The Theory of General Reltaivity in a Physically Flat Space.[1]

By

Bunsaku Arakatsu.

(Received February 2, 1925)

ABSTRACT.

Starting from a new definition for the general covariant derivatives of tensors in a space defined by $ds^2 = g_{\mu\nu} dx_\mu dx_\nu$, the present writer proposes a theory of general relativity from the conception that the physical world must be a flat space in his view of the rate of change of any vector. The condition for the physically flat space involves the two important equations, (i) Maxwell's equations of electrodynamics, (ii) Einstein's equation of gravitation.

I. Introduction.

According to the principle of general relativity the rate of change of a physical entity should be expressed by a tensor of some kind in order that it may enter into the general physical laws.

Furthermore in order to agree with the customary definition in elementary cases, the above rate of change should be reduced to that of its rectangular components when the co-ordinates are Galilean.

Since the ordinary covariant derivative of a contravariant vector A^μ defined by

$$\left(A^\mu\right)_\nu \equiv \frac{\partial A^\mu}{\partial x_\nu} + \{\alpha\nu,\mu\} A^\alpha, \qquad (1\cdot1)$$

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu, \qquad (1\cdot2)$$

is a tensor, and reduces to $\frac{\partial A^\mu}{\partial x_\nu}$ in the Galilean space, this was taken

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1 This is a part of the paper read before the annual meeting of the Physico-Mathematical Society of Japan on April 1st, 1924.

as the rate of change of a physical entity in the theory of general relativity.

As is well known, the necessary and sufficient condition of integrability of the differential equations

$$(A^\mu)_\nu \equiv \frac{\partial A^\mu}{\partial x_\nu} + \{\varepsilon\nu,\mu\} A^\epsilon = 0, \quad \text{.......................} \quad (1\cdot 3)$$

is

$$B^\epsilon_{\mu\nu\sigma} = 0, \quad \text{..} \quad (1\cdot 4)$$

where

$$B^\epsilon_{\mu\nu\sigma} \equiv \{\mu\sigma,\alpha\}\{\alpha\nu,\varepsilon\} - \{\mu\nu,\alpha\}\{\alpha\sigma,\varepsilon\}$$

$$+ \frac{\partial}{\partial x_\nu}\{\mu\sigma,\varepsilon\} - \frac{\partial}{\partial x_\sigma}\{\mu\nu,\varepsilon\}, \quad \text{.......................} \quad (1\cdot 5)$$

and therefore this equation (1·4) expresses the condition for building up a uniform vector field.

When the condition (1·4) is satisfied the space (1·2) is said to be flat.

When there exists no matter and no field the equations (1·3) are integrable everywhere, so that the space is flat. But when matter and field are present

$$B^\epsilon_{\mu\nu\sigma} \neq 0, \quad \text{..} \quad (1\cdot 6)$$

and in this case the differential equations (1·3) are not integrable except along some special curves, and so the space is distorted.

In such a case the space has an irreducible curvature, and so we can not construct a uniform vector field by the parallel displacement defined by (1·3). The gravitational field and the law of energy are expressed in terms of this curvature of the space.

In order to express the coexistence of the electromagnetic field and of the matter in terms of the curvature of the space, the idea to geometralize the physical space is further extended by Weyl, Eddington, and Einstein.

On the return voyage from Japan, Prof. A. Einstein, by adopting Eddington's idea of the extended parallel conditions for the displacement vector A^μ, namely,

$$\frac{\partial A^\mu}{\partial x_\nu} = -\Gamma^\mu_{\epsilon\nu} A^\epsilon, \quad \text{...................................} \quad (1\cdot 7)$$

where

$$\Gamma^{\mu}_{\epsilon\nu} = \Gamma^{\mu}_{\nu\epsilon}, \qquad \text{(1·8)}$$

deduced an extended curvature equation, and applying Hamilton's principle, he finally arrived at Maxwell's equations of electrodynamics, as the first approximation of his extended field equations.

The aim of the writer is now to extend the theory in a different way.

II. Covariant Derivative of a Vector.

Let the covariant derivative of any contravariant vector A^{μ} be defined by

$$\left[A^{\mu}\right]_{\nu} \equiv \frac{\partial A^{\mu}}{\partial x_{\nu}} + \Gamma^{\mu}_{\epsilon\nu} A^{\epsilon}, \qquad \text{(2·1)}$$

where the affine condition

$$\Gamma^{\mu}_{\epsilon\nu} = \Gamma^{\mu}_{\nu\epsilon} \qquad \text{(2·2)}$$

is retained, and $\left[A^{\mu}\right]_{\nu}$ must be, of course, a mixed tensor.

Instead of Eddington-Einstein's extended conception, we confine ourselves only to the space defined by

$$ds^2 = g_{\mu\nu} dx_{\mu} dx_{\nu}. \qquad \text{(2·3)}$$

Since

$$\left(A^{\mu}\right)_{\nu} \equiv \frac{\partial A^{\mu}}{\partial x_{\nu}} + \left\{ {\mu \atop \epsilon\nu} \right\} A^{\epsilon} \qquad \text{(2·4)}$$

is a covariant derivative of a vector A^{μ}, it is easily inferred that $\left[A^{\mu}\right]_{\nu}$ may be generally expressed by

$$\left[A^{\mu}\right] \equiv \frac{\partial A^{\mu}}{\partial x_{\nu}} + \left\{ {\mu \atop \epsilon\nu} \right\} A^{\epsilon} + \Phi S^{\mu}_{\epsilon\nu} A^{\epsilon}, \qquad \text{(2·5)}$$

where Φ is an invariant and $S^{\mu}_{\epsilon\nu}$ is a mixed tensor in the space defined by (2·3), and consequently $\Phi S^{\mu}_{\epsilon\nu} A^{\epsilon}$ is a tensor of equal rank with the ordinary covariant derivative.

This definition of the covariant derivative expresses the most general rate of change of a physical entity in the space (2·3); it fulfils the conditions mentioned at the outset of the paper.

Since $\left\{ {\nu \atop \varepsilon\nu} \right\}$ is not a tensor and $\Phi S^{\mu}_{\epsilon\nu}$ has the tensor properties, it is evident that in general

$$\Gamma^{\mu}_{\epsilon\nu} \equiv \left\{ {\mu \atop \varepsilon\nu} \right\} + \Phi S^{\mu}_{\epsilon\nu} \quad \text{..........} \quad (2\cdot6)$$

does not vanish, except in the case where each of the members vanishes separately.

We may call $\dfrac{\partial A^{\mu}}{\partial x_{\nu}}$ the *analyitical* rate of change, $\dfrac{\partial A^{\mu}}{\partial x_{\nu}} + \left\{ {\mu \atop \varepsilon\nu} \right\} A^{\epsilon}$ the *geometrical* rate of change and $[A^{\mu}]_{\nu} \equiv \dfrac{\partial A^{\mu}}{\partial x_{\nu}} + \left(\left\{ {\mu \atop \varepsilon\nu} \right\} + \Phi S^{\mu}_{\epsilon\nu} \right) A^{\epsilon}$ the *physical* rate of change of any vector A^{μ} in our space.

Corresponding to the definition of the covariant derivative of a contravariant vector A^{μ} (2·5), the covariant derivative of any covariant vector A_{μ} is naturally defined by

$$[A_{\mu}]_{\nu} \equiv \frac{\partial A_{\mu}}{\partial x_{\nu}} - \left\{ {\varepsilon \atop \mu\nu} \right\} A_{\epsilon} + \Phi S^{\beta}_{\alpha\nu} g^{\alpha\epsilon} g_{\beta\mu} A_{\epsilon}, \quad \text{..........} \quad (2\cdot7)$$

and also those of the tensors are appropriately defined as follows:

$$[A^{\mu\nu}]_{\sigma} \equiv (A^{\mu\nu})_{\sigma} + \Phi\left(S^{\mu}_{\alpha\sigma} A^{\alpha\nu} + S^{\nu}_{\alpha\sigma} A^{\mu\alpha} \right), \quad \text{..........} \quad (2\cdot8)$$

$$[A^{\nu}_{\mu}]_{\sigma} \equiv (A^{\nu}_{\mu})_{\sigma} + \Phi\left(S^{\mu}_{\sigma\epsilon} A^{\epsilon}_{\mu} + S^{\beta}_{\alpha\sigma} g^{\alpha\epsilon} g_{\beta\mu} A^{\nu}_{\epsilon} \right), \quad \text{..........} \quad (2\cdot9)$$

etc.

Now let us consider the derivatives of the fundamental tensors of our space. We have by (2·9)

$$[g^{\nu}_{\mu}]_{\sigma} = \Phi S^{\nu}_{\sigma\mu} + \Phi S^{\beta}_{\alpha\sigma} g^{\alpha\nu} g_{\beta\mu}, \quad \text{..........} \quad (2\cdot10)$$

But since $g^{\nu}_{\mu} = 0$ when $\mu \neq \nu$,

and $\quad g^{\nu}_{\mu} = 1$ when $\mu = \nu$, ... (2·11)

$[g^{\nu}_{\mu}]_{\sigma}$ must be zero everywhere, and, therefore, we must have

$$S^{\nu}_{\sigma\mu} - = S^{\beta}_{\alpha\sigma} g^{\alpha\nu} g_{\beta\mu}, \quad \text{..........} \quad (2\cdot12)$$

and consequently,

$$S_{\sigma\mu\cdot\nu} = - S_{\nu\sigma\cdot\mu}. \quad \text{..........} \quad (2\cdot12)$$

Hence we have, as might be expected,

$$\left[A_\mu\right]_\nu \equiv \frac{\partial A_\mu}{\partial x_\nu} - \left(\begin{Bmatrix}\varepsilon \\ \mu\nu\end{Bmatrix} + \Phi S^{\epsilon}_{\mu\nu}\right)A_\epsilon, \quad \ldots\ldots\ldots\ldots \quad (2\cdot13)$$

$$\left[A^{\nu}_{\mu}\right]_\sigma \equiv \left(A^{\nu}_{\mu}\right)_\sigma + \Phi\left(S^{\nu}_{\sigma\epsilon}A^{\epsilon}_{\mu} - S^{\epsilon}_{\mu\sigma}A^{\nu}_{\epsilon}\right), \quad \ldots\ldots\ldots\ldots \quad (2\cdot14)$$

$$\left[A_{\mu\nu}\right]_\sigma \equiv \left(A_{\mu\nu}\right)_\sigma - \Phi\left(S^{\alpha}_{\mu\sigma}A_{\alpha\nu} + S^{\alpha}_{\nu\sigma}A_{\mu\alpha}\right). \quad \ldots\ldots\ldots\ldots \quad (2\cdot15)$$

From (2·14) we have

$$\left[A^{\nu}_{\nu}\right]_\sigma = \left(A^{\nu}_{\nu}\right)_\sigma = \frac{\partial A^{\nu}_{\nu}}{\partial x_\sigma},$$

hence

$$\left[\,I\,\right]_\sigma \equiv \frac{\partial I}{\partial x_\sigma}. \quad \ldots\ldots\ldots\ldots \quad (2\cdot16)$$

We can easily show that the differentiation of a product of tensors can be carried on by the relation

$$\left[A_\mu B^\nu\right]_\sigma = \left[A_\mu\right]_\sigma B^\nu + A_\mu\left[B^\nu\right]_\sigma. \quad \ldots\ldots\ldots\ldots \quad (2\cdot17)$$

Now

$$\left[g_{\mu\nu}\right]_\gamma = -\Phi\left(S^{\alpha}_{\mu\sigma}\,g_{\alpha\nu} + S^{\alpha}_{\nu\alpha}\,g_{\mu\alpha}\right) \quad \ldots\ldots\ldots\ldots \quad (2\cdot18)$$

$$= -\Phi\left(S_{\mu\sigma\cdot\nu} + S_{\nu\sigma\cdot\mu}\right)$$

$$= 0, \qquad \text{(by 2·12).} \quad \ldots\ldots\ldots\ldots \quad (2\cdot19)$$

Similarly, we can show that

$$\left[g^{\mu\nu}\right]_\sigma = 0. \quad \ldots\ldots\ldots\ldots \quad (2\cdot20)$$

Thus we see that our extended covariant derivatives have properties similar to those of the ordinary covariant derivatives.

III. Physically Flat World.

We may consider that the space, which was initially flat, must have some sorts of the properties of flatness, so to speak, even after it has been distorted by the presence of matter and fields, and the physical world is constructed by some vector fields which have some kinds of the properties of uniformity.

Therefore, let us conceive that the physical world is such a space that, when we take, at any point in it, a displacement vector which constitutes an element of the space, and transfer it successively to an adjoining point without any physical rate of change, then we obtain a unique vector field independent of the path of its transference. Thus we may regard the physical world to be constituted of systems of uniform vector fields, and it may be called physically flat in an extended sense.

IV. Field Equations I:—Conditions of Physical Flatness.

Let us take a displacement vector A^μ in the space defined by

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu . \qquad (4\cdot1)$$

In order that the space be physically flat, we must have the differential equation

$$\left[A^\mu\right]_\nu \equiv \frac{\partial A^\mu}{\partial x_\nu} + \Gamma^\mu_{\epsilon\nu} A^\epsilon = 0, \qquad (4\cdot2)$$

where

$$\Gamma^\mu_{\epsilon\nu} = \left\{ {\mu \atop \epsilon\nu} \right\} + \Phi S^\mu_{\epsilon\nu}, \qquad (4\cdot3)$$

and

$$\Gamma^\mu_{\epsilon\nu} = \Gamma^\mu_{\nu\epsilon}, \; S^\mu_{\epsilon\nu} = S^\mu_{\nu\epsilon} . \qquad (4\cdot3)$$

The necessary and sufficient condition for integrability of the above equation (4·2) everywhere in this physical space is

$$\left[B^\epsilon_{\mu\nu\sigma}\right] = 0, \qquad (4\cdot4)$$

where

$$\left[B^\epsilon_{\mu\nu\epsilon}\right] \equiv -\frac{\partial}{\partial x_\sigma}\Gamma^\epsilon_{\nu\mu} + \frac{\partial}{\partial x_\nu}\Gamma^\epsilon_{\sigma\mu} + \Gamma^\alpha_{\sigma\mu}\Gamma^\epsilon_{\nu\alpha} - \Gamma^\alpha_{\nu\mu}\Gamma^\epsilon_{\sigma\alpha}, \quad \ldots\ldots \qquad (4\cdot5)$$

or

$$\left[B^\epsilon_{\mu\nu\sigma}\right] \equiv B^\epsilon_{\mu\nu\sigma} - \left(\Phi S^\epsilon_{\mu\nu}\right)_\sigma + \left(S\Phi^\epsilon_{\mu\sigma}\right)_\nu + \Phi^2 S^\alpha_{\mu\sigma} S^\epsilon_{\nu\alpha} - \Phi^2 S^\alpha_{\mu\nu} S^\epsilon_{\sigma\alpha} = 0, \quad (4\cdot6)$$

$$\therefore \; B^\epsilon_{\mu\nu\sigma} = \left(\Phi S^\epsilon_{\mu\nu}\right)_\sigma - \left(\Phi S^\epsilon_{\mu\sigma}\right)_\nu - \Phi^2 S^\alpha_{\mu\sigma} S^\epsilon_{\nu\alpha} + \Phi^2 S^\alpha_{\mu\nu} S^\epsilon_{\sigma\alpha} . \qquad (4\cdot7)$$

where the suffix $(\quad)_\sigma$ and $(\quad)_\nu$ represent the ordinary covariant differentiation.

If we express the right hand side of (4·7) in the form of the extended physical covariant differentiation we have

$$B^\epsilon_{\mu\nu\sigma} = \left[\Phi S^\epsilon_{\mu\nu}\right]_\sigma - \left[S\Phi^\epsilon_{\mu\sigma}\right]_\nu + \Phi^2 S^\alpha_{\mu\sigma} S^\epsilon_{\nu\alpha} - \Phi^2 S^\alpha_{\mu\nu} S^\epsilon_{\sigma\alpha} \ldots\ldots \qquad (4\cdot8)$$

The expression of $\left[B_{\mu\gamma\sigma}^{\ \ \ \epsilon}\right]$ of the equation (4·5) may be symbolically written in a form similar to (4·8), namely,

$$\left[B_{\mu\nu\sigma}^{\ \ \ \epsilon}\right] \equiv \left[\Gamma_{\mu\sigma}^{\ \ \epsilon}\right]_{\nu} - \left[\Gamma_{\mu\nu}^{\ \ \epsilon}\right]_{\sigma} + \Gamma_{\mu\nu}^{\ \ \alpha}\Gamma_{\sigma\alpha}^{\ \ \epsilon} - \Gamma_{\mu\sigma}^{\ \ \alpha}\Gamma_{\nu\alpha}^{\ \ \epsilon}. \quad \text{(4·9)}$$

As the expression of the right hand side of (4·8) is thus analogous to the extended Riemann-Christoffel tensor, we will call it a "Riemann-Christoffel tensor of the tensor type" and denote it by the symbol

$$-\left[\mathbf{B}_{\mu\gamma\sigma}^{\ \ \ \epsilon}\right] \equiv \left[\Phi S_{\mu\nu}^{\ \ \epsilon}\right]_{\sigma} - \left[\Phi S_{\mu\sigma}^{\ \ \epsilon}\right]_{\nu} + \Phi^2 S_{\mu\sigma}^{\ \ \alpha} S_{\gamma\alpha}^{\ \ \epsilon} - \Phi^2 S_{\mu\nu}^{\ \ \alpha} S_{\sigma\alpha}^{\ \ \epsilon}. \quad \text{(4·10)}$$

Then the condition of the physical flatness of the world becomes

$$B_{\mu\nu\sigma}^{\ \ \ \epsilon} = -\left[\mathbf{B}_{\mu\nu\sigma}^{\ \ \ \epsilon}\right]. \quad \text{(4·11)}$$

This is the field equation of the physical world.

V. Field Equation II :—(a). Maxwell's Equation of Electrodynamics.

Contracting $\left[B_{\mu\nu\sigma}^{\ \ \ \epsilon}\right] = 0$ we obtain the covariant equation of the second rank

$$\left[B_{\mu\gamma\epsilon}^{\ \ \ \epsilon}\right] \equiv \left[G_{\mu\nu}\right] = 0, \quad \text{(5·1)}$$

and

$$\left[B_{\epsilon\nu\sigma}^{\ \ \ \epsilon}\right] \equiv \left[F_{\gamma\sigma}\right] = 0. \quad \text{(5·2)}$$

The first equation (5·1) becomes

$$G_{\mu\nu} = -\left[\mathbf{G}_{\mu\nu}\right], \quad \text{(5·3)}$$

but, since $G_{\mu\nu}$ is symmetrical with respect to μ and γ, the antisymmetrical part of $\left[\mathbf{G}_{\mu\nu}\right]$ must drop off. Therefore, we have

$$G_{\mu\nu} = -\frac{1}{2}\left(\left[\mathbf{G}_{\mu\nu}\right] + \left[\mathbf{G}_{\gamma\mu}\right]\right) \quad \text{(5·4)}$$

and

$$\left[\mathbf{G}_{\mu\nu}\right] - \left[\mathbf{G}_{\nu\mu}\right] = 0, \quad \text{(5·5)}$$

where

$$-\left[\mathbf{G}_{\mu\nu}\right] \equiv \left[\Phi S_{\mu\nu}^{\ \ \epsilon}\right]_{\epsilon} - \left[\Phi S_{\mu\epsilon}^{\ \ \epsilon}\right]_{\nu} + \Phi^2 S_{\mu\epsilon}^{\ \ \alpha} S_{\gamma\alpha}^{\ \ \epsilon} - \Phi^2 S_{\mu\nu}^{\ \ \alpha} S_{\epsilon\alpha}^{\ \ \epsilon}. \quad \text{(5·6)}$$

The second equation (5·2) gives the same equation as (5·5), and therefore (5·4) and (5·5) or

$$G_{\mu\nu}=-\left[\mathbf{G}_{\mu\nu}\right] \quad \cdots\cdots \quad (5\cdot7)$$

and

$$\left[\mathbf{G}_{\mu\nu}\right]=\left[\mathbf{G}_{\nu\mu}\right] \quad \cdots\cdots \quad (5\cdot8)$$

are the physical laws expressed by the tensors of the second rank.

The equation (5·8) is reduced to

$$\left[\Phi S_{\mu\alpha}^{\ \alpha}\right]_\nu-\left[\Phi S_{\nu\alpha}^{\ \alpha}\right]_\mu=0, \quad \cdots\cdots \quad (5\cdot9)$$

and if we have

$$S_{\mu\alpha}^{\ \alpha}=\Psi\kappa_\mu \quad \cdots\cdots \quad (5\cdot10)$$

where Ψ is an invariant, then

$$\left[\kappa_\mu\Psi\Phi\right]_\nu-\left[\kappa_\nu\Psi\Phi\right]_\mu=0,$$

therefore

$$\left[\kappa_\mu\right]_\nu-\left[\kappa_\nu\right]_\mu=\kappa_\nu\frac{\partial log(\Phi\Psi)}{\partial x_\mu}-\kappa_\mu\frac{\partial log(\Phi\Psi)}{\partial x_\nu}. \quad \cdots\cdots \quad (5\cdot11)$$

Putting

$$\frac{\partial log(\Phi\Psi)}{\partial x_\nu}=\lambda_\nu \quad \cdots\cdots \quad (5\cdot12)$$

we have

$$\left[\kappa_\mu\right]_\nu-\left[\kappa_\nu\right]_\mu=\kappa_\nu\lambda_\mu-\kappa_\mu\lambda_\nu \quad \cdots\cdots \quad (5\cdot13)$$

or

$$\frac{\partial\kappa_\mu}{\partial x_\nu}-\frac{\partial\kappa_\nu}{\partial x_\mu}=\kappa_\nu\lambda_\mu-\kappa_\mu\lambda_\nu \quad \cdots\cdots \quad (5\cdot14)$$

Let us denote the antisymmetrical tensor of the left hand side of this equation by $F_{\mu\nu}$, thus

$$F_{\mu\nu}\equiv\left[\kappa_\mu\right]_\nu-\left[\kappa_\nu\right]_\mu, \quad \cdots\cdots \quad (5\cdot14)$$

$$\equiv\frac{\partial\kappa_\mu}{\partial x_\nu}-\frac{\partial\kappa_\nu}{\partial x_\mu} \quad \cdots\cdots \quad (5\cdot14)$$

then the equation (5·13) becomes

$$F_{\mu\nu}=\kappa_\nu\lambda_\mu-\kappa_\mu\lambda_\nu\ . \quad \cdots\cdots \quad (5\cdot15)$$

From (5·14) it is evident that Maxwell's equation

$$\frac{\partial F_{\mu\nu}}{\partial x_\sigma}+\frac{\partial F_{\nu\sigma}}{\partial x_\mu}+\frac{\partial F_{\sigma\mu}}{\partial x_\nu}=0 \quad \text{(5·16)}$$

is satisfied.

Further, if we denote the divergence of the raised tensor of $\left(\kappa_\nu\lambda_\mu-\kappa_\mu\lambda_\nu\right)$ by J^μ,

$$J^\mu\equiv\left[\kappa^\nu\lambda^\mu-\kappa^\mu\lambda^\nu\right]_\nu \quad \text{(5·17)}$$

we have the other part of the Maxwell equation

$$\left[F^{\mu\nu}\right]_\nu=J^\mu . \quad \text{(5·18)}$$

Thus we see that the equation (5·13) includes the law of electrodynamics, and, therefore, the equation (5·9), or rather (5·8), expresses exactly Maxwell's equations of electrodynamics. The laws of electrodynamics are the conditions for the symmetry of "Einstein's curvature of the tensor type."

(b). Einstein's Equation of Gravitation.

Since

$$G^{\mu\nu}=-\left[\mathbf{G}^{\mu\nu}\right],$$

we have

$$G^\nu_\mu=-\left[\mathbf{G}^\nu_\mu\right] \quad \text{(5·19)}$$

and

$$G=-\left[\mathbf{G}\right]. \quad \text{(5·20)}$$

Consider the tensor

$$G^\nu_\mu-\tfrac{1}{2}g^\nu_\mu G+\lambda g^\nu_\mu \quad \text{(5·21)}$$

where λ is a constant.

This must be equal to the tensor

$$\left[\mathbf{G}^\nu_\mu\right]-\tfrac{1}{2}g^\nu_\mu\left[\mathbf{G}\right]-\lambda g^\nu_\mu \quad \text{(5·22)}$$

If we denote this by $8\pi\left[T^\nu_\mu\right]$, *i.e.*,

$$\left[\mathbf{G}^\nu_\mu\right]-\tfrac{1}{2}g^\nu_\mu\left[\mathbf{G}\right]-\lambda g^\nu_\mu\equiv8\pi\left[T^\nu_\mu\right], \quad \text{(5·23)}$$

we have

$$G_{\mu}^{\nu}-\tfrac{1}{2}g_{\mu}^{\nu}G+\lambda g_{\mu}^{\nu}=-8\pi\left[T_{\mu}^{\nu}\right] \qquad (5\cdot 24)$$

which is the form of Einstein-de Sitter's equation in which $\left[T_{\mu}^{\nu}\right]$ represents the energy tensor of all kinds of our physical phenomena.

In conclusion the author's thanks are due to Prof. K. Tamaki for his kind advice throughout this work.