idw_i = 1,$$

$$(i = 1, 2, 3\ \ldots\ldots\ n)$$

we obtain, after reduction

$$M_n(H^2) = m^2\Big\{\sum_{i=1}^{n}\int(v_iz_i - w_iy_i)^2\cdot p\cdot\rho\, du_idv_idw_idx_idy_idz_i + +$$

$$+ 2\sum_{\substack{h,k=1\\h\neq k}}^{n}\int(v_hz_h - w_hy_h)\cdot p\cdot\rho\, du_hdv_hdw_hdx_hdy_hdz_h \times$$

$$\int(v_kz_k - w_ky_k)\,p\cdot\rho\, du_kdv_kdw_kdx_kdy_kdz_k + +\Big\},$$

and further, by virtue of the abbreviations in (13) and (14),

$$M_n(H^2) = m^2\Big\{n(\mu_1^2 + \mu_2^2 + \mu_3^2) + n(n-1)(\nu_1^2 + \nu_2^2 + \nu_3^2)\Big\}. \qquad (15)$$

If we confine ourselves to the case of spherical symmetry, as assumed in our present problem, and write, for brevity, H^2 instead of $M_n(H^2)$, we obtain at last

$$H^2 = 3nm^2c^2k^2 = \frac{3c^2k^2M^2}{n}, \qquad (16)$$

$$\text{or} \qquad n = \frac{3c^2k^2M^2}{H^2}. \qquad (16)$$

This is a remarkable result of great importance. After we had obtained the above relation, we have noticed that a similar formulae was also found by Jeans[1] as early as in 1905.

To recapitulate: In a primordial swarm of meteorites, let

n = the number of meteorites,

M = the total mass of the swarm,

k = the radius of gyration about an axis through the center of mass,

c = a "mean" value of the "mean square" velocity of individual meteorites;

then the angular momentum of such a swarm is not in general zero, but may be expected to be of the magnitude

$$H = \sqrt{\frac{3}{n}} \cdot ckM.$$

If this primordial swarm be left to itself, widely separated from other celestial bodies, and hence free from any tidal action due to external causes, then it will retain its angular momentum forever constant, throughout its whole career of evolution.

In passing, it may be remarked that if n increases indefinitely, M remaining constant, then H tends to zero. Physically interpreted, this amounts to saying that if a gaseous nebula with spherical symmetry be left to itself and condenses according to its own gravitation, the resulting body would probably show no sign of rotation.

Numerical Calculation.

As a numerical example, let us consider the following problem:—

> If we were to assume that our solar system had evolved from a primordial swarm of meteorites, isolated from other celestial bodies, what would have been the size of the individual meteorites in the primordial swarm?

We take for the mass and angular momentum of the primordial swarm, the present values of our solar system, so that

$$M = 1, \qquad H = 0.022.$$

[1] Jeans, Ap. J., 22, 101, 1905.

As to the size of the primordial swarm and the "mean square" velocity of the meteorites, there seems to be no appropriate measure from which to estimate their probable order of magnitude; they might vary according to different opinions. We put, as a rude trial,

$$c = 1 \left(=5 \frac{km}{\text{sec}}\right),$$

$$k = 10^5 \,(=0.5 \text{ parsec}).$$

Putting, then, these values in the formula (17), we obtain

$$n = ca\, 10^{14},$$

and hence

$$m = \frac{\odot}{10^{14}} = \frac{\text{earth's mass}}{3 \times 10^8}.$$

Thus, if we assume the meteorites to have about the same density as our earth, they should be in size about 20 km in diameter, i.e., about the size of the asteroids now circulating between the orbits of Mars and Jupiter.

Theory Proposed.

In the light of all that has been stated above, we propose a theory of celestial rotation as follows :—

The celestial bodies are looked upon as having evolved from primordial swarms of meteorites, isolated from one another, gaseous nebulae being thereby decidedly excluded, since we consider the individual meteorites to have been about the size of the asteroids in our solar system. Although the constituent meteorites are moving at random, just like gaseous molecules, yet such a swarm as a whole is seen to have a finite amount of angular momentum; and the latter would manifest itself as a sensible rotation, as the primordial swarm gradually condenses, by virtue of its own mutual gravitation.

The size of the meteorites in one swarm may very probably have varied from those in another. Swarms consisting of larger meteorites would have, in general, a larger angular momentum; they would very probably condense, in the stage of their evolution, into two or more bodies, and thus form double or multiple systems. Those consisting of medium-sized meteorites would have, in general, a medium angular momentum; they might have first condensed into single bodies, and then have divided themselves by Poincaré-Darwin procedure, and thus evolved to most spectroscopic binaries. Lastly, those consisting of smaller meteorites would have, in general, a smaller angular momentum, and hence, unable to divide themselves by rotation, they would condense into single bodies,—probably leaving, by the way of evolution, small remnant planets, here and there, and thus would have evolved to the so-called planetary systems, such for instance as our solar system.

Although there is undoubtedly a general tendency toward equality in both the masses and angular momenta of the primordial swarms, yet it is enevitable that there were always some small differences in individual cases. Such small differences, which might have arisen either accidentally or according to the situation of the birth-place in the stellar universe, would control the further evolution of those swarms.

Resumé.

1. For 76 visual systems and 10 spectroscopic binaries, the masses and angular momenta have been calculated, and it has been found that they are, broadly speaking, of about the same order of magnitude.

   Further details of the observed facts have been recapitulated above.

2. The probable amount of the angular momentum of a primordial swarm of meteorites, having spherical symmetry, and isolated from all other external influences have been theoretically calculated.

3. As the result of comparison of the theoretical considerations with the observed facts, a theory of the origin of celestial rotation has been proposed.

4. Probable division of celestial bodies into binary and planetary systems has been accounted for, from the consideration of their angular momenta.