# On the Theory of Reverberation

By

**Keiji Yamashita**

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ABSTRACT

This paper consists of four parts. In Part I two differential equations generally used for interpreting the reverberation and investigating the acoustic properties of building materials are discussed. In Part II the growth of sound in a single room when a source is sounding and its decay after the emission is stopped are studied, and a new formula for reverberation is obtained. In Part III the growth and decay of sound in two adjacent rooms are considered, and a new method of measuring the acoustic properties of building materials is proposed. In Part IV the method used by E. A. Eckhardt and V. L. Chrisler[1] for measuring the transmission of sound is discussed, and it is proved that the " reduction factor " defined by them is not a pure constant but a function depending generally on the rooms where the observations are made.

## Part I

If $e_1$ be the sound energy incident on the surface of a body in a room, for instance, the wall, the ceiling, the floor, or a piece of furniture, and if $e_2$ be the part of $e_1$ which is preserved as sound energy in the room after the incidence, then we define the " coefficient of reduction " of the body by

$$r \equiv \frac{e_1 - e_2}{e_1}, \qquad (1.1)$$

and assume that it is a constant depending on the nature of the body, but not on the incident energy $e_1$.

Let us consider the case where the room and the furniture are made

1 E. A. Eckhardt & V. L. Chrisler, B. S., Sci. Pap., **21**, 37 (1926).

of $\nu$ different kinds of material whose coefficients of reduction are $r_1$, $r_2$, .........$r_\nu$; and whose respective surface areas are $S_1$, $S_2$,......$S_\nu$.

Let $\tau$ represent the mean value of the time-intervals between successive incidences of the sound energy on all the bodies in the room, and $N$ the reciprocal of $\tau$, *i. e.* the mean number of incidences in unit time.

Let us further assume that (A) the incidence of the sound energy in the room on the bodies takes place at equal intervals $\tau$, and that (B) the incident amount of energy is divided proportionally among their respective surface areas.

When there is a source of sound in the room which can emit energy continuously at a constant rate $\varepsilon$, and if its emission is started, the sound energy in the room increases; and then if the emission is stopped, it begins to decrease. While if the emission is continued, the sound energy in the room tends to have a limiting value, and the state becomes steady.

Let $E_0^{(\infty)}$ be the sound energy in the room in this steady state. Then the part of $E_0^{(\infty)}$ which strikes the surface of the area $S_i$ in an interval $\tau$ is $E_0^{(\infty)}\dfrac{S_i}{S}$, and consequently the loss of sound energy on that surface in an interval $\tau$ is $E_0^{(\infty)}\dfrac{S_i}{S}r_i$; where

$$S \equiv S_1 + S_2 + \dots\dots\dots\dots + S_\nu.$$

Therefore, the loss of sound energy from the room in an interval $\tau$ in the steady state is given by

$$\left.\begin{aligned} &\sum_{i=1}^{\nu} E_0^{(\infty)}\frac{S_i}{S}r_i = E_0^{(\infty)}R, \\ \text{where}\quad &R \equiv \frac{S_1r_1+S_2r_2+\dots\dots+S_\nu r_\nu}{S}; \end{aligned}\right\}\dots\dots\dots\dots\dots\dots(1.2)$$

and the sound energy lost in unit time is given by

$$\frac{E_0^{(\infty)}R}{\tau} = NRE_0^{(\infty)}\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1.3)$$

Thus we see that in the steady state the loss of sound energy from the room in unit time is proportional to the sound energy in the room at that instant, and that its proportional constant is given by $NR$.

Let us now inquire into the loss which would take place when the sound energy in the room is in a state of increasing or of decreasing. Is it reasonable to assume that (a) in a state of increasing or of decreasing the loss of sound energy from the room in unit time is still proportional to the sound energy in the room, and that (b) for the proportional constant the same proportional constant $NR$ in the steady state may be used? In other words, if $E'$, $E$ be the sound energies in the room in the increasing and the decreasing states, are the two differential equations

$$\frac{dE'}{dt} = \varepsilon - NRE', \quad \text{(1. 4)}$$

$$\frac{dE}{dt} = -NRE \quad \text{(1. 5)}$$

satisfied by $E'$ and $E$? The main object of Part I is to answer the above question.

If we assume these equations to be correct, we must have

$$E' = \frac{\varepsilon}{NR}\{1 - e^{-NRt}\}, \quad \text{(1. 6)}$$

$$E = E_0 e^{-NRt}; \quad \text{(1. 7)}$$

where $E_0$ is the sound energy when the emission is stopped, and $t$ in (1. 6) is measured from the instant when the sound from the source starts, and in (1. 7) from the instant when it ceases to emit. These expressions are identical with those obtained by G. Jäger[1], E. A. Eckhardt[2], S. Nakamura[3], and E. Buckingham[4]. The relation (1. 7) was already found by W. C. Sabine[5] before them.

Jäger also obtained the expression $N = \frac{cS}{4V}$; where $c$ is the speed of sound and $V$ the volume of the room, assuming that the sound energy is uniformly distributed throughout the room, both with regard to position and direction of propagation.

If we assume that the sound energy in the room after the source has ceased to emit sound is given by (1. 7), then the sound energy in the room at $t$ and $t + \tau$ are given by

$$E_t = E_0 e^{-NRt}, \quad E_{t+\tau} = E_0 e^{-NR(t+\tau)},$$

and therefore we have

$$E_{t+\tau} = E_t e^{-NR\tau} = E_t e^{-K} \quad \text{(1. 8)}$$

But the loss of sound energy from the room in the interval $\tau$ from $t$ to

1 G. Jäger, Wiener Sitz. Ber., **120**, 613 (1911).
2 E. A. Eckhardt, Frank. Inst., J., **195**, 799 (1923).
3 S. Nakamura, 日本學術協會報告, **1**, 452 (1925).
4 E. Buckingham, B. S., Sci. Pap., **20**, 193 (1925).
W. C. Sabine, Collected Papers on Acoustics, pp. 34-37, 39.

$t+\tau$ must be $E_tR$, which is obtained by the same method as (1.2); and we must have

$$E_{t+\tau} = E_t - E_tR = E_t(1 - R) \quad \dots\dots\dots\dots\dots\dots (1.9)$$

The expressions (1.8) and (1.9) are incompatible unless $R$ is zero. Hence we may conclude that (1.7) is not adaptable to the assumptions (A) and (B).

The inadaptability of (1.6) can similarly be inferred.

Now let us examine the differential equations (1.4) and (1.5). Let the curve in Fig. 1 show the decay of sound energy in the room after the emission is stopped. Let $X_1M_1$, $Y_1N_1$ represent the amounts of sound energy in the room at $t$ and $t+\tau$ respectively, and $X_1Y_1'$ the tangent to the curve at the point $X_1$, then we have

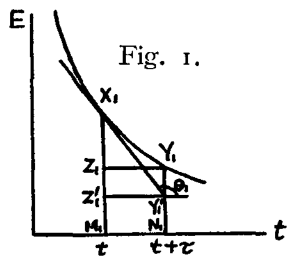



Fig. 1.

$$\begin{aligned} X_1Z_1 &= X_1M_1 - Y_1N_1 \\ &= E_t - E_{t+\tau} \\ &= E_tR \ . \end{aligned}$$

Dividing this by $\tau$, we get

$$\frac{X_1Z_1}{\tau} = NE_tR$$

and this is exactly equal to the absolute value of the right hand side of (1.5). Namely we have

$$-NRE = -\frac{X_1Z_1}{\tau} \quad \dots\dots\dots\dots\dots\dots (1.10)$$

But the left hand side of (1.5) is

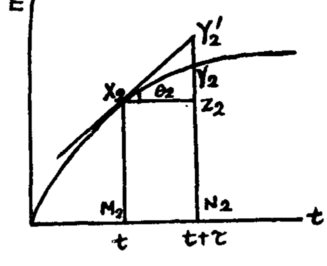
$$\frac{dE}{dt} = \tan\theta_1 = -\frac{X_1Z_1'}{Z_1'Y_1'} = -\frac{X_1Z_1'}{\tau}, \dots\dots (1.11)$$

and $X_1Z_1$ is generally unequal to $X_1Z_1'$.

Next, let Fig. 2 show the growth of sound energy in the room, then we have

$$\begin{aligned} Y_2Z_2 &= Y_2N_2 - X_2M_2 \\ &= E'_{t+\tau} - E'_t \\ &= (\varepsilon\tau + E'_t - E'_tR) - E'_t \\ &= \varepsilon\tau - E'_tR, \end{aligned}$$

Fig. 2.




and

$$\frac{Y_2Z_2}{\tau} = \varepsilon - NRE'_t \dots\dots\dots (1.12)$$

That is, the right hand side of (1.4) is

$\frac{Y_2Z_2}{\tau}$; but the left hand side of (1. 4) is

$$\frac{dE'}{dt} = \tan\theta_2 = \frac{Y'_2Z_2}{\tau}, \qquad \text{(1. 13)}$$

and $Y_2Z_2$ is generally unequal to $Y'_2 Z_2$.

Lastly, let $N$ increase to infinity and $R$ decrease to zero, while their product $NR$ retains a finite constant value $k$. Then at the limit, the left hand sides of (1. 5) and (1. 4), or (1. 10) and (1. 12) become

$$-kE = -\lim_{\tau\to 0}\frac{X_1Z_1}{\tau} = \tan\theta_1 = \frac{dE}{dt},$$

$$\varepsilon - kE' \equiv \lim_{\tau\to 0}\frac{Y_2Z_2}{\tau} = \tan\theta_2 = \frac{dE'}{dt}$$

respectively, and the reduced forms of (1. 4) and (1. 5), i. e.,

$$\frac{dE'}{dt} = \varepsilon - kE',$$

$$\frac{dE}{dt} = -kE$$

are satisfied. The case where $N$ tends to infinity and $R$ to zero, while $NR$ remains constant, will occur when the sound energy is lost by a certain continuous process, but not by such a discontinuous one as incidence.

When the sound energy in the room is discussed as a whole, the sum of all the fractions of sound energy which may be different from each other with regard to position or direction of propagation is considered. Since the sum is considered, the loss of sound energy from the room may appear to be continuous.

But, if the loss of some fractions of sound energy occur discontinuously by incidence, it is not reasonable to assume (1. 4) and (1. 5) to give the growth and decay of the sound energy in the room in a strict sense.

## Part II.

If $e_1$ be the sound energy incident on the surface of a body in a room, and if $e_2$ be the part of $e_1$ which is preserved as sound energy in the room after the incidence, then let the quantity $p$ given by

$$p \equiv \frac{e_2}{e_1} \qquad \text{(2. 1)}$$

be called the "coefficient of preservation" of the sound by the body, and assume that it is a constant depending on the nature of the body, but not on the incident energy $e_1$.

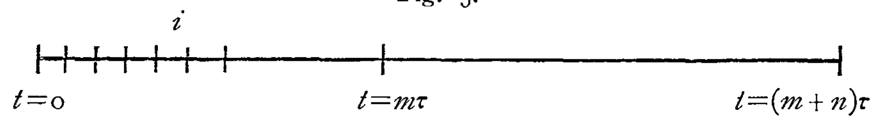
It should be noticed that the coefficient of preservation thus defined is different from the coefficient of reflection ordinarily used. Let $e_1$ be the incident sound energy and $e_1'$, $e_1''$, $e_1'''$ be the parts of $e_1$ which are reflected, transmitted, and dissipated by the body respectively. Then when the body struck by the sound wave is the wall of a room, $e_2=e_1'$ and the coefficient of preservation is the same as the coefficient of reflection; but if the body is a screen, $e_2=e_1'+e_1''$ and the coefficient of preservation is equal to the sum of the coefficient of reflection and that of transmission; while, if the screen last mentioned were used as a door, it is clear that we have $e_2=e_1'$.

Let $\tau$ represent the mean value of the time-intervals between successive incidences of the sound energy on all the bodies in the room, and $\varepsilon$ the rate of emission, assumed constant, of a continuous sound source in the room.

Let the source start emitting in the previously soundless room at the time $t=0$ and cease to sound at the time $t=m\tau$, then let us obtain the sound energy in the room at the time $t=(m+n)\tau$, where $m$ is a positive integer and $n$ a positive integer or zero.

Divide the time-interval from $t=0$ to $t=m\tau$ into $m$ equal short intervals and consider the sound energy emitted in the $i$-th short interval from the beginning, assuming that it strikes the bodies $(m+n-i)$ times by the time $t=(m+n)\tau$.

Fig. 3.




First, consider the simplest case where the coefficient of preservation $p$ is constant for all incidences. An empty room made of a single material, with neither windows nor entrances is an example. In this case the sound energy under consideration will be reduced to

$$\varepsilon\tau p^{m+n-i} \qquad \text{(2. 2')}$$

by $t=(m+n)\tau$ after $(m+n-i)$ incidences. Therefore, the sound energy $E$ in the room at $t=(m+n)\tau$ is given by

$$E = \sum_{i=1}^{m} \varepsilon\tau p^{m+n-i}$$

$$= \varepsilon\tau \frac{1-p^m}{1-p} p^n \qquad \text{(2. 3')}$$

Next, consider a more general case which would occur ordinarily; the room and the furniture are made of $\nu$ different kinds of material whose coefficients of preservation are $p_1, p_2, \ldots\ldots p_\nu$ and whose respective surface areas are $S_1, S_2, \ldots\ldots S_\nu$.

The sound energy $\varepsilon\tau$ emitted by the source in the $i$-th short interval is to strike the bodies once in the next interval. By assumption (B) we have

$$\varepsilon\tau \frac{S_1p_1+S_2p_2+\ldots\ldots+S_\nu p_\nu}{S}$$

as the reduced sound energy of $\varepsilon\tau$ at $t=(i+1)\tau$, where

$$S\equiv S_1+S_2+\ldots\ldots+S_\nu.$$

This sound energy will again be reduced to

$$\varepsilon\tau\left(\frac{S_1p_1+S_2p_2+\ldots\ldots+S_\nu p_\nu}{S}\right)^2$$

by $t=(i+2)\tau$, and to

$$\varepsilon\tau\left(\frac{S_1p_1+S_2p_2+\ldots\ldots+S_\nu p_\nu}{S}\right)^{m+n-i} \quad \ldots\ldots\ldots\ldots\ldots(2.2)$$

by $t=(m+n)\tau$. Hence the sound energy $E^{(m)}$ in the room at $t=(m+n)\tau$ is given by

$$\left.\begin{aligned} E^{(m)} &= \sum_{i=1}^{m}\varepsilon\tau\left(\frac{S_1p_1+S_2p_2+\ldots\ldots+S_\nu p_\nu}{S}\right)^{m+n-i} \\ &= \sum_{i=1}^{m}\varepsilon\tau\, P^{m+n-i} \\ &= \varepsilon\tau\frac{1-P^m}{1-P}P^n, \\ \text{where} \qquad P &\equiv \frac{S_1p_1+S_2p_2+\ldots\ldots+S_\nu p_\nu}{S} < 1. \end{aligned}\right\} \ldots\ldots\ldots(2.3)$$

The sound energy $E_0^{(m)}$ in the room, just when the source is stopped, is given by putting $n=0$ in (2.3), namely

$$E_0^{(m)} = \varepsilon\tau\frac{1-P^m}{1-P}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

which is exactly the same as obtained by W. C. Sabine.[1]

The sound energy in the room, when the state becomes steady by prolonged use of the source, may be given by the limiting value of (2.4) when $m$ tends to infinity, namely

1 W. C. Sabine, *op. cit.*, p. 44.

$$E_0^{(\infty)} = \varepsilon\tau \frac{1}{1-P} \quad \text{......................................(2. 5)}$$

This may be used to give the sound energy in the room even a few seconds after the source has started, when $P$ is very small.

By (2. 3) and (2. 4), we have

$$E^{(m)} = E_0^{(m)} P^n \quad \text{...................................... (2. 3. 1)}$$

This expression or (2. 3) gives the sound energy in the room at the time of an integral multiple of $\tau$, and not at any other time. But between $n$ and $t$ there exists a relation

$$t = (m + n)\tau.$$

Hence by puting

$$T \equiv t - m\tau,$$

we have

$$T = n\tau, \quad \text{or} \quad n = \frac{T}{\tau};$$

$T$ represents the time measured from the instant when the source is stopped. Therefore we may write (2. 3. 1) as

$$E^{(m)} = E_0^{(m)} p^{\frac{T}{\tau}} \quad \text{......................................(2. 3. 2)}$$

Now, let us assume that this represents the sound energy in the room at any instant $T$, which may be considered as a continuous variable.

Differentiating (2. 3. 2), we get

$$\frac{dE^{(m)}}{dt} = \frac{E_0^{(m)} \log P}{\tau} P^{\frac{T}{\tau}}$$

$$\left.\begin{aligned} &= -AE^{(m)}, \\ \text{where} \qquad A &\equiv -\frac{\log P}{\tau}. \end{aligned}\right\} \quad \text{...............................(2. 6)}$$

It follows from this that the rate of decay of the sound energy in the room at any instant after the emission has ceased is proportional to the sound energy in the room at that instant. This was experimentally found by W. C. Sabine[1].

By a similar reasoning it may be assumed that the sound energy in the room at any instant when the source is sounding is given by the following continuous function of $t$:

1 W. C. Sabine, *op. cit.*, pp. 34–37.

$$E_0^{\left(\frac{t}{\tau}\right)} = \varepsilon\tau \frac{1-P^{\frac{t}{\tau}}}{1-P}, \quad\text{..............................}\quad (2.4.1)$$

which is obtained from (2.4) by replacing $m$ by $\dfrac{t}{\tau}$. Differentiating this, we get

$$\frac{dE_0^{\left(\frac{t}{\tau}\right)}}{dt} = -\varepsilon \frac{\log P}{1-P} P^{\frac{t}{\tau}} \quad\text{........................}\quad (2.7)$$

This gives the rate of growth of the sound energy in the room when the source is sounding.

Let $D$ be the sound energy which is lost from the room in unit time in the steady state. Since in the steady state the loss of sound energy just balances the emission of the source, we must have

$$D = \varepsilon.$$

Hence, eliminating $\varepsilon$ from this and (2.5), we get

$$\left.\begin{aligned} D &= \frac{1-P}{\tau} E_0^{(\infty)} \\ &= A^{(\infty)} E_0^{(\infty)}, \\ \text{where} \quad A^{(\infty)} &\equiv \frac{1-P}{\tau}. \end{aligned}\right\} \quad\text{....................................}\quad (2.8)$$

Thus we see that in the steady state the loss of sound energy from the room in unit time is proportional to the sound energy in the room. This has been already obtained in Part I.

It is important to notice that $A$ in (2.6) is not equal to $A^{(\infty)}$ in (2.8), but they are approximately equal only when $P$ takes a value sufficiently near to 1; and in such a case (2.7) may be reduced to

$$\begin{aligned} \frac{dE_0^{\left(\frac{t}{\tau}\right)}}{dt} &\doteqdot \varepsilon P^{\frac{t}{\tau}} \\ &= \varepsilon - A^{(\infty)} E_0^{(\infty)} \quad\text{..........................}\quad (2.7.1) \end{aligned}$$

Since $\dfrac{dE_0^{\left(\frac{t}{\tau}\right)}}{dt}$ is the rate of growth of the sound energy in the room and $\epsilon$ is the emission of sound energy from the source in unit time, $A^{(\infty)}E_0^{(\infty)}$ represents the rate of the loss of sound energy from the room. Hence, if $P$ takes a value near to 1, the loss of sound energy from the room in unit time, when the source is sounding, is proportional to the sound energy in the room.

From these it follows that both when the sound energy in the room is increasing and decreasing, the loss of sound energy from the room in unit time may be approximately given by the produce of $A^{(\infty)}$

and the sound energy in the room, provided that $P$ takes a value sufficiently near to 1.

Let $T_2$ represent the time-interval required to reduce the sound energy in the room, after the emission is stopped, to one half of its initial value, then from (2. 3. 2) we get

$$\frac{1}{2} E_0^{(m)} = E_0^{(m)} P^{\frac{T_2}{\tau}},$$

or

$$T_2 = \frac{\log 2}{-\log P}\tau \qquad \text{(2. 9)}$$

Thus we see that, if $P$ remains constant, $T_2$ is proportional to $\tau$. This shows the reason why the reverberation is strong in large rooms.

Let $T_1$ be the time when $E^{(m)}$ in (2. 3. 2) becomes a given value $E_1$, then we have

$$E_1 = E_0^{(m)} \left( \frac{S_1 p_1 + S_2 p_2 + \ldots\ldots + S_\nu p_\nu}{S} \right)^{\frac{T_1}{\tau}}.$$

Keeping $E_1$ and $E_0^{(m)}$ as fixed values, let $S_1$ be increased by $S_{1,2}$ and $S_2$ be decreased by $S_{1,2}$, then we have

$$E_1 = E_0^{(m)} \left\{ P + \frac{(p_1 - p_2) S_{1,2}}{S} \right\},$$

and, therefore,

$$\tau \log \frac{E_0^{(m)}}{E_1} = - T_1 \left\{ \log \left( 1 + \frac{(p_1 - p_2) S_{1,2}}{PS} \right) + \log P \right\}.$$

If $\left| \frac{(p_1 - p_2) S_{1,2}}{PS} \right|$ be sufficiently small compared with 1, this may be written

$$\tau \log \frac{E_0^{(m)}}{E_1} \doteqdot T_1 \left\{ \frac{p_2 - p_1}{PS} S_{1,2} - \log P \right\} \ldots\ldots \text{(2. 10)}$$

This shows that there exists between $S_{1,2}$ and $T_1$ a relation of a rectangular hyperbola, which was found by W. C. Sabine[1] by experiments with cushions and open windows.

## Part III

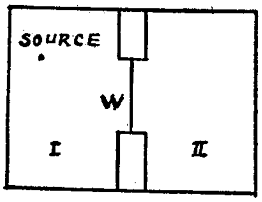
Let there be two adjacent rooms I and II which are in acoustic communication only through an incompletely soundproof panel $W$, which is set up as a part of the otherwise soundproof partition between the

1 W. C. Sabine, *op. cit.*, p. 20

rooms; and let the two rooms have the same acoustic properties, i. e. let $P$, $S$, and $\tau$ defined in Part II be the same for the two rooms.

Let us assume that the coefficient of transmission

Fig. 4.




$$q \equiv \frac{e_1''}{e_1}, \qquad \text{.....................(3. 1)}$$

where $e_1$ is the incident sound energy and $e_1''$ the part of the energy which passes through the body, is a constant independent of $e_1$. The coefficient of transmission $q$ takes, of course, different values for different bodies.

Let a uniform and continuous sound source be started at the time $t=0$ to emit the energy at the rate $\varepsilon$ in room I and the emission be stopped at the time $t=m\tau$. And let it be required to obtain the respective sound energies in the two rooms at the time $t=(m+n)\tau$, where $m$ is a positive integer and $n$ a positive integer or zero.

Dividing the time-interval from $t=0$ to $t=(m+n)\tau$ into $(m+n)$ equal short intervals, we shall calculate what amount of sound energy emitted by the source in the $i$-th $[i \leq m]$ short interval would exist in room I at $t=(m+n)\tau$.

The sound energy $\varepsilon\tau$, which was emitted by the source in the $i$-th short interval and has never been in room II, reduces to

$$\varepsilon\tau\, P^{m+n-i}$$

by $t=(m+n)\tau$ after $(m+n-i)$ incidences in room I.

The sound energy, which went out into room II and came back again into room I passing through the panel twice, will undergo $(m+n-i-2)$ reflections in the time interval $(m+n-i)\tau$. So the part of sound energy $\varepsilon\tau$, which was emitted by the source in the $i$-th short interval and passed through the panel twice, reduces to

$$\varepsilon\tau\left(\frac{S_v}{S}q\right)^2 P^{m+n-i-2},$$

where $S_v$ is the area of the panel $W$ and $q$ its coefficient of transmission. Hence

$$\varepsilon\tau_{m+n-i}C_2\left(\frac{S_v}{S}q\right)^2 P^{m+n-i-2},$$

where ${}_{m+n-i}C_2$ shows the combination of 2 among $(m+n-i)$, gives the part of the sound energy which exists in room I at $t=(m+n)\tau$ having passed through the panel twice.

For the same reason,

$$\varepsilon\tau_{m+n-i}C_4\left(\frac{S_\nu}{S}q\right)^4 P^{m+n-i-4}, \qquad \varepsilon\tau_{m+n-i}C_6\left(\frac{S_\nu}{S}q\right)^6 P^{m+n-i-6}, \dots\dots\dots$$

give the parts of energy which are present in room I at $t=(m+n)\tau$ having passed through the panel four times, six times, etc. respectively.

Therefore, the sound energy which was emitted by the source in the $i$-th short interval and exists in room I at $t=(m+n)\tau$ is

$$\varepsilon\tau\left\{P^{m+n-i}+{}_{m+n-i}C_2\left(\frac{S_\nu}{S}q\right)^2 P^{m+n-i-2}+{}_{m+n-i}C_4\left(\frac{S_\nu}{S}q\right)^4 P^{m+n-i-4}\right.$$
$$\left.+{}_{m+n-i}C_6\left(\frac{S_\nu}{S}q\right)^6 P^{m+n-i-6}+\dots\right\}, \dots\dots\dots\dots \quad (3.2)$$

where ${}_\mu C_\lambda=0$ when $\mu<\lambda$.

Summing up (3.2) from $i=1$ to $i=m$, the sound energy $E_{\mathrm{I}}^{(m)}$ in room I at $t=(m+n)\tau$ is given by

$$E_{\mathrm{I}}^{(m)}=\varepsilon\tau\left\{\sum_{i=1}^{m}P^{m+n-i}+Q^2\sum_{i=1}^{m}{}_{m+n-i}C_2P^{m+n-i-2}\right.$$
$$\left.+Q^4\sum_{i=1}^{m}{}_{m+n-i}C_4P^{m+n-i-4}+Q^6\sum_{i=1}^{m}{}_{m+n-i}C_6P^{m+n-i-6}+\dots\dots\right\}$$
$$=\varepsilon\tau\left\{\frac{1-P^m}{1-P}P^n+Q^2\sum_{j=n}^{m+n-1}{}_jC_2P^{j-2}+Q^4\sum_{j=n}^{m+n-1}{}_jC_4P^{j-4}\right.$$
$$\left.+Q^6\sum_{j=n}^{m+n-1}{}_jC_6P^{j-6}+\dots\dots\right\},$$
$$\text{where} \quad Q\equiv\frac{S_\nu}{S}q. \qquad \dots\dots\dots\dots \quad (3.3)$$

From this we can easily obtain the sound energy $E_{\mathrm{I}0}^{(m)}$ in room I when the emission of the sound energy is stopped, and the sound energy $E_{\mathrm{I}0}^{(m)}$ in room I when the source has sounded long enough to establish a steady state in both rooms as follows:

$$E_{\mathrm{I}0}^{(m)}=\varepsilon\tau\left\{\frac{1-P^m}{1-P}+Q^2\sum_{j=2}^{m-1}{}_jC_2P^{j-2}+Q^4\sum_{j=4}^{m-1}{}_jC_4P^{j-4}\right.$$
$$\left.+Q^6\sum_{j=6}^{m-1}{}_jC_6P^{j-6}+\dots\dots\right\}, \dots\dots\dots\dots\dots\dots\dots \quad (3.4)$$

$$E_{\mathrm{I}0}^{(\infty)}=\varepsilon\tau\left\{\frac{1}{1-P}+\frac{Q^2}{(1-P)^3}+\frac{Q^4}{(1-P)^5}+\frac{Q^6}{(1-P)^7}\right.$$
$$\left.+\dots\dots\dots\dots\right\}$$
$$=\varepsilon\tau\frac{1-P}{(1-P)^2-Q^2} \dots\dots\dots\dots\dots\dots\dots\dots \quad (3.5)$$

Similarly we obtain $E_{\mathrm{II}}^{(m)}$, $E_{\mathrm{II}0}^{(m)}$, and $E_{\mathrm{II}0}^{(\infty)}$, the corresponding energy amounts in room II, as follows:

$$E_{\mathrm{II}}^{(m)} = \varepsilon\tau\left\{Q\sum_{j=n}^{m+n-1}{}_jC_1P^{j-1} + Q^3\sum_{j=n}^{m+n-1}{}_jC_3P^{j-3} + Q^5\sum_{j=n}^{m+n-1}{}_jC_5P^{j-5} + \cdots\cdots\right\}, \quad\cdots\cdots(3.\ 6)$$

$$E_{\mathrm{II}\,0}^{(m)} = \varepsilon\tau\left\{Q\sum_{j=1}^{m-1}{}_jC_1P^{j-1} + Q^3\sum_{j=3}^{m-1}{}_jC_3P^{j-3} + Q^5\sum_{j=5}^{m-1}{}_jC_5P^{j-5} + \cdots\cdots\right\}, \quad\cdots\cdots(3.\ 7)$$

$$E_{\mathrm{II}\,0}^{(\infty)} = \varepsilon\tau\left\{\frac{Q}{(1-P)^2} + \frac{Q^3}{(1-P)^4} + \frac{Q^5}{(1-P)^6} + \cdots\cdots\right\}$$

$$= \varepsilon\tau\frac{Q}{(1-P)^2 - Q^2} \quad\cdots\cdots(3.\ 8)$$

If the emission of the source be cut off after the sound energies in the two rooms have reached the steady values $E_{\mathrm{I}\,0}^{(\infty)}$, $E_{\mathrm{II}\,0}^{(\infty)}$ respectively, then the sound energies $E_{\mathrm{I}}^{(\infty)}$, $E_{\mathrm{II}}^{(\infty)}$ in the two rooms after the time $n\tau$ are given by

$$E_{\mathrm{I}}^{(\infty)} = \varepsilon\tau\frac{1-P}{(1-P)^2-Q^2}\left\{P^n + {}_nC_2Q^2P^{n-2} + {}_nC_4Q^4P^{n-4} + {}_nC_6Q^6P^{n-6} + \cdots\cdots\right\} + \varepsilon\tau\frac{Q}{(1-P)^2-Q^2}\left\{{}_nC_1QP^{n-1} + {}_nC_3Q^3P^{n-3} + {}_nC_5Q^5P^{n-5} + \cdots\cdots\right\}, \quad\cdots\cdots(3.\ 9)$$

$$E_{\mathrm{II}}^{(\infty)} = \varepsilon\tau\frac{Q}{(1-P)^2-Q^2}\left\{P^n + {}_nC_2Q^2P^{n-2} + {}_nC_4Q^4P^{n-4} + {}_nC_6Q^6P^{n-6} + \cdots\cdots\right\} + \varepsilon\tau\frac{1-P}{(1-P)^2-Q^2}\left\{{}_nC_1QP^{n-1} + {}_nC_3Q^3P^{n-3} + {}_nC_5Q^5P^{n-5} + \cdots\cdots\right\}, \quad\cdots\cdots(3.\ 10)$$

which are obtained by the same method as (3. 3), (3. 6).

If $Q$ be so small that we may neglect the terms containing its higher powers, we get

$$E_{\mathrm{I}}^{(\infty)} \doteqdot \varepsilon\tau\frac{P^n}{1-P}, \quad\cdots\cdots(3.\ 9.\ 1)$$

$$E_{\mathrm{II}}^{(\infty)} \doteqdot \varepsilon\tau\frac{\{P+n(1-P)\}P^{n-1}}{(1-P)^2}Q \quad\cdots\cdots(3.\ 10.\ 1)$$

In these expressions $n$ denotes a positive integer or zero, but, by the same reasoning as we obtained (2. 3. 2) and (2. 4. 1), it may be assumed that (3. 9. 1) and (3. 10. 1) are transformed into the following continuous functions of $T$:

$$E_{\mathrm{I}}^{(\infty)} \doteqdot \varepsilon\tau \frac{P^{\frac{T}{\tau}}}{1-P}, \qquad (3.\ 9.\ 2)$$

$$E_{\mathrm{II}}^{(\infty)} \doteqdot \varepsilon\tau \frac{\left\{P+\frac{T}{\tau}(1-P)\right\}^{\frac{T}{\tau}-1}}{(1-P)^2} Q \qquad (3.\ 10.2)$$

The decay of sound energy in two adjacent rooms was studied by E. Buckingham[1] and A. H. Davis[2] on the assumption that the sound energy in each room was uniformly distributed throughout the room, both with regard to position and direction.

We shall now propose a method of measuring the coefficients of preservation and transmission using two adjacent rooms.

Keeping $P$ in (3. 5) as a fixed value, let $Q$ become zero and let $E_0^{(\infty)}$ denote the value of $E_{\mathrm{I}\,0}^{(\infty)}$ in this case, then we get

$$E_0^{(\infty)} = \varepsilon\tau \frac{1}{1-P}, \qquad (3.\ 11)$$

which is identical with (2. 5).

From (3. 5), (3. 8) we get

$$\frac{E_{\mathrm{I}\,0}^{(\infty)}}{E_{\mathrm{II}\,0}^{(\infty)}} = \frac{1-P}{Q}, \qquad (3.\ 12)$$

and

$$\sqrt{\{E_{\mathrm{I}\,0}^{(\infty)}\}^2 - \{E_{\mathrm{II}\,0}^{(\infty)}\}^2} = \varepsilon\tau; \qquad (3.\ 13)$$

from (3. 11), (3. 13)

$$1 - P = \frac{\sqrt{\{E_{\mathrm{I}\,0}^{(\infty)}\}^2 - \{E_{\mathrm{II}\,0}^{(\infty)}\}^2}}{E_0^{(\infty)}} \equiv k; \qquad (3.\ 14)$$

and from (3. 12), (3. 14)

$$Q = k \frac{E_{\mathrm{II}\,0}^{(\infty)}}{E_{\mathrm{I}\,0}^{(\infty)}} \qquad (3.\ 15)$$

Since $E_{\mathrm{I}\,0}^{(\infty)}$, $E_{\mathrm{II}\,0}^{(\infty)}$, and $E_0^{(\infty)}$ are all values in steady states, they and their ratios are measurable quantities; and, therefore, if they or their ratios are measured, $P$ and $Q$ may be obtained by (3. 14), (3. 15).

Let $S_1$ in each room be increased by $S_{1,2}$ and $S_2$ be decreased by $S_{1,2}$, then we get

1 E. Buckingham, *loc. cit.*

2 A. H. Davis, Phil. Mag., **50**, 75 (1925).

$$1 - P - \frac{(p_1 - p_2)S_{1,2}}{S} = k'.$$

$k'$ can be measured by the same method as $k$. Eliminating $1 - P$ from this and (3.14), we get

$$\frac{(p_1 - p_2)S_{1,2}}{S} = k' - k,$$

or

$$p_1 - p_2 = (k' - k)\frac{S}{S_{1,2}} \quad \cdots\cdots\cdots\cdots \quad (3.16)$$

If either $p_1$ or $p_2$ is known, the other may be obtained by (3.16). If $S_2$ is the area of an open window, and if its coefficient of preservation is assumed to be zero, (3.16) becomes

$$p_1 = (k' - k)\frac{S}{S_{1,2}} \quad \cdots\cdots\cdots\cdots \quad (3.17)$$

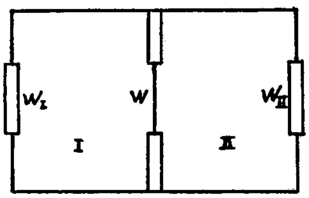
From (3.15) we get

$$q = \frac{S}{S_v}Q = k\frac{S}{S_v}\frac{E_{\text{II}\,0}^{(\infty)}}{E_{\text{I}\,0}^{(\infty)}} \quad \cdots\cdots\cdots\cdots \quad (3.18)$$

The coefficient of preservation and that of transmission may be obtained by (3.17) and (3.18).

In the course of this experiment it is necessary to make $Q$ become zero without changing the value of $P$. This will be done, if the windows of room II are all opened, so that the sound energy may pass through the panel from room I to room II, but not back from room II to room I; or if a substance whose coefficient of preservation is very small is placed near the panel $W$ in room II. There is another method. The rooms I and II are each provided with soundproof windows $W_{\text{I}}$ and $W_{\text{II}}$ as shown in Fig. 5, which have the same areas as $W$. After the values of $E_{\text{I}\,0}^{(\infty)}$, $E_{\text{II}\,0}^{(\infty)}$ have been obtained we measure $E_0^{(\infty)}$ by interchanging the material of the window $W$ with that of $W_{\text{I}}$.

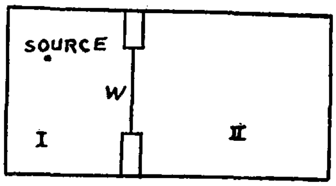
Fig. 5.




## Part IV

Let there be two adjacent rooms I and II which are in acoustic communication only through a window $W$ built in the partition between the rooms. Let $F$, $H$ represent the respective average intensities in the

Fig. 6.




two rooms, with the window open when a uniform and continuous source in room I has sounded long enough to establish a steady state in both rooms. If an incompletely soundproof panel is fitted in the window $W$, the value of $F$ is increased, say, to $F+f$, and that of $H$ decreased, say, to $H-h$. If the average intensity were $F+f$ in room I with the window open, the average intensity in room II would be $H\frac{F+f}{F}$. Here we see that the average intensity in room II is reduced by the panel from $H\frac{F+f}{F}$ to $H-h$. E. A. Eckhardt and V. L. Chrisler[1] called the ratio of $H-h$ to $H\frac{F+f}{F}$, or

$$\frac{H-h}{H}\frac{F}{F+f} \qquad\qquad (4.1)$$

the "reduction factor" of the panel and they observed this value for many kinds of building material.

Is the "reduction factor" which is defined by E. A. Eckhardt and V. L. Chrisler a constant depending only on the panel, but not on the rooms where the observations are made? In order to denote an acoustic property of the panel by the "reduction factor," it is necessary that the "reduction factor" be such a constant. The object of Part IV is to examine whether it is such a constant or not, under the same assumption as we have made before.

Let $\tau_{I}$, $P_{I}$, $Q_{I}$, and $\tau_{II}$, $P_{II}$, $Q_{II}$ be the values of $\tau$, $P$, $Q$ for rooms I and II.

If $E_{I}$, $E_{II}$ be the sound energies in the two rooms with the panel in place in the steady state, then the sound energies lost in unit time from room I and room II are

$$\frac{E_{I}(1-P_{I})}{\tau_{I}}, \quad \frac{E_{II}(1-P_{II})}{\tau_{II}}$$

respectively; and the sound energy received in unit time by room I from room II through the panel and that received by room II from room I are

$$\frac{E_{II}Q_{II}}{\tau_{II}}, \quad \frac{E_{I}Q_{I}}{\tau_{I}}.$$

Since the state is steady in both rooms, we must have the following two

1 E. A. Eckhardt & V. L. Chrisler, *loc. cit.*

equations :

$$\frac{E_{\rm I}(1-P_{\rm I})}{\tau_{\rm I}} = \varepsilon + \frac{E_{\rm II}Q_{\rm II}}{\tau_{\rm II}},$$

$$\frac{E_{\rm II}(1-P_{\rm II})}{\tau_{\rm II}} = \frac{E_{\rm I}Q_{\rm I}}{\tau_{\rm I}},$$

where $\varepsilon$ is the rate of emission of the source. Solving these equations, we get

$$E_{\rm I} = \frac{\varepsilon\tau_{\rm I}(1-P_{\rm II})}{(1-P_{\rm I})(1-P_{\rm II})-Q_{\rm I}Q_{\rm II}}, \qquad (4.\ 2)$$

$$E_{\rm II} = \frac{\varepsilon\tau_{\rm II}Q_{\rm I}}{(1-P_{\rm I})(1-P_{\rm II})-Q_{\rm I}Q_{\rm II}}. \qquad (4.\ 3)$$

These expressions give the sound energies in rooms I and II with the panel in place. (4. 2) and (4. 3) are the generalized forms of (3. 5) and (3. 8).

Next, let $E'_{\rm I}$, $E'_{\rm II}$ be the sound energies in rooms I and II with the window open; and $P'_{\rm I}$, $Q'_{\rm I}$, $P'_{\rm II}$, $Q'_{\rm II}$ be the values of $P_{\rm I}$, $Q_{\rm I}$, $P_{\rm II}$, $Q_{\rm II}$ in this case. Then we have

$$E'_{\rm I} = \frac{\varepsilon'\tau_{\rm I}(1-P'_{\rm II})}{(1-P'_{\rm I})(1-P'_{\rm II})-Q'_{\rm I}Q'_{\rm II}}, \qquad (4.\ 4)$$

$$E'_{\rm II} = \frac{\varepsilon'\tau_{\rm II}Q'_{\rm I}}{(1-P'_{\rm I})(1-P'_{\rm II})-Q'_{\rm I}Q'_{\rm II}}, \qquad (4.\ 5)$$

as the expressions corresponding to (4. 2), (4. 3); where $\varepsilon'$ is the rate of emission of the source which is adjusted to satisfy the condition

$$E_{\rm I} = E'_{\rm I} \qquad (4.\ 6)$$

It is clear that the "reduction factor" defined by E. A. Eckhardt and V. L. Chrisler is equal to $\frac{E_{\rm II}}{E'_{\rm II}}$.

From (4. 3), (4. 5) we get

$$\frac{E_{\rm II}}{E'_{\rm II}} = \frac{\varepsilon}{\varepsilon'}\frac{(1-P'_{\rm I})(1-P'_{\rm II})-Q'_{\rm I}Q'_{\rm II}}{(1-P_{\rm I})(1-P_{\rm II})-Q_{\rm I}Q_{\rm II}}\frac{Q_{\rm I}}{Q'_{\rm II}}; \ldots \qquad (4.\ 7)$$

and from (4. 2), (4. 4), (4. 6)

$$\frac{\varepsilon(1-P_{\rm II})}{(1-P_{\rm I})(1-P_{\rm II})-Q_{\rm I}Q_{\rm II}} = \frac{\varepsilon'(1-P'_{\rm II})}{(1-P'_{\rm I})(1-P'_{\rm II})-Q'_{\rm I}Q'_{\rm II}},$$

or

$$\frac{\varepsilon}{\varepsilon'}\frac{(1-P'_{\rm I})(1-P'_{\rm II})-Q'_{\rm I}Q'_{\rm II}}{(1-P_{\rm I})(1-P_{\rm II})-Q_{\rm I}Q_{\rm II}} = \frac{1-P'_{\rm II}}{1-P_{\rm II}}.$$

Substituting this in (4. 7) we get

$$\frac{E_{II}}{E'_{II}} = \frac{1-P'_{II}}{1-P_{II}} \frac{Q_I}{Q'_I} \qquad (4.8)$$

For the values of $1-P_{II}$ and $Q_I$ we have

$$1 - P_{II} = \frac{(1-p_1)S_1 + \ldots\ldots + (1-p_{\nu-1})S_{\nu-1} + (1-p_\nu)S_\nu}{S_{II}}, \qquad (4.9)$$

$$Q_I = \frac{S_\nu}{S_I} q ; \qquad (4.10)$$

where $S_I$, $S_{II}$ are the surface areas of rooms I and II, and $p_\nu$, $q$ the coefficients of preservation and of transmission of the panel. Since for the open window we may assume that its coefficients of preservation and of transmission are zero and unity respectively, we have

$$1 - P'_{II} = \frac{(1-p_1)S_1 + \ldots\ldots + (1-p_{\nu-1})S_{\nu-1} + S_\nu}{S_{II}}, \qquad (4.11)$$

$$Q'_I = \frac{S_\nu}{S_I} \qquad (4.12)$$

Comparing (4.11) with (4.9) we get

$$1 - P'_{II} = (1 - P_{II}) + p_\nu \frac{S_\nu}{S_{II}} \qquad (4.13)$$

Substituting (4.10), (4.12) into the expression (4.8) we get

$$\frac{E_{II}}{E'_{II}} = \frac{1-P'_{II}}{1-P_{II}} q \qquad (4.13)$$

If $1-P_{II}$ were equal to $1-P'_{II}$, the "reduction factor" is the same as the coefficient of transmission. But, as (4.13) shows, $1-P'_{II}$ differs from $1-P_{II}$ by

$$p_\nu \frac{S_\nu}{S_{II}};$$

and since $P_{II}$ and $p_\nu$ will often take values near to 1, we can in general not neglect this difference compared with $1-P_{II}$.

Therefore, we may conclude that the "reduction factor" defined by E. A. Eckhardt and V. L. Chrisler depends generally on the values of $P'_{II}$ and $\frac{S_\nu}{S_{II}}$, *i. e.* on the rooms where the observations are made.

The writer wishes to express his sincere thanks to Professor K. Tamaki by whose suggestion this investigation was undertaken.