The Spark Potential Difference between Spheres

By

Usaburo Yoshida

(Received April 25, 1921)

When a sufficiently large potential difference is applied between two metallic spheres the electric spark will pass between them. The maximum intensity of the electric field in front of one of the electrodes in this case was calculated by Schuster[1] and Russel[2] from the observed values of the spark potential between the electrodes. According to Schuster, the maximum electric intensity became smaller with the same electrode as the spark length was increased, and became larger with the same spark length as the diameter of the electrode was reduced. Next Russel arrived at a very simple conclusion that the maximum electric intensity in front of the electrode was independent of the spark length and the diameter of the electrode and attained to a value of about 38 kilovolts per centimetre in the air at ordinary pressure. He extended his calculation to the case of spark discharge between parallel cylindrical electrodes, and the same value for the maximum electric intensity was also found to be true in this case.

Now let V be the spark potential difference between two equal spherical electrodes, either of which is not grounded, then we may regard that the potential of one of the electrodes is $V/2$ and that of the other is $-V/2$. The spark potential V will be a function of the maximum electric field R_m in front of either of the electrodes, the radius of the electrode a and the shortest distance between the elec-

---

[1] Schuster, Phil. Mag., **29**, 182, (1890).

[2] Russel, Phil. Mag., **11**, 237, (1906).

trodes namely the spark length x. The actual relation of these quantities is given by Russel by the equation

$$V=R_m\frac{x}{f},$$

where f is a function of x/a as is shown numerically in the Table I.

When one of the electrodes is earthed, the intensity of the electric field is maximum just in front of the isolated sphere. In this case the relation between the maximum electric field and the potential difference between the electrodes is represented by a similar expression as before,

$$V=R_m\frac{x}{f_1},$$

where f_1 is also a function of x/a and its numerical value calculated by Schuster and Russel is shown in the Table I.

TABLE I.—Values of f and f_1

$\frac{x}{a}$	f	f_1	$\frac{x}{a}$	f	f_1
0.0	1.000	1.000	1.5	1.559	1.910
0.1	1.034	1.034	2.0	1.770	2.338
0.2	1.068	1.068	3.0	2.214	3.252
0.3	1.102	1.106	4.0	2.677	4.200
0.4	1.137	1.150	5.0	3.151	5.172
0.5	1.173	1.199	6.0	3.632	6.144
0.6	1.208	1.253	7.0	4.117	7.126
0.7	1.245	1.313	8.0	4.604	8.112
0.8	1.283	1.378	9.0	5.095	
0.9	1.321	1.446	10.0	5.586	
1.0	1.359	1.517	100.0	50.51	

Russel regarded the potential difference of 0.8 kilovolts, which was nearly equal to twice the minimum spark potential difference in the air, as the "lost volts" between the electrodes. The calculation of the maximum electric field in front of one of the electrodes was then carried on by assigning the spark potential difference minus the "lost volts" as the potential difference between the electrodes. The value of the maximum electric field obtained by Russel from the observed values of spark potential differences by many authors was nearly constant, and the mean value of 38 kilovolts per centimetre was given to the air at ordinary pressure as its "dielectric strength."

When, however, a wider range in the values of the spark length and the diameter of the electrode than those employed by Russel was considered, the maximum electric field in front of the electrode was no more constant as shown in Table II.

TABLE II.—Values of R_m in K. V. per c.m.

$\frac{x}{a}$	$a=2.5$ c.m.	$a=1.0$ c.m.	$a=0.5$ c.m.
0.5	36.7	—	—
1.0	38.2	40.9	—
2.0	40.1	41.8	46.3
4.0	—	43.6	47.1
5.0	—	44.8	46.9
6.0	—	—	48.6

Here the value of the maximum electric field R_m was calculated in the same way as Russel from the spark potential difference between two equal isolated spheres observed by Algermissen.[1]

In the calculation of the dielectric strength of a gas, made by Schuster and Russel, the effect of the presence of leading-in-wires was disregarded. But the experiment made by Paschen[2] indicated that the spark potential betweeen small spherical electrodes was somewhat higher with a thicker leading-in-wire than with a thinner one. Judging from this it might be inferred that the mere presence of a leading-in-wire, even though small, would have some influence upon the spark potential difference. As the mathematical calculation of this effect upon the spark potential between spheres seemed to be difficult, the writer attempted, for the present, to obtain an empirical formula showing the actual relation between the spark potential and the spark length for any value of the radius of the spherical electrode. The following formula (1) was found to express fairly well the Algermissen's value of the spark potential between two equal isolated spheres.

$$(1) \qquad V=4+30\times\frac{1+0.2\left(\frac{x}{a}\right)^{\frac{1}{2}}}{f},$$

where V was the spark potential difference between the electrodes expressed in kilovolt, and f a function of x/a as already shown in

---

[1] Algermissen, Ann. d. Physik, **19**, 1007, (1906).

[2] Paschen, Wied. Ann., **37**, 69, (1889).

Table I. The calculated spark potential difference and those observed by Algermissen are given in Table III, the agreement between them being very good.

TABLE III.

Spark potential in K. V., observed by Algermissen.

$a=2.5$ c.m.			$a=1.0$ c.m.			$a=0.5$ c.m.		
x in c.m.	Vcalc.	Vobs.	x in c.m.	Vcalc.	Vobs.	x in c.m.	Vcalc.	Vobs.
1.0	33.6	33.0	1	30.5	31.0	0.75	22.0	23.0
1.25	40.5	40.0	1.5	40.0	40.0	1.0	25.8	27.0
1.50	47.0	47.0	2.0	47.5	48.0	2.0	35.4	36.0
1.75	53.0	53.0	3.0	58.8	59.0	3.0	40.9	41.0
2.00	59.4	59.0	4.0	66.9	66.0	4.0	44.9	44.0
2.50	70.3	70.5	5.0	72.9	72.0	4.5	46.5	45.0
3.75	94.0	94.0	5.5	75.8	74.5	5.0	47.8	46.5
4.50	106.1	107.0						

For Heydweiller's[1] value of the spark potential between two equal isolated spheres the same formula as before was seen also to hold good for the most part of the spark lengths except for very short ones, and the calculated values came out somewhat larger than observed ones for the very short sparks as shown in Table IV.

TABLE IV.

Spark potential in K. V., observed by Heydweiller.

$a=2.5$ c.m.			$a=1.0$ c.m.			$a=0.5$ c.m.		
x in c.m.	Vcalc.	Vobs.	x in c.m.	Vcalc.	Vobs.	x in c.m.	Vcalc.	Vobs.
0.5	19.3	18.4	0.1	7.1	4.7	0.1	7.1	4.8
0.6	22.3	21.6	0.2	10.1	8.1	0.2	9.9	8.4
0.7	25.2	24.5	0.4	15.9	14.5	0.3	12.6	11.4
0.8	28.1	27.3	0.6	21.2	20.4	0.4	15.1	14.6
0.9	30.8	30.1	0.8	26.2	26.1	0.5	17.2	17.3
1.0	33.7	32.8	1.0	30.4	31.3	0.6	19.2	20.0
1.2	39.2	38.2	1.2	34.5	35.5	0.7	21.1	22.1
1.4	44.4	43.6	1.4	38.1	38.7	0.8	22.8	24.1
1.6	49.7	48.8	1.6	41.6	41.2	1.0	25.7	27.0

---

[1] Heydweiller, Wied. Ann., **48**, 213, (1893).

Baille's[1] observation on the spark potential between two equal spheres, one charged and insulated and the other earthed, is again formulated in the similar manner as before, but with f_1 instead of f as given in the following equation.

$$(2) \qquad V=5+98.5\times\frac{1+0.297\left(\frac{x}{a}\right)^{\frac{1}{2}}}{f_1},$$

where V represents the spark potential in electrostatic units. The calculated and the observed values are compared in Table V, and it may be seen from this table that the discrepancy between the calculated and the observed values is within the limit of experimental errors.

TABLE V.

Spark potential V in C.G.S.E.S.U., observed by Baille.

	x in c.m.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$a=3$ c.m.	Vcalc.	15.2	25.6	36.3	46.8	57.6	67.6	78.0	88.1	98.0	108.9
	Vobs.	14.8	25.6	36.1	46.3	55.1	65.2	75.4	88.0	97.4	112.9
	x in c.m.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$a=1.5$ c.m.	Vcalc.	15.4	25.9	36.1	46.5	56.1	66.1	74.9	84.1	93.0	99.6
	Vobs.	15.0	26.0	36.6	45.9	55.0	65.2	73.8	84.8	94.6	104.7

Paschen[2] and Orgler[3] measured the spark potential difference between two equal spheres, one being earthed, in some gases under various pressures below the atmospheric one. Orgler experimented on the gases air, hydrogen, carbon dioxide, nitrogen and oxygen with the spherical electrodes of radius 1.25 c.m. His result may be represented by the following equation (3) throughout all ranges of spark lengths and pressures, and for every kind of gases.

$$(3) \qquad V=C+R_m\times\frac{1+0.46\left(\frac{x}{a}\right)^{\frac{1}{2}}}{f_1},$$

---

[1] Baille, J. J. Thomson, Conduction of Electricity through Gases, 461, (1906).

[2] Paschen, Wied. Ann., **37**, 69, (1889).

[3] Orgler, Ann. d. Physik, **1**, 159, (1900).

where C and R_m are independent of the spark length as in the former cases, and they depend only on the pressure and the nature of the gas. Their numerical values corresponding to each kind of gases and various values of its pressure are given in the Table XI. And in the tables from Table VI to X the values of the spark potential difference observed and calculated by the equation (3) are tabulated. The coincidence between them is within the limit of experimental errors.

TABLE VI.

Air, $a=1.25$ c.m., V in C.G.S.E.S.U., p the pressure in c.m. of mercury, observed by Orgler.

x in c.m.		0.04	0.06	0.08	0.1	0.2	0.3	0.4	0.5
p=75 c=3.8 R_m=100.0	Vcalc.	8.1	10.3	12.5	14.8	26.4	37.7	49.0	59.9
	Vobs.	7.5	10.1	12.7	15.2	27.3	38.6	48.8	58.2
p=55 c=3.3 R_m=76.4	Vcalc.	6.7	8.3	10.0	11.7	20.5	29.1	37.8	46.3
	Vobs.	6.1	8.1	10.1	12.0	21.3	29.8	37.6	44.9
p=35 c=2.7 R_m=52.2	Vcalc.	4.9	6.1	7.2	8.5	14.5	20.4	26.4	32.1
	Vobs.	4.6	6.1	7.4	8.7	15.0	20.9	26.2	31.1
p=15 c=2.1 R_m=25.6	Vcalc.		3.8	4.3	4.9	7.9	10.8	13.7	16.5
	Vobs.		3.5	4.3	5.0	8.2	11.0	13.6	16.2
p=10 c=1.5 R_m=19.2	Vcalc.		2.74	3.17	3.61	5.83	8.00	10.15	12.28
	Vobs.		2.57	3.10	3.64	6.13	8.16	10.05	11.93
p=6 c=1.20 R_m=13.2	Vcalc.		2.06	2.35	2.65	4.17	5.68	7.17	8.60
	Vobs.		1.94	2.30	2.65	4.31	5.80	7.13	8.35
p=4 c=1.05 R_m=9.82	Vcalc.		1.69	1.90	2.13	3.27	4.39	5.49	6.56
	Vobs.		1.61	1.89	2.16	3.39	4.50	5.48	6.38

TABLE VII.

Carbon dioxide, $a=1.25$ c.m., V in C.G.S.E.S.U., p the pressure in c.m. of mercury, observed by Orgler.

x in c.m.		0.04	0.06	0.08	0.1	0.2	0.3	0.4	0.5	0.6
$p=75$ $c=4.7$ $R_m=85.7$	Vcalc.	8.4	10.3	12.2	14.1	24.1	33.8	43.5	52.8	62.0
	Vobs.	8.0	10.2	12.4	14.5	24.8	34.7	43.9	52.4	60.5
$p=55$ $c=4.4$ $R_m=66.1$	Vcalc.	7.2	8.7	10.2	11.7	19.3	26.8	34.2	41.5	48.6
	Vobs.	6.7	8.5	10.2	11.9	20.1	27.6	34.5	41.1	47.4
$p=35$ $c=3.6$ $R_m=45.2$	Vcalc.	5.6	6.5	7.5	8.6	13.8	19.0	24.1	29.0	33.8
	Vobs.	5.2	6.5	7.7	8.9	14.4	19.6	24.3	28.8	33.1
$p=15$ $c=3.0$ $R_m=22.3$	Vcalc.		4.45	4.94	5.45	8.03	10.58	13.08	15.55	17.95
	Vobs.		4.13	4.79	5.42	8.29	10.80	13.18	15.52	17.85
$p=10$ $c=2.5$ $R_m=16.4$	Vcalc.		3.57	3.93	4.31	6.21	8.08	9.94	11.74	13.50
	Vobs.		3.28	3.77	4.25	6.50	8.35	10.09	11.79	13.40
$p=6$ $c=2.1$ $R_m=11.2$	Vcalc.			3.08	3.34	4.63	5.90	7.17	8.40	9.60
	Vobs.			3.00	3.34	4.78	6.11	7.29	8.40	9.54

TABLE VIII.

Hydrogen, $a=1.25$ c.m., V in C.G.S.E.S.U., p the pressure in c.m. of mercury, observed by Orgler.

x in c.m.		0.05	0.08	0.1	0.2	0.3	0.5	0.6
$p=75$ $c=3.4$ $K_m=53.0$	Vcalc.	6.3	8.0	9.2	15.4	21.4	33.1	39.8
	Vobs.	5.5	7.8	9.2	15.9	22.0	32.9	37.5
$p=55$ $c=2.8$ $R_m=41.1$	Vcalc.	5.0	6.4	7.3	12.1	16.7	25.9	30.2
	Vobs.	4.6	6.3	7.4	12.6	17.4	25.7	29.3
$p=35$ $c=2.1$ $R_m=28.7$	Vcalc.	3.66	4.60	5.26	8.60	11.85	18.25	21.30
	Vobs.	3.48	4.62	5.38	8.98	12.37	18.23	20.72
$p=15$ $c=1.7$ $R_m=14.2$	Vcalc.	2.47	2.94	3.26	4.90	6.52	9.68	11.20
	Vobs.	2.27	2.91	3.31	5.11	6.60	9.57	11.01
$p=10$ $c=1.3$ $R_m=10.55$	Vcalc.	1.87	2.22	2.46	3.68	4.88	7.22	8.34
	Vobs.	1.78	2.25	2.54	3.86	4.99	7.16	8.28
$p=6$ $c=1.31$ $R_m=6.91$	Vcalc.			2.07	2,87	3.66	5.19	5.93
	Vobs.			2.01	2.90	3.73	5.18	5.90

TABLE IX.

Nitrogen, $a=1.25$ c.m., V in C.G.S.E.S.U., p the pressure in c.m. of mercury, observed by Orgler.

x in c.m.		0.06	0.1	0.2	0.3	0.5
$p=75$ $c=4.8$ $R_m=101.4$	Vcalc.	11.4	16.0	27.7	39.2	61.8
	Vobs.	10.9	16.3	28.7	40.0	60.4
$p=55$ $c=4.0$ $R_m=78.7$	Vcalc.	9.1	12.7	21.8	30.7	49.2
	Vobs.	8.8	13.1	23.0	31.6	47.3
$p=35$ $c=3.5$ $R_m=54.3$	Vcalc.	6.5	9.5	15.8	21.9	34.0
	Vobs.	6.6	9.5	16.1	22.3	33.2
$p=15$ $c=2.3$ $R_m=27.2$	Vcalc.	4.1	5.3	8.5	11.6	17.6
	Vobs.	3.9	5.4	8.7	11.7	17.3
$p=10$ $c=2.10$ $R_m=19.40$	Vcalc.		4.23	6.47	8.68	13.00
	Vobs.		4.06	6.62	8.82	12.80

TABLE X.

Oxygen, $a=1.25$ c.m., V in C.G.S.E.S.U., p the pressure in c.m. of mercury, observed by Orgler.

x in c.m.		0.06	0.1	0.2	0.3	0.5
$p=75$ $c=3.8$ $R_m=88.0$	Vcalc.	9.5	13.5	23.6	33.6	53.3
	Vobs.	9.2	13.6	24.1	34.3	52.3
$p=55$ $c=3.4$ $R_m=67,3$	Vcalc.	7.8	10.8	18.6	26,2	41.2
	Vobs.	7.4	11.0	19.0	26.7	40.5
$p=35$ $c=2.85$ $R_m=45.3$	Vcalc.	5.80	7.83	13.07	18.25	28.35
	Vobs.	5.54	7.95	13.42	18.62	27.75
$p=15$ $c=2.0$ $R_m=22.6$	Vcalc.	3.46	4.48	7.10	9.65	14.65
	Vobs.	3.30	4.53	7.40	9.81	14.40
$p=10$ $c=1.86$ $R_m=15.7$	Vcalc.		3.59	5.41	7.19	10.69
	Vobs.		3.42	5.58	7.32	10.57

TABLE XI.

Orgler, $a = 1.25$ c.m., the values of C and R_m, p the pressure in c.m. of mercury.

p	Air		H_2		CO_2		N_2		O_2	
	C	R_m	C	R_m	C	R_m	C	R_m	C	R_m
4	1.05	9.82								
6	1.2	13.2	1.31	6.91	2.10	11.2	2.0	12.5	1.60	10.55
10	1.5	19.2	1.3	10.55	2.5	16.4	2.1	19.4	1.86	15.7
15	2.1	25.6	1.7	14.2	3.0	22.3	2.3	27.2	2.0	22.6
25	2.2	39.6	1.7	21.75	3.3	34.8	2.9	41.0	2.4	34.4
35	2.7	52.2	2.1	28.7	3.6	45.2	3.5	54.3	2.85	45.3
45	3.1	64.3	2.4	35.2	4.0	55.8	3.8	66.5	3.1	56.2
55	3.3	76.4	2.8	41.1	4.4	66.1	4.0	78.7	3.4	67.3
65	3.4	88.5	2.9	47.25	4.7	75.5	4.6	90.1	3.5	78.3
75	3.8	100.0	3.4	53.0	4.7	85.7	4.8	101.4	3.8	88.0

Paschen's experiment was made on air, hydrogen and carbon dioxide with the spherical electrodes of radius 1 c.m. His result is also represented by a similar equation as (3), but with a somewhat different constants as shown in the following equation (4).

$$(4) \qquad V = C + R_m \times \frac{1 + 0.356(\frac{x}{a})^{\frac{1}{2}}}{f_1}.$$

The values of C and R_m, which were so determined as to fit the Paschen's observation, are given in Table XII. The spark potentials calculated by the equation (4) assigning proper values to C and R_m, as given in Table XII, are seen to be in fair agreement with the observed values. The values of R_m for gases at different pressures obtained from the experiments of Orgler and Paschen are plotted in Fig. 1.

Here it must be especially remarked that the expressions $\{1 + 0.46(\frac{x}{a})^{\frac{1}{2}}\}/f_1$ and $\{1 + 0.356(\frac{x}{a})^{\frac{1}{2}}\}/f_1$ in the equations (3) and (4) corresponding to the observation of Orgler and Paschen respectively are independent of the pressure and the nature of the gas.

For sparks, which are not very short, between parallel plates the relation between the spark potential and the spark length is a linear one. If V is the spark potential measured in electrostatic units and

x the spark length at atmospheric pressure measured in centimeters, then

$$(5) \qquad V = C + R_m x.$$

The values of C and R_m obtained from the experiments made by Baille[1] and Liebig[2] are given in Table XIII.

TABLE XII.

Paschen, $a = 1$ c.m., the values of C and R_m, p the pressure in c.m. of mercury.

p	Air		H_2		CO_2	
	C	R_m	C	R_m	C	R_m
2	1.68	5.58	1.1	3.03	2.6	5.14
6	1.80	13.00	1.7	7.0	3.2	12.0
10	2.70	18.57	1.9	10.33	4.2	16.2
20	4.0	31.5	2.3	18.0	6.1	26.8
30	4.5	44.9	2.8	24.8	7.1	37.8
40	5.1	57.5	3.2	31.4	8.1	48.0
50	5.3	66.9	3.7	37.5	8.1	59.9
60	5.7	81.8	4.2	43.5	8.4	70.8
70	—	—	4.9	49.0	8.8	80.9
75	6.6	98.3	4.8	52.3	9.0	85.6

TABLE XIII.

	Air		H_2		CO_2	
	C	R_m	C	R_m	C	R_m
Baille	5.0	99.6	—	—	—	—
Liebig	7.7	100.5	11.0	53.0	12.0	90.0

All the expressions from (1) to (4) may be extended to the case of sparks between parallel planes, by making the value of the radius a of the electrode infinite. Thus putting x/a equal to zero and f, f_1

---

[1] Baille, loc. cit.

[2] Liebig, Phil. Mag., **24**, 106, (1887).

each equal to 1, all these expressions take the form given in (5). If the spark potential is represented in electrostatic units, and the spark length in centimeter, the constants C and R_m thus obtained for gases at atmospheric pressure take the values shown in Table XIV.

TABLE XIV.

	Air		H_2		CO_2		N_2		O_2	
	C	R_m	C	R_m	C	R_m	C	R_m	C	R_m
Algermissen and Heydweiller ...	13.3	100.0								
Baille	5.0	98.5								
Orgler	3.8	100.0	3.4	53.0	4.7	85.7	4.8	101 4	3.8	88.0
Paschen	6.6	98.3	4.8	52.3	9.0	85.6	—	—	—	—
Mean	7.1	99.2	4.1	52.7	6.9	85.7	4.8	101.4	3.8	88.0

In this table, the numerical values of C and R_m for Algermissen's and Heydweiller's experiments are taken from the equation (1), and those for Baille are taken from the equation (2); and those for Orgler's and Paschen's experiments the values at the pressure of 75 c.m.s of mercury as shown in Table XI and XII. The values of R_m for a gas shown in Table XIV are in fair agreement to each other, and they are also equal to those given in Table XIII, where they were deduced from the observations on the spark potentials between two nearly parallel plates. As to the values of C, we notice that they differ from each other by a pretty amount. This may possibly due to some unknown causes. But, as this value of C is small compared with the total spark potential, it is very difficult to determine its value accurately. Thus we may conclude that the value deduced from the experiments of different authors is, at least, of the same order of magnitude.

The spark potential difference between two parallel plates increases linearly with the spark length but not proportionally, on account of the constant term C as stated before. Perhaps this may be due to the fact that there is a certain special difference of potential between the electrode and the gas just in contact with it, somewhat similar to the anode or cathode fall of potential in the case of vacuum discharges. This special difference of potential between the electrode and the gas just in contact with it would perhaps be caused by the accumulation of ions generated in the gas by the preliminary ionisation in the stage of the so-called "lag" of the spark. If we interpret the constant term C in such a way, the meaning of the other constant $R_m = dV/dx$ becomes now clear; indeed it represents the intensity of the electric field in the gas when the spark discharge just occurs and it may be termed "the dielectric strength" of the gas as was commonly called.

It was already stated that, in the case of the spark discharge between spheres, the spark potential can be represented by a general formula of the following type:—

$$(6) \qquad V - C = R_m \times \frac{1 + \alpha\left(\frac{x}{a}\right)^{\frac{1}{2}}}{f'}$$

where C, R_m and α are constants and f' represents either f or f_1 which are the functions of $\frac{a}{x}$ and correspond respectively to the case of the two isolatad electrodes and the case of the one electrode earthed. If we assume that the constant term C is the potential difference between the electrode and the gas just in contact with it, and that, as the ideal case, the spherical electrodes have no leading-in-wire, then the relation between the spark potential V and the maximum electric field R_m in the gas beyond the region of the cathode or anode fall of potential is connected by the expression

$$(7) \qquad V - C = R_m \frac{x}{f'}.$$

When the right hand member of this equation is multiplied by the factor $1 + \alpha\left(\frac{x}{a}\right)^{\frac{1}{2}}$, the equation becomes identical to the formula (6). It was already stated that the constant α in this factor was independent of the pressure and the nature of the gas. Thus considering it seems not to be absurd to imagine that the above factor is a correction to be applied to $1/f'$ in the equation (7) in virtue of the presence of leading-in-wires and some other bodies in the neighborhood of the spark gap.

If this consideration is allowed, the constant R_m in the empirical formula (6) will have the same meaning as that in the equation (7) and it will represent the maximum electric field in the gas beyond the region of the cathode or anode fall of potential; and this important constant may be called "the dielectric strength" of the gas.

Under such consideration, the spark discharge seems to occur between spheres when there is a certain amount of potential fall between the electrode and the gas just in contact with it perhaps due to the accumulation of ions toward the electrodes, and when the intensity of the electric field in the gas just beyond this region of the cathode or anode fall of potential has attained to a certain value characteristic to the pressure and the nature of the gas. When once the electric field of this kind was established between the electrodes, the brush discharge would start and grow from either of the two electrodes or from the both simultaneously; and the spark will pass, at last, across the electrodes along the path made by the brush discharge.

In conclusion the writer wishes to express his hearty thanks to Prof. Mizuno for his interest in the research.

Fig. 1.

• Observed by Orgler, Table XI.

× Observed by Paschen, Table XII.