On the Regularity in the Stark Effect on the Spectral Lines of Hydrogen and Helium

By

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Since Stark's discovery of the effect of an intense electric field on the spectral lines of hydrogen helium and lithium, this effect has been investigated by many experimenters. On the one hand, many positive results were obtained in the spectral lines belonging to the other elements; and on the other, the effect on the spectral lines of hydrogen and helium became very clear in their details. As to the first four Balmer lines of hydrogen, Stark has succeeded in photographing the manner of decomposition in "Feinzerlegung," and obtained a number of components in each of the four lines above-mentioned. His first communication on the "Feinzerlegung" in the electric field of 74000 volts per centimeter is described in his "Electrische Spektral Analyse Chemischer Atome." And a better result obtained by employing the electric field of 104000 volts per centimeter is reported in Ann. d. Phys., Bd. 48, S. 193, 1915. According to this latter communication the manner of decomposition of each of the four lines above mentioned was quite symmetrical in reference to the undisplaced position of the original line. A small displacement of about one Å.U. of the central perpendicular component of H_γ toward the red side was observed by T. Takamine and N. Kokubu[1] in a strong electric field of about 130000 volts per centimeter, but this minute dissimilarity is disregarded in the following.

[1] Takamine and Kokubu, Mem. Coll. Sci., Kyoto, **3**, 271 (1919).

TABLE I, parallel components.

Wave length	Term number of the series, m.	$\Delta\lambda$ in A. U. obs. by Stark at $10{\cdot}4\times10^4$ volt/cm.	Difference of $\Delta\lambda$	$\Delta\nu$ obs. by Stark at $10{\cdot}4\times10^4$ volt/cm.	Difference of $\Delta\nu$ obs.	p	$\Delta\lambda$ calc. at $10{\cdot}4\times10^4$ volt/cm.
6563·1 (Hα)	3	11·5 8·8 6·2 — —	2·7 2 6	26·7 20.4 14·4	6·3 6 o	8 7 6 5 4	12·0 9·0 6·0 3 0 0·0
4861·6 (Hβ)	4	19·4 16·4 13·2 10·0 6·7 3·3 0·0	3·0 3·2 3·2 3·3 3·4 3·3	82·0 69·4 55 9 42·3 28·3 14·0 0·0	12·6 13·5 13·6 14·0 14·3 14·0	8 7 6 5 4 3 2	19·8 16·6 13·2 9·9 6 6 3·3 0·0
4340·5 (Hγ)	5	29·4 — 23·9 19·9 15 9 10·6 6·6 2·7	5·5 4·0 4·0 5·3 4·0 3·9	156 127 105·9 84·4 56·3 35·1 14·4	29 21·1 21·5 28·1 21·2 20·7	9 8 7 6 5 4 3 2	30·4 26·4 22·4 18·5 14·5 10·6 6·6 2·6
4102·0 (Hδ)	6	37·5 33·4 28·6 24·2 19·6 14·4 9·6 5·2 —	4·1 4·8 4·4 4·6 5·2 4·8 4·4	223 199 170 144 116·5 85·7 57·1 31·0	24 29 26 27·5 30·8 28 6 26·1	9 8 7 6 5 4 3 2 1	37·7 33·0 28·3 23·6 18·9 14·2 9·4 4·7 0·0

TABLE 2, perpendicular components.

Wave length	Term number of the series, m.	Δλ obs. in Å. U. at 10·4×10⁴ volt/cm.	Difference of Δλ	Δν obs. at 10·4×10⁴ volt/cm.	Difference of Δν	p	Δλ calc. at 10·4×10⁴ volt/cm.
6563·1 (Hα)	3	2·6 0·0	2·6	6·2 0·0	6.2	7 6	3·0 0·0
4861·6 (Hβ)	4	19·3 16·4 13·2 9·7 6·6 3·4 0·0	2·9 3·2 3·5 3·1 3·2 3·4	81·8 69·4 56·0 41·0 28·0 14·4 0·0	12·4 13·4 15·0 13·0 13·6 14·4	9 8 7 6 5 4 3	19·9 16·6 13·3 9·9 6·6 3·3 0·0
4340·5 (Hγ)	5	26·3 22·8 — 17·3 13·3 9·7 3·9 0·0	3·5 5·5 4·0 3·6 5·8 3·9	140 121 92·0 70·8 51·6 20·7 0·0	19 29 21.2 19·2 30·9 20·7	9 8 7 6 5 4 3 2	27·7 23·7 19·8 15·8 11·9 7·9 4·0 0·0
4102·0 (Hδ)	6	34·8 30·4 25·8 21·2 17·2 11·9 7·4 2·4	4·4 4·6 4·6 4·0 5·3 4·5 5·0	207 181 154 126 106 71·0 44·0 14·3	26 27 28 20 35 27 30	9 8 7 6 5 4 3 2	35·4 30·6 25·9 21·2 16·5 11·8 7·1 2·4

Already from the results in "Grobzerlegung" Stark and others have pointed out that the displacements of the components of Balmer lines were proportional to the strength of the electric field. In the case of "Feinzerlegung" this law was also followed fairly well by most of the components of Hβ and Hγ. Although a slight deviation from this law was observed in some of the components of Hδ, yet this may be ascribed to experimental errors. With regard to H_α, comparing Stark's result obtained at 27500 volts per centimeter with that at 104000 volts per centimeter, the separation of its outermost parallel components was observed to increase proportionally with the strength of the electric field. Thus considered, it seems to be a general law that the displacement of each component of the Balmer lines of hydrogen increases proportionally with the strength of the electric field. In the following, this law was considered to be true of all the components of the Balmer lines of hydrogen.

In the tables, Table 1 and Table 2, the values of the displacements of the components of the Balmer lines observed by Stark at 104000 volts per centimeter are given. As the manner of decomposition of each of the Balmer lines is quite symmetrical with respect to their original undisplaced positions, only the result on the red-side-components is given in the tables.

The first regularity of the manner of decomposition which may easily be gathered from the tables is that the distance between any two successive components of either the parallel or the perpendicular component of a line is constant, so far as they belong to the same line. Some exceptions from this law are of course present, especially in the case of H_γ. Next it is highly interesting to note that all the perpendicular components of H_β occupy exactly the same positions as those of the parallel ones.

Now in order to see whether there is a simple regularity or not among the electric components of the different Balmer lines, the writer calculated the displacements in wave number corresponding to those in wave length given by Stark as shown in Table 1, and Table 2.

As was stated before, the difference of displacements between any two neighbouring components of a line is constant. This is nearly true in the case of H_α, H_β and H_δ. And these constant differences in wave number of the neighbouring two components of these three lines are 6, 14 and 28 respectively, which are nearly integral multiples of 7. As regards the components of H_γ, though with some exceptions, most of the differences of the two neighbouring components are nearly equal to 21 in wave number, which is equal to three times 7. Moreover the displacements in wave number of the components which are nearest to the original lines are nearly equal to some whole multiples of 7 in all the cases observed.

Observing these regularities and trying to express the displacements in wave number of the components of the Balmer lines in a formula, the writer at last obtained the following two formulae.

$$\begin{cases} \Delta\nu_p = \frac{7}{10\cdot4}\left\{p(m-2)-4\right\}E = 0\cdot673\left\{p(m-2)-4\right\}E, \\ \Delta\nu_s = \frac{7}{10\cdot4}\left\{p(m-2)-6\right\}E = 0\cdot673\left\{p(m-2)-6\right\}E, \end{cases}$$

where $\Delta\nu_p$ and $\Delta\nu_s$ represent the displacement in wave number of a component in the parallel and in the perpendicular component respectively, E is the strength of the electric field in 10^4 volt per centimeter, and p positive integers beginning from one and m the term number of the series. The numerical values of $\Delta\lambda_p$ and of $\Delta\lambda_s$ at the field of $10\cdot4 \times 10^4$ volts per centimeter calculated by these formulae for different values of p, and for each value of m corresponding to the first four lines of the Balmer series are given in Table 1 and Table 2. As may be seen from these tables the calculated values of $\Delta\lambda$ are in fair agreement with the observed values except for a few components of H_γ. This is also shown graphically in Fig. 1. Here the value of $\Delta\nu$ and the term number of the series m are taken as the ordinate and abscissa respectively. Each straight line drawn in these figures represents the relation between the calculated value $\Delta\nu$ and the term number of the series m corresponding to each value of p. The observed values of the displacements of the components expressed in wave number are represented by the points.

Recently the writer became aware that P. S. Epstein[1] had obtained a similar formula theoretically. But, so far as the writer knows, this is different in some points from those obtained by the writer.

Next, the Stark effect on the spectral lines of helium is somewhat more complicated than that of hydrogen. As is evident from the investigations made by Takamine and Yoshida[2], Nyquist[3] and Takamine and Kokubu[4], most of the spectral lines belonging to the diffuse series of helium have isolated components, in the electric field, on their immediate violet sides.

According to Takamine and Kokubu the line λ 4472 has three such isolated components in the parallel and in the perpendicular

1 Epstein, Phys. Z.S., **17**, 148, (1916).
2 Takamine and Yoshida, Mem. Coll. Sci., Kyoto, **2**, 325, (1917).
3 Nyquist, Phys. Rev., **10**, 226, (1917).
4 Takamine and Kokubu, Mem. Coll. Sci., Kyoto, **3**, 81, (1918); and **3**, 275, (1919).

component respectively. And by exterpolating the displacements of these components, it was observed that all these six isolated components started from a common point in the zero field, so that they might be regarded as if they were originated from a rather hypothetical spectral line, so to speak, at λ 4470.0 at the zero field. The term "isolated line" employed in the following description was used in such sense. The next member of the series, viz the line λ 4026, had four and five isolated components in the parallel and in the perpendicular component respectively, of which the first three, both in the parallel and in the perpendicular component in the region of shorter wave length, started from an isolated spectral line at λ 4025.4, and the remaining three isolated components originated also from an other isolated line at λ 4025.9. In the case of the diffuse series of parhelium many such isolated components which might be regarded as due to the isolated lines at the zero field were also present, and the isolation of these isolated lines in the region of shorter wave length from the original lines became smaller as the term number of the series was increased. This was also the case in the isolated lines of helium. Moreover, the displacement in the electric field of the components of the original lines of the diffuse series of helium and parhelium was toward the region of longer wave length. The only one exception from this rule was the line λ 5876 which was the first member of the diffuse series of helium. Next, the displacement of most of the isolated components of parhelium was toward the region of shorter wave length. This was also the case in helium, the only exception was that of the isolated perpendicular component originating from an isolated line λ 4025.8 at the zero field, whose wave length was shorter by about 0·5 Å. U. than that of the original line at λ 4026.3.

In the case of the line λ 3819.9, which is the fourth member of the diffuse series of helium, four components were observed in the parallel and in the perpendicular components respectively. One half of these four components were in the region of shorter wave length, and the other half were in the region of longer wave length of the original line. All these components started from the original line at the zero field. and displaced proportionally with the intensity of the electric field. From the analogy of the manner of decomposition of the other lines in the electric field, the writer regarded the two p- and the two s-components which displaced toward the region of shorter wave length from the original line as due to an isolated spectral line, whose isolation from the original line became very small as the term number of this line became larger. The remaining two p- and two s-components which displaced toward the region of longer wave length was considered to be the components of the original spectral line λ 3819.9 in the electric field.

In the case of the line 4026.3, two isolated lines at λ 4025.4 and λ 4025.8 were imagined. It is, of course, not clear that the isolated line at λ 3819.9 as was considered above corresponds to either of the two isolated lines of λ 4026.3. But in the following the writer supposed, as a trial, that the components in the region ot shorter wave length of λ 3819.9 and all of those of the other lines of this series correspond to the components of the isolated line at λ 4025.4 of the line λ 4026.3.

In the case of the Balmer lines of hydrogen it was observed, as was described above, that the displacement of a component in the electric field increased proportionally with the intensity of the electric field. This was also the case in all the components of the first four lines of the diffuse series of helium.

Recently G. Liebert[1] investigated, using the method employed first by Stark, the effect of an electric field on the spectral lines of helium. His investigation was mainly concerned with the spectrum lines in the ultraviolet region. With regard to the lines of the diffuse series of helium, his results on the lines of the higher term number is especially important. He succeeded in observing many components of the lines at λ 3705.15 and 3634.52. According to the same investigator, the separations of the outer components of these two lines were not proportional to the electric field

In the case of the line λ 3705.15, the separation of the outer components increased nearly proportional to the insensity of the electric field; and a slight deviation of it from being proportional to the electric field might be ascribed to experimental errors, as the images of most of the outer components appeared broadened as was stated by Liebert. Next, with regard to the line λ 3634.52, Liebert observed 6 p- and 6 s-components in the electric field of 17900 volts per centimeter, and 10 p- and 10 s- components in the field of 27500 volts per centimeter. As it appeared to the writer, that there was some

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1 Liebert, Ann. d. Phys., **56**, 589, (1918).

ambiguity about the correspondency between the outer components of the line in the two different electric fields, it was natural to consider that another component in one of the electric fields might correspond to the outer component in the other field. Taking the displacement of a component and the intensity of the electric field as the ordinate and the abscissa respectively, and plotting the values observed by Liebert at the field of 17900 and of 27500 volts per centimeter, it was found that each of the components of the line λ 3634.52 at the field of 17900 volts per centimeter was nearly on a straight line passing through the origin and the point representing the displacement of a certain component in the field of 27500 volts per centimeter, as was shown in Fig. 3. This was also the case in all the components of the line 3705.15, as was shown in Fig. 2. Judging from these facts it seems natural to consider that the displacement of all the electric components of these two lines should increase proportionally with the intensity of the electric field as in the case of the other lines of the same series.

The electric components in the region of longer wave length of these two lines were regarded as due to the original spectrum lines of the diffuse series of helium, and those in the region of shorter wave length as due to the isolated spectrum lines as was considered in the case of the line λ 3819.75 of the same series.

In the case of the diffuse series of parhelium, the isolated lines of this series were found by Liebert to form a new series of parhelium. With regard to the isolated lines of the diffuse series of helium, no such relation was as yet observed. But the writer supposes, at the present, that they are connected by a similar relation, and the same term number of the series as that of the diffuse series was affixed, for the sake of convenience, to the corresponding isolated line.

Trying to express the manner of decomposition of the lines of the diffuse series of helium and those of the series of the isolated lines separately by the formulae similar to those of the Balmer lines of hydrogen, the writer obtained the following formulae,

$$\begin{cases} \Delta\nu_p = 0{\cdot}673\ \{p(m-2)-2\}\ E, \\ \Delta\nu_s = 0{\cdot}673\ \{p(m-2)-6\}\ E \end{cases}$$

for the lines of the diffuse series of helium, and

$$\begin{cases} \Delta\nu_p = 0{\cdot}673\ \{p(m-2)-1\}\ E \\ \Delta\nu_s = 0{\cdot}673\ \{p(m-2)-3\}\ E \end{cases}$$

for the lines supposed to belong to a series of the isolated lines corresponding to the diffuse series of helium. In these formulae, E represents the intensity of the electric field in 10^4 volt per centimeter, p positive integers beginning from 1, m the term number of the series, $\Delta\nu_p$ the displacement in wave number of a parallel component and $\Delta\nu_s$ represents the displacement of a perpendicular component.

The calculated values of $\Delta\lambda$ of the components of the lines belonging to the diffuse series of helium are tabulated in the tables from Table 3 to Table 7, and those values of $\Delta\lambda$ of the components of the

TABLE 3.

λ 4472, $m=4$, diffuse series of helium.

Parallel Component			Perpendicular Component		
p	Δλ calc. in Å. U. at 10×10⁴ volt/cm.	Δλ obs. in Å. U. at 10×10⁴ volt/cm.	p	Δλ calc. at 10×10⁴ volt/cm.	Δλ obs. at 10×10⁴ volt/cm.
1	0·0	0·0	3	0·0	0·0
2	2·7	2·4	4	2·7	3·2
3	5·4	5·3	5	5·4	6·0
4	8·1	7·5	6	8·1	8·4
5	10.8		7	10·8	

TABLE 4.

λ 4026, $m=5$, diffuse series of helium.

Parallel Component			Perpendicular Component		
p	Δλ calc. at 10×10⁴ volt/cm.	Δλ obs. at 10×10⁴ voltc/m.	p	Δλ calc. at 10×10⁴ volt/cm.	Δλ obs. at 10×10⁴ volt/cm.
1	1·1	—	2	0·0	—
2	4·4	—	3	3·3	—
3	7·7	9·4	4	6·6	—
4	10·9	11·3	5	9·8	8·4
5	14·2	14·3	6	13·1	13·4
6	17·5	—	7	16·4	17·0
			8	19·7	

TABLE 5.

λ 3819.9, $m=6$, diffuse series of helium.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at 5×10^4 volt/cm.	$\Delta\lambda$ obs. at 5×10^4 volt/cm.	p	$\Delta\lambda$ calc. at 5×10^4 volt/cm.	$\Delta\lambda$ obs. at 5×10^4 volt/cm.
1	1·0	—	2	1·0	—
2	3·0	3·8	3	3·0	2·5
3	4·9	—	4	4·9	—
4	6·9	—	5	6·9	—
5	8·8	8·6	6	8·8	9·0
6	10·8		7	10·8	

TABLE 6.

λ 3705.15, $m=7$, diffuse series of helium.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at $2{\cdot}75\times10^4$ volt/cm.	$\Delta\lambda$ obs. at $2{\cdot}75\times10^4$ volt/cm.	p	$\Delta\lambda$ calc. at $2{\cdot}75\times10^4$ volt/cm.	$\Delta\lambda$ obs. at $2{\cdot}75\times10^4$ volt/cm.
1	0·8	—	2	1·0	—
2	2·0	—	3	2·3	—
3	3·3	3·35	4	3·6	3·60
4	4·6	—	5	4·8	—
5	5·9	—	6	6·1	—
6	7·1	7·66	7	7·4	—
7	8·4	—	8	8·7	8·19

TABLE 7.

λ 3634.52, $m=8$, diffuse series ot helium.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at 2.75×10^4 volt/cm.	$\Delta\lambda$ obs. at 2.75×10^4 volt/cm.	p	$\Delta\lambda$ calc. at 2.75×10^4 volt/cm.	$\Delta\lambda$ obs. at 2.75×10^4 volt/cm.
1	1·0	0·18	1	0·0	0.18
2	2·5	2·13	2	1·5	2.13
3	3·9	—	3	2·9	—
4	5·4	5·58	4	4·4	—
5	6·9	—	5	5·9	5·58
6	8·3	—	6	7·3	—
7	9·8	9·58	7	8·8	8·72

TABLE 8.

λ 4470.0, $m=4$, series of isolated lines.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at 10×10^4 volt/cm.	$\Delta\lambda$ obs. at 10×10^4 volt/cm.	p	$\Delta\lambda$ calc. at 10×10^4 volt/cm.	$\Delta\lambda$ obs. at 10×10^4 volt/cm.
1	1·3	—	2	1·3	—
2	4·0	4·2	3	4·0	3·9
3	6·7	7·7	4	6·7	6·4
4	9·4	10·1	5	9·4	9·8
5	12·1		6	12·1	

TABLE 9.

λ 4025.4, $m=5$, series of isolated lines.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at 10×10^4 volt/cm.	$\Delta\lambda$ obs. at 10×10^4 volt/cm.	p	$\Delta\lambda$ calc. at 10×10^4 volt/cm.	$\Delta\lambda$ obs. at 10×10^4 volt/cm.
1	2·2	—	1	0·0	—
2	5·5	—	2	3·3	—
3	8·7	—	3	6·5	—
4	12·0	12·2	4	9·8	9·3
5	15·3	15·6	5	13·1	13·2
6	18·5	—	6	16·4	16·3
7	21·8	20·5	7	19·6	

TABLE 10.

λ 3820, $m=6$, series of isolated lines.

Parallel Component			Perpendicular Component		
p	$\Delta\lambda$ calc. at 5×10^4 volt/cm.	$\Delta\lambda$ obs. at 5×10^4 volt/cm.	p	$\Delta\lambda$ calc. at 5×10^4 volt/cm.	$\Delta\lambda$ obs. at 5×10^4 volt/cm.
1	1·5	—	1	0·5	—
2	3·4	3·6	2	2·5	2·5
3	5·4	—	3	4·4	—
4	7·4	—	4	6·4	—
5	9·3	9·6	5	8·3	8·6
6	11·3	—	6	10·3	—

TABLE 11.

λ 3705.2, $m=7$, series of isolated lines.

Parallel Component			Perpendicular Component		
p	Δλ calc. at 2·75×10⁴ volt/cm.	Δλ obs. at 2·75×10⁴ volt/cm.	p	Δλ calc. at 2·75×10⁴ volt/cm.	Δλ obs. at 2·75×10⁴ volt/cm.
1	1·0	1.09	1	0·5	0·01
2	2·3	—	2	1·8	—
3	3·6	—	3	3·1	—
4	4·8	5·40	4	4·3	4·42
5	6·1	—	5	5·6	—
6	7·4	—	6	6·9	—
7	8·6	9·14	7	8·1	8·09
8	9·9	—	8	9·4	—

TABLE 12.

λ 3634.5, $m=8$, series of isolated lines.

Parallel Component			Perpendicular Component		
p	Δλ calc. at 2·75×10⁴ volt/cm.	Δλ obs. at 2·75×10⁴ volt/cm.	p	Δλ calc. at 2·75×10⁴ volt/cm.	Δλ obs. at 2·75×10⁴ volt/cm.
1	1·2	—	1	0·7	—
2	2·7	2·28	2	2·2	2·28
3	4·2	—	3	3·7	—
4	5·6	5·47	4	5·1	5·47
5	7·1	7·66	5	6·6	—
6	8·6	—	6	8·1	7·78
7	10·0	—	7	9·5	—
8	11·5	11·85	8	11·0	—
9	13·0	—	9	12·4	11·92
10	14·4	14·39	10	13·9	14·57
11	15·9	—	11	15·4	—
12	17·4	—	12	16·8	—
13	18·8	18·25	13	18·3	18·25
14	20·2	—	14	19·8	—

lines supposed to belong to a series of the isolated lines are given in tables from Table 8 to Table 12. And these results are also shown graphically in Fig. 4 and Fig. 5, which are drawn similarly as in the case of the Balmer lines of hydrogen. Here the values of $\Delta\nu$ at the field of 5×10^4 volt per centimeter are employed.

For the lines λ 4472, λ 4026 and λ 3820, the values observed by

Takamine and Kokubu were utilised, and for the lines λ 3705 and λ 3634 those observed by Liebert were employed.

As is seen from the tables, the agreement between the observed and calculated values is fairly good in the case of most of the electric components of the 10 spectral lines before-mentioned. But there are, of course, some components which showed large deviations from the calculated values. Moreover many vacant spaces are left in the observed values. It was already stated that the line λ 4026 had two isolated lines, and that the components belonging to the isolated line 4025.8 which was nearer to the original line λ 4026 were disregarded; and the case might occur that some components which were really due to isolated lines corresponding to the isolated one of the 4026 at 4025·8 just mentioned were taken as those due to the others. All of those ambiguities above mentioned are to be made clear by future investigations, and a finer electric decomposition of the spectrum lines of helium like that of the Balmer lines obtained by Stark are highly desirable.

Lastly it is especially noteworthy that the constant 0·673 of the formulae for the lines of helium is entirely identical with that of the formulae for the Balmer series. This is also nearly equal to 0·660 for the Balmer lines obtained by Epstein theoretically.

In conclusion the writer's hearty thanks are due to Profs. T. Mizuno and M. Kimura for the interest they have taken in the research, and the writer also expresses his sincere thanks to Mr. M. Kiuchi of the University of Tokyo for his kind information of the results obtained by Stark on hydrogen.

Physical Laboratory,
Kyoto University,
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FIG. 1. (Balmer series)

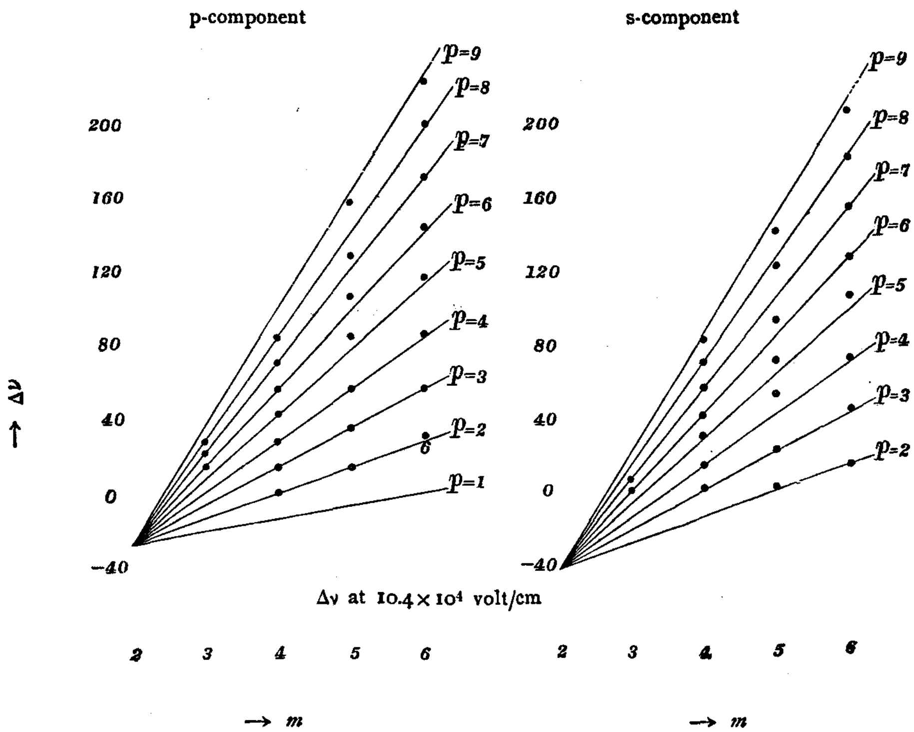

FIG. 2.

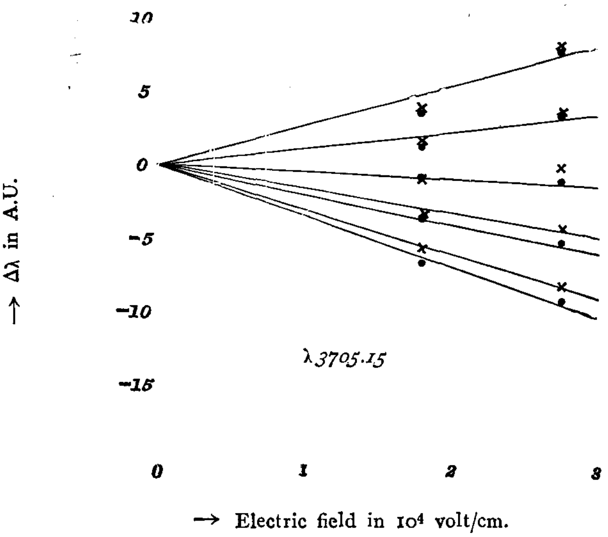

FIG. 3.

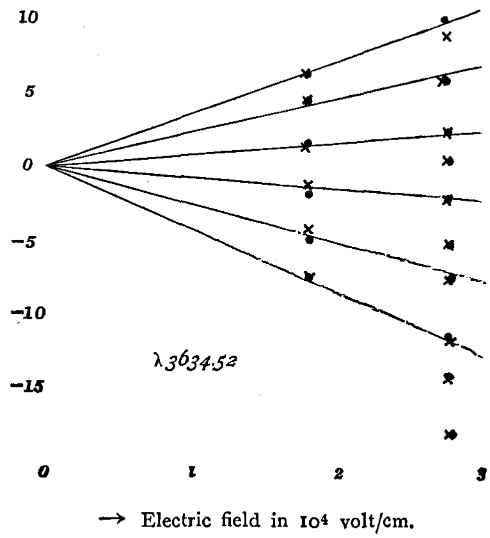

FIG. 4. (Diffuse series of helium)

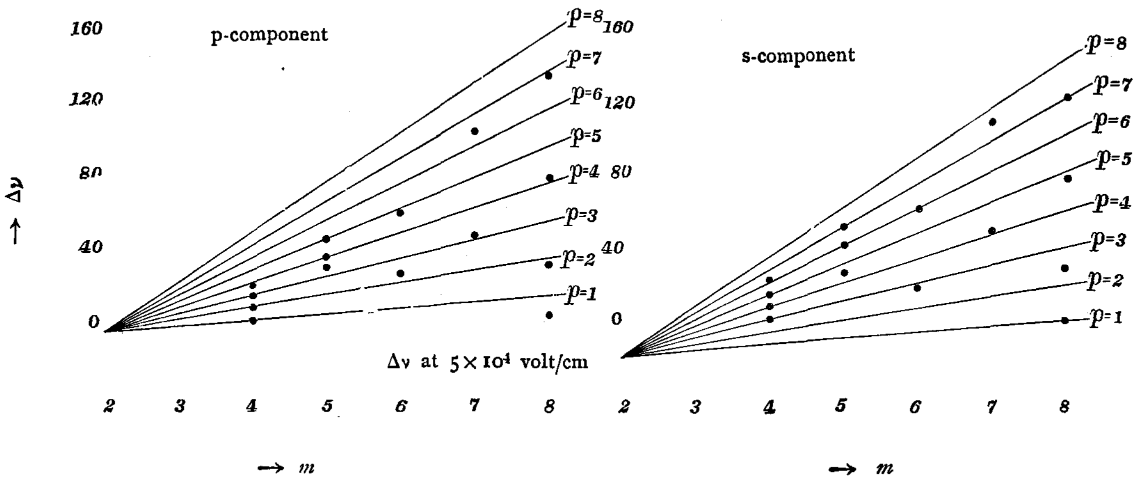

FIG. 5. (Series of isolated lines of helium)