Laue-Photograph taken with Convergent X-Rays

By

Takeo Fujiwara.

(Received May 23, 1928)

Adstract

Some time ago, the writer had determined the orientation of the crystals in tungsten wire by finding the spectrum of the characteristic K–radiation of the Mo-target in some of the ordinary Laue-spots. Now he has improved some parts of the former method. He utilized the convergent X-rays starting from the focus of the Mo-target of a Coolidge tube. These convergent X-rays were made to pass through a circular pin hole, and then to illuminate a small portion of a specimen. The photographic plate was set perpendicular to the line connecting the centres of the focus and of the slit. In all photographs taken with convergent X-rays, we found that each of many Laue-spots contained an intense nucleus at its centre, and that some of the spots were accompanied by the spectrum lines of the K–radiation of molybdenum. As to the positions of the spectrum lines, they did not, generally coincide with those of the corresponding centres of the Laue-spots. From the positions of these spectrum lines and of the intense centres the orientation of the crystal was determined. In obtaining the exact position of the centre of the central spot, which was impressed by the direct beam of the X-rays, two methods were employed. One was to give a correction, by calculation, of its rough position estimated provisionarily; and the other was to use crossed wires before the photographic plate.

Introduction

Some time ago, the writer[1] had determined the orientation of the crystals in tungsten wire by finding the characteristic K–radiation in some of the ordinary Laue-spots.

1 T. Fujiwara, Mazda-Kenkyu-Jiho, **1**, No. 2, (1926)

At that time the X-rays passing through the slits of two pin-holes were used, the specimen and the photographic plate were generally placed perpendicular to the beam of the X-rays. Recently the same method has been improved in certain particulars by the writer and is described in the following pages.

General Idea of using Convergent X-Rays

If a crystal be placed in a position satisfying the following Bragg's equation in the path of monochromatic X-rays of wave length λ, it will reflect them according to

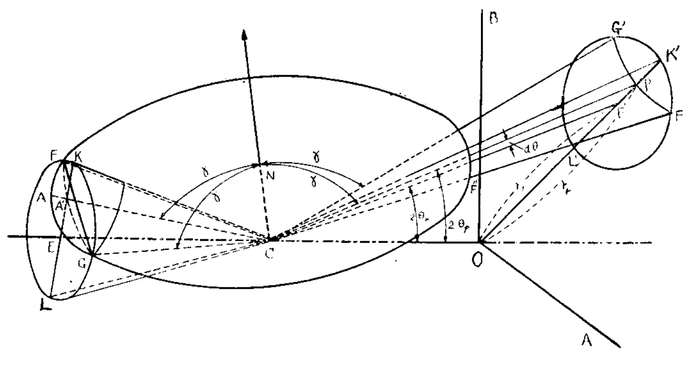
$$n\lambda = 2d \sin\theta, \quad \text{(1)}$$

where d is the grating constant of the crystal, and θ is the angle between the atomic plane of the crystal and the incident ray, and n is an integer which determines the order of the spectrum. But if a crystal be placed in the path of the "white" X-rays, it will reflect some of them at any position of the atomic plane, as there will be some X-rays of a certain wave length in the incident X-rays, which will satisfy equation (1). The diffraction pattern thus taken is so-called Laue-spots.

Therefore if the "White" X-rays containing a characteristic radiation of strong intensity strike the crystal plane in such a position that the wave length of the characteristic radiation satisfies equation (1), the spectrum line of that radiation will be found in the ordinary Laue-spots.

Now let convergent "white" X-rays accompanying a characteristic radiation of strong intensity strike a crystal plane and produce a Laue diffraction pattern on the photographic plate in the manner shown in Fig. 1. In this figure CKL is convergent rays starting from a

Fig. 1



circular focus KFLG, E is the centre of it, and the plane POB is the photographic plate which is placed perpendicularly to the line EC.

Now we assume any characteristic radiation AC which is contained in the plane ECN and is reflected by an atomic plane of the crystal in the direction CP. CN is the normal to the atomic plane, γ the angle between the ray and the normal to the plane, and P the image on the photographic plate which is impressed by the reflected characteristic radiation. Then, in addition to the ordinary Laue-spot, the same characteristic radiation which strikes the crystal at the same glancing angle as the former one, will also be reflected by the same atomic plane, and the characteristic radiations whose paths are the generators of the cone CFAGF" will be reflected by the plane and will make a spectrum line G'PF' in the ordinary Laue-spot. The vertex of the cone CFAGF" is C, its axis CN and the semivertical angle γ.

The characteristic radiations whose paths are not lying on the surface of the cone CFAGF", do not satisfy equation (1), they will not be reflected by the same atomic plane. Thus, the shape of the spectrum line produced on the photographic plate will be, strictly speaking, a curved line G′PF′, but as the angle γ is very near to $\frac{\pi}{2}$, so the spectrum line appears nearly as a straight line and is perpendicular to the radial line OP.

Compared with the case of using parallel rays, the sharpness of the Laue-spots is of course very much destroyed by using convergent X-rays; but the chance of obtaining the spectrum lines in the Laue-spots is very much increased with convergent X-rays, because the chance of covering the focus by the conical surface CFAGF" is greatly increased by using a broad focus tube.

Outline of the Method

As stated above the writer has improved some parts of the former method. He utilized convergent X-rays starting from the focus on the Mo-target of a Coolidge tube. These convergent X-rays were made to pass through a circular pin hole, and then to illuminate a small portion of a specimen of a single crystal of tungsten. The photographic plate was set perpendicular to the line connecting the centres of the focus and of the slit. For the present purpose an X-ray tube with a broad focus and a circular conical slit are preferable. The focus of the tube was about 9 mm in diameter and the applied voltage was about 40 K.V.s. The semivertical angle of the conical slit was about five degrees, slightly larger than that of the convergent X-rays, and the diameter of its opening was about 0.4 mm.

A lead screen with a circular hole of proper diameter was placed between the X-ray tube and the slit so as to cut off the weak X-rays starting from any other part of the anode than the focus. By the use of this lead screen the Laue-spots and the central spot impressed by the direct beam of the X-rays became sharper and thinner than those obtained without a screen.

In setting the photographic plate perpendicularly to the line joining the centres of the focus and of the slit, a fluorescent screen provided with a point-mark was so placed, just at the position where the photographic plate would be placed afterward, that the line connecting the centre of the slit and the point-mark became perpendicular to the plane of the fluorescent screen. Then the position of the X-ray tube was so adjusted as to project the image of the centre of the focus at the point-mark on the screen: and then it was replaced by the photographic plate exactly in the same position.

The Results

With this apparatus Laue diffraction patterns of thin tungsten wires composed of single crystals were taken as shown in Figs. 1 & 3 of Plate I, and the diffraction pattern of the same crystal taken with the parallel X-rays is shown in Fig. 2 of Plate I. As comparison, the Laue patterns in Figs. 1 & 2 were taken with the same tungsten crystal in the form of a thin wire, which was exactly in the same position. The potential difference applied on the X-ray tube was about 40 K.V.s, the current about 10 milliamperes and the exposure of the photographic plates about five hours in both cases.

Comparing these two photographs we can see that some more spectrum lines of the characteristic K-radiation of Mo appeared in Fig. 1, and that the lines are longer in this case. With regard to the Laue-spots produced by the white radiation they are larger in Fig. 1 than in Fig. 2.

By means of convergent X-rays and with a crystal of tungsten the writer took many photographs with different X-ray tubes having foci of different shapes; and it was observed, as a matter of course, that the shapes of the Laue-spots were respectively similar to those of the corresponding central spots which are nothing but the ordinary images of the foci obtained by the pin hole slit. This is illustrated by the photographs, shown in Figs. 1, 3, 4, 5, 6, 7, & 8 of Plates I. & II. In the cases of Figs. 1, 3 & 4, Plate I, the same anode with a nearly circular focus was used, the shape of which is shown in Fig. 7, Plate II. With Fig. 4, Plate I and Fig. 5, Plate II, the same tungsten crystal was used, but with the others the tungsten crystals used were different in each instance.

In all photographs taken with convergent X-rays we can detect that every one of the Laue-spots has an intense nucleus at the centre. This is, of course, due to the presence of an intense nucleus at the centre of the focus of the target; and it is very convenient in determining the position of the Laue-spot, and consequently in determining the orientation of the crystal. As to the position of the spectrum lines of the K-radiation, they are not generally coincident with the corresponding intense nuclei at the centres of the Laue-spots. The glancing angle θ of the X-rays which are responsible for the spectrum line G′PF′ in Fig. 1 does not coincide with the half of the angle PCO, but it will be given by the following equation.

$$2\theta = 2\theta_n \pm 2d\theta, \qquad \text{(2)}$$

were θ_n is the glancing angle of the intense nucleus at the centre of the Laue-spot E′, $d\theta$ is the difference between the angles $PCO = 2\theta_p$ and $E'CO = 2\theta_n$, and the double signs are to be properly selected according to the position of the spectrum line against the point E′. This circumstance must be taken into consideration in determining the orientation of the crystal.

The determination of the orientation of the axes of the crystal with these spectra and spots will be carried out as follows:

At first, from the measured values of r_n and r_p which are the distances between OE′ and OP respectively, the values of the angles E′CO, and PCO and their difference $d\theta$ are calculated. The glancing angle θ corresponding to the spectrum line is found by equation (2), and the indices of the atomic plane of the crystal by whose reflection the spectrum line and Laue-spot are caused, are determined in the same way as in the former method. From the value of θ and the observed value of the angle between the lines OE′ and OB which is drawn on the photographic plate in the direction parallel to the axis of the wire, the orientation of the crystal is determined in the ordinary manner.

Table I

No. of spots.		r (obs.) in mm.	$2\theta_n$ or $2\theta_p$ in degree		$d\theta$ in degree	θ (calc.) in degree	Grating constant d in 10^{-8} cm.		Davey's Data: d in 10^{-8} cm.	Davey's Data: Indices of atomic plane	Indices of the atomic plane.
1	r_n	18·19	θ_n	31°39.'5							
	r_{p1}	16·39	θ_{p1}	29°3.'4	−2°36.'1	13°13.'65	$d\alpha'$	1·56	1·58	(100)	(100)$_\alpha$
							$d\alpha$	1·54	1·58	(100)	
							$d\beta$	1·38			
							$d\gamma$	1·35			
	r_{p2}	15·46	θ_{p2}	27°42.'3	−3°57.'2	11°52.'6	$d\alpha'$	1·73			(100)$_\beta$
							$d\alpha$	1·72			
							$d\beta$	1·53	1·58	(100)	
							$d\gamma$	1·51			
2	r_n	27·55	θ_n	43°2.'6							
	r_{p1}	30·76	θ_{p1}	46°11.'9	+3°9.'3	24°40.'6	$d\alpha'$	0·853	0·843	(321)	(321)$_\alpha$
							$d\alpha$	0·848	0·843	(321)	
							$d\beta$	0·756			
							$d\gamma$	0·742			
	r_{p2}	27·77	θ_{p2}	43°16.'1	+ 13.'5	21°39.'8	$d\alpha'$	0·964			(321)$_\beta$
							$d\alpha$	0·958			
							$d\beta$	0·855	0·843	(321)	
							$d\gamma$	0·839			
3	r_n	29·00	θ_n	44°30.'6							
	r_p	25·21	θ_p	40°31'	−3°59.'6	18°15.'65	$d\alpha'$	1·14	1·12	(110)^2	(110)$^2_\alpha$ or (310)$_\beta$
							$d\alpha$	1·13	1·12	(110)^2	
							$d\beta$	1·01	0·997	(310)	
							$d\gamma$	0·989	0·997	(310)	

Table I (cont.)

No. of spots.		r (obs.) in mm.	$2\theta_n$ or $2\theta_p$ in degree		$d\theta$ in degree	θ (calc.) in degree	Grating constant d in 10^{-8} cm.		Davey's Data: d in 10^{-8} cm.	Davey's Data: Indices of atomic Plane	Indices of the atomic plane
4	r_n	25·18	θ_n	40°29′							
	r_{p1}	26·15	θ_p	41°33′3	+1°4·′3	21°18·′8	$d\alpha'$	0·980			$(321)_\gamma$
							$d\alpha$	0·974			
							$d\beta$	0·862			
							$d\gamma$	0·853	0 843	(321)	
	r_{p2}	26·54	θ_p	41°58·′55	+1°29·′55	21°44·′05	$d\alpha'$	0·961			$(321)_\beta$
							$d\alpha$	0·956			
							$d\beta$	0·852	0·843	(321)	
							$d\gamma$	0·837			
5	r_n	21·28	θ_n	53°48·′3							
	r_{p1}	19·63	θ_p	33°38·′5	−2°9·′8	15°44·′35	$d\alpha'$	1·31	1·29	(211)	$(211)_\alpha$ or $(110)^2_\gamma$
							$d\alpha$	1·31	1·29	(211)	
							$d\beta$	1·16			
							$d\gamma$	1·14	1·12	$(110)^2$	
	r_{p2}	18·29	θ_p	31°48′	−4°0·′3	13°53·′85	$d\alpha'$	1·48			$(211)_\beta$
							$d\alpha$	1·47			
							$d\beta$	1·31	1·29	(211)	
							$d\gamma$	1·29	1·29	(211)	
6	r_n	9·55	θ_n	17°56·′2							
	r_p	9·55	θ_p	17°56·′2	0	8°58·′1	$d\alpha'$	2·28	2·23	(110)	$(110)_\alpha$
							$d\alpha$	2·27	2·23	(110)	
							$d\beta$	2·02			
							$d\gamma$	1·99			

From the diffraction patterns shown in Eig. 3, Plate I, the writer determined the indices of the atomic planes by whose reflections the spectrum lines and the Laue-spots are caused, and are given in Table I. In this table the spots 1, 2, 3, etc., refer respectively to those marked in Fig. 3, Plate I. The values of r_p and r_n corresponding to the spectrum lines and the intense nuclei at the centres of the Laue-spots respectively were measured on the negative plate with Hilger's travelling micro-meter and are represented in the 3rd column. For the value of the distance between the specimen and the photographic plate $a=29.50$ mms. was used. The angles $2\theta_n$ and $2\theta_p$ in the 4th column correspond respectively to the intense nucleus and the spectrum line in the same horizontal row. The symbols $d_{\alpha'}$, d_α, d_β & d_γ in the 8th solumn represent the value of d obtained by assuming that the spectrum tabulated in the same horizontal row is due to any one of the $K_{\alpha'}$, K_α, K_β and K_γ lines of molybdenum respectively. For the value of the wave length of the $K_{\alpha'}$, K_α, K_β and K_γ lines the writer has taken $\lambda_{\alpha'}=0.712\times10^{-8}$ cm., $\lambda_\alpha=0.708\times10^{-8}$ cm., $\lambda_\beta=0.631\times10^{-8}$ cm. and $\lambda_\gamma=0.620\times10^{-8}$ cm.[1] respectively. In the 10th and the 11th columns, the values of d and the indices of the corresponding reflecting atomic plane obtained by Davey[2] are represented. The symbols $(100)_\alpha$, $(100)_\beta$, etc., in the last column show that the corresponding line is the spectrum of the K_α or K_β line caused by the reflection from the atomic plane represented by the indices given in the bracket, and the symbol $(110)^2$ denotes that the spectrum line is the second order spectrum caused by the crystal.

Correction for the Determination of the Position of the Centre of the Central Spot

In the former case the photographic plate was set perpendicular to the line connecting the centres of the nucleus of the focus and of the slit exactly, and the distance between the photographic plate and the specimen was accurately measured. The centre of the central spot, which was impressed by the direct beam of the X-rays, was determined on the negative plate by drawing two diametral lines on the central spot. The shape of this central spot was nearly a circle and could clearly be seen on the original plate, though it was barely visible in the reprinted figure.

Generally the centre thus obtained does not coincide exactly with

1 W. Duane and Kang-Fuh-Hu: Phys. Rev., **9**, 489 (1918); **14**, 369 (1919).
2 W.P. Davey: Phys. Rev., **25**, 678 (1925).

the intense nucleus of the central spot, and in the former experiment its accuracy was much less than the other factors stated above. Therefore the writer obtained the position of the centre of the central spot, which is more close to the real one than that obtained in the former, by making a correction. But if we are doubtful of the accuracies of the distance between the photographic plate and the specimen, and of the setting of the photographic plate perpendicularly to the line connecting the centres of the nucleus of the focus and of the slit, besides the correction for the centre of the central spot, as a matter of course, we must consider the corrections for these factors. The correction for the centre of the central spot was made as follows:

Let the real centre of the intense nucleus of the central spot which is impressed by the direct beam of X-rays on the photograph be O, its approximate position be O′, the centre of the intense nucleus of the reflected Laue-spot be E′, and the reflected spectrum line of a known wave length be P′P as shown in Fig. 2. In this figure all symbols have the same meaning as in the case of Fig. 1.

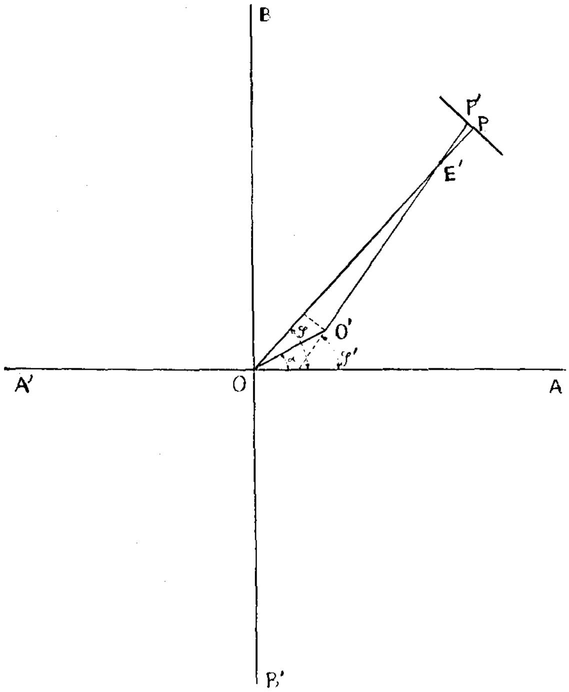


Fig. 2

If we put $OO'=c$, angle $O'OA=\alpha$, $OE'=r_n$, $OP=r_p$, $O'E'=r'_n$ and $O'P'=r'_p$, then we have the following relations

$$\left.\begin{aligned} \tan 2\theta_n &= \frac{r_n}{a} \\ \tan 2\theta_p &= \frac{r_p}{a} \\ \tan 2\theta'_n &= \frac{r'_n}{a} \\ \tan 2\theta'_p &= \frac{r'_p}{a} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

$$\left.\begin{aligned} r_n &\doteqdot r'_n + c\cos(\varphi'-\alpha) \\ r_p &\doteqdot r'_p + c\cos(\varphi'-\alpha), \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

where θ'_n and θ'_p are the values corresponding to the centre O', the symbols without dash represent the values corresponding to the real centre and φ' is the angle between the lines $E'O'$ and OA.

From the equations (3) and (4) we have

$$\tan 2\theta_n = \frac{r'_n}{a} + \frac{c}{a}\cos(\varphi'-\alpha),$$

$$\text{i. e. } \tan 2\theta_n - \tan 2\theta'_n = \frac{c}{a}\cos(\varphi'-\alpha). \quad \cdots\cdots\cdots\cdots\cdots\cdots (5)$$

Now, at first, take the approximate position of the centre of the central spot as O', and after finding the values θ'_n and $d\theta'$, determine the indices of the atomic plane by whose reflection the spectrum line $P'P$ is caused in the manner as was stated before. Then by using Davey's data, calculate the glancing angle θ of the above determined spectrum line by equation (1). With this value of θ and that of $d\theta'$ which was obtained before, the value of θ_n corresponding to that spectrum line is calculated by the following equation

$$\theta_n = \theta \mp d\theta' \quad \cdots\cdots\cdots\cdots\cdots\cdots (6);$$

and then the value of $\tan 2\theta_n$ will be immediately obtained.

Generally the difference $\tan 2\theta_n - \tan 2\theta'_n$ does not vanish and it takes the value $\frac{c}{a}\cos(\varphi'-\alpha)$.

If we could find the values of c and α which satisfy equation (5) for the various spots, we can obtain the corrected values of r_n and r_p, and then the real value of θ of the spectrum line.

For the purpose of obtaining the values c and α, at first the relations between φ' and $\tan 2\theta_n - \tan 2\theta'_n$ for various spots are required; and then by taking the value of $\tan 2\theta_n - \tan 2\theta'_n$ as the radius vector and φ' as the vectorial angle these relations are plotted for various spots. If a circle passing approximately through the origin and the plotted points is drawn, and the length of the diameter of this circle which passes through the origin and the vectorial angle of this diameter are taken as $\frac{c}{a}$ and α respectively, then these values of $\frac{c}{a}$ and α will approximately satisfy equation (5). Thus the approximate values of c and α which satisfy equation (5) can be obtained.

If we denote these approximate values of c and α by c_0 and α_0 respectively, the real values of c and α will be represented as follows

$$c = c_0 + \varDelta c \qquad \alpha = \alpha_0 + \varDelta\alpha, \qquad\ldots\ldots(7)$$

where $\varDelta c$ and $\varDelta\alpha$ are very small.

Substituting these relations into equation (5) the following relations are derived,

$$\tan 2\theta_n - \tan 2\theta_n' = \frac{c_0 + \varDelta c}{a} \cos(\varphi' - \alpha_0 - \varDelta\alpha)$$

$$= \frac{c_0 + \varDelta c}{a} \cos(\varphi' - \alpha_0) \cos(\varDelta\alpha) + \frac{c_0 + \varDelta c}{a} \sin(\varphi' - \alpha_0) \sin(\varDelta\alpha).$$

By putting $\cos(\varDelta\alpha) = 1$, and $\sin(\varDelta\alpha) = \varDelta\alpha$ the above equation becomes

$$\tan 2\theta_n - \tan 2\theta_n' = \frac{c_0}{a} \cos(\varphi' - \alpha_0) + \frac{\varDelta c}{a} \cos(\varphi' - \alpha_0)$$

$$+ \frac{c_0 + \varDelta c}{a} (\varDelta\alpha) \sin(\varphi' - \alpha_0). \qquad\ldots\ldots(8)$$

Since α_0 and c_0 are known, the values of $\frac{c_0}{a} \cos(\varphi' - \alpha_0)$ and $(\varphi' - \alpha_0)$ can be calculated for all points. By putting

$$\left\{ \tan 2\theta_n - \tan 2\theta_n' \right\} - \frac{c_0}{a} \cos(\varphi' - \alpha_0) = \Delta \qquad\ldots\ldots(9)$$

the following relation is obtained from equation (8)

$$\frac{\Delta}{\cos(\varphi' - \alpha_0)} = \frac{\varDelta c}{a} + \frac{c_0}{a}(\varDelta\alpha) \tan(\varphi' - \alpha_0). \qquad\ldots\ldots(10)$$

By putting

$$\frac{\Delta}{\cos\,(\varphi'-\alpha_0)}=y \quad \text{and} \quad \tan\,(\varphi'-\alpha_0)=x,$$

the values of x and y are calculated for all spots, and if the relation between x and y is plotted on a graph it will be represented nearly by a straight line. By drawing this line properly the values of $\frac{\Delta c}{a}$ and α can be obtained; consequently the corrected values of c, α, r_n, r_p and φ will be obtained where φ is the angle between the lines E′O and OA in Fig. 2.

With these corrected values of r_n and r_p the real value of θ is calculated and the indices of the atomic plane by whose reflection the spectrum line is caused is determined and the orientation of the crystal can be determined exactly.

According to the principle stated above, the actual corrections were made with the data represented in Table I, and they are represented in Tables II, III and IV. In these tables the data which are reproduced from Table I are denoted with dashes.

Table II

No. of spots		2θ in degree	$2d\theta'$	$2\theta_n$ $(2\theta \mp 2d\theta')$	mean value of $2\theta_n$	$2\theta'_n$	$\tan 2\theta_n - \tan 2\theta'_n$ in 10^{-4}	φ' in degree	x in 10^{-2}	y in 10^{-4}
	α	25° 55·′5	−5°12·′2	31°7·′7						
1 (100)					31°3·′1	31°39·′5	−145·2	1°	−13·2	+1·8
	β	23° 4·′1	−7°54·′4	30°58·′5						
	α	49° 39′	+6°18·′6	43°20·′4						
2 (321)					43°25·′7	43°2·′6	126·6	217°	+54·2	−0·6
	β	43° 58′	+ 27·′0	43°31′						
3 (100)²	α	36° 57′	−7°57·′3	44°54·′3	44°54·′3	44°30·′6	136·4	196°	+13·1	−7·0
	α	43° 8·′3	+2° 8·′6	40°59·′7						
4 (321)					40°59·′4	40°29·′0	145·5	175°	−24·1	+5·1
	β	43° 58·′2	+2°59·′1	40°59·′1						

Table II (Cont.)

No. of spots		2θ in degree	$2d\theta'$	$2\theta_n$ $(2\theta \mp 2d\theta')$	mean value of $2\theta_n$	$2\theta'_n$	$\tan 2\theta_n - \tan 2\theta'_n$ in 10^{-4}	φ' in degree	x in 10^{-2}	y in 10^{-4}
5 (211)	α	31° 55′	−4°19·′6	36°14·′6	36°18·′9	35°48·′3	136·2	164°	−45·6	+5·3
	β	28° 22·′7	−8° 0·′6	36°23·′3						
6 (110)	α	18° 15·′3	0	18°15·′3	18°15·′3	17°56·′2	62·5	133° 30′	−143·0	−35·1

$$\frac{c_0}{a} = 144.6 \times 10^{-4}, \quad \frac{\Delta c}{a} = -3.2 \times 10^{-4}, \quad \frac{c}{a} = 141.4 \times 10^{-4}$$

$$\alpha_0 = 188^\circ 32', \quad \Delta\alpha = 1^\circ 53', \quad \alpha = 190^\circ 25'$$

Table III

c=0·417 mm. α=190°25′

No. of spots	$\varphi' - \alpha$ in degree	$dc = c \cos(\varphi' - \alpha)$ in mm.	Φ (corrected) in degree
1	−189° 25′	−0·41	49′
2	+26° 35′	+0·37	216° 37′
3	+ 5° 35′	+0·42	195° 55′
4	−15° 25′	+0·40	175° 15′
5	−26° 25′	+0·37	164° 29′
6	−56° 55′	+0·23	135° 32′

Table IV

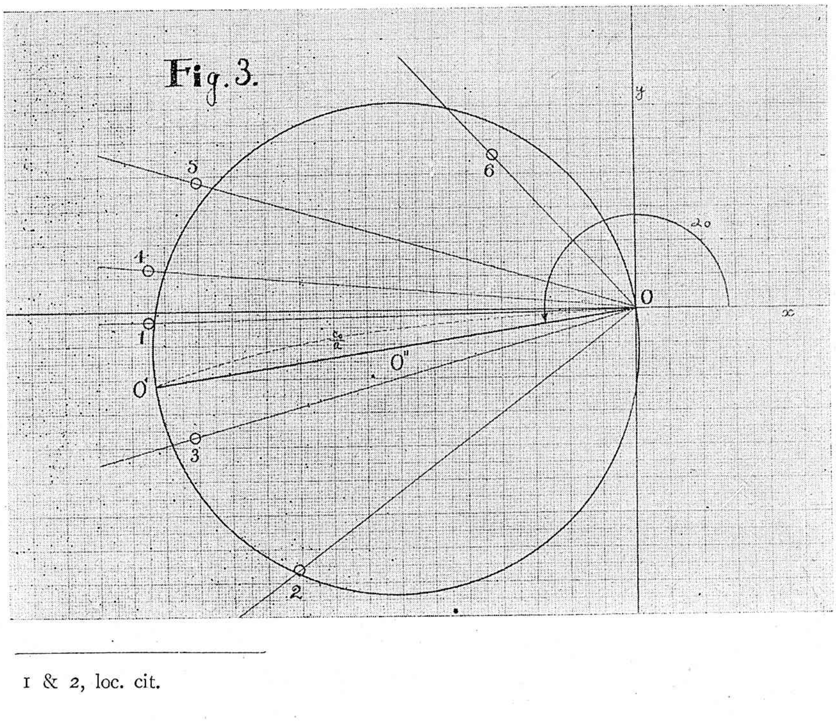
No. of spots		r' (obs.) in mm.	dc in mm.	r (corrected) in mm.	2θ (calc.) in degree	$d\theta$	θ (calc.)	Grating constant d in 10^{-8}cm.			Davey's date: d in 10^{-8} cm	Davey's date: Indices of atomic plane	Indices of the atomic plane
1	r_n	18·19		17·78	θ_n 31° 47′								
	r_{p1}	16·39		15·98	θ_{p1} 28°26·′7	2°38′	12°54·′35	dα′	1·59	(1·56)	1·58	(100)	(100)α
								dα	1·58	(1·54)	1·58	(100)	
								dβ	1·41	(1·38)			
			−0·41					dγ	1·39	(1·35)			
	r_{p2}	15·46		15·05	θ_{p2} 27°1·′8	4°2·′9	11°29·′45	dα′	1·79	(1·73)			(100)β
								dα	1·78	(1·72)			
								dβ	1·58	(1·53)	(1.58)	(100)	
								dγ	1·56	(1·51)			
2	r_n	27·55		27·92	θ_n 43°25·′4								
	r_{p1}	30·76		31·13	θ_{p1} 46°32·′4	3° 7′	24°49·′7	dα	0·848	(0·853)	0·843	(321)	(321)α
								dα	0·843	(0·848)	0·843	(321)	
								dβ	0·752	(0·756)			
			+0·37					dγ	0·738	(0·742)			
	r_{p2}	27·77		28·14	θ_{p2} 43°28·′9	13·′5	21°56·′2	dα′	0·953	(0·964)			(321)β
								dα	0·947	(0·958)			
								dβ	0·845	(0·855)	0·843	(321)	
								dγ	0·829	(0·839)			
3	r_n	29·00		29·42	θ_n 44°55·′4								
	r_p	25·21	+0·42	25·63	θ_p 40° 59′	3°56·′4	18°31·′3	dα′	1·12	(1·14)	1·12	$(110)^2$	$(110)^2_\alpha$
								dα	1·12	(1·13)	1·12	$(110)^2$	
								dβ	0·994	(1·01)			
								dγ	0·976	(0·989)			

Table IV (cont.)

No. of spots		r' (obs.) in mm.	dc in mm.	r (corrected) in mm.		2θ (calc.) in degree	$d\theta$	θ (calc.)	Grating constant d in 10^{-8}cm.			Davy's date d in 10^{-8} cm	Davy's date Indices of atomic plane	Indices of the atomic plane
4	r_n	25·18	+0·40	25·58	θ_n	40°51·′8								
	r_{p1}	26·15		26·55	θ_{p1}	41°59·′3	1°3·′5	21°31·′4	$d_{\alpha'}$	0·970	(0·980)			$(321)_\gamma$
									d_α	0·965	(0·974)			
									d_β	0·860	(0·862)			
									d_γ	0·845	(0·853)	0·843	(321)	
	r_{p2}	26·54		26·94	θ_{p2}	42°24·′2	1°28·′4	21°56·′3	$d_{\alpha'}$	0·953	(0·961)			$(321)_\beta$
									d_α	0·947	(0·956)			
									d_β	0·845	(0·852)	0·843	(321)	
									d_γ	0·829	(0·837)			
5	r_n	21·28	+0·37	21·65	θ_n	36°16·′5								
	r_{p1}	19·63		20·00	θ_{p1}	34°8·′2	2°8·′3	15°59·′95	$d_{\alpha'}$	1·29	(1·31)	1·29	(211)	$(211)_\alpha$
									d_α	1·28	(1·31)	1·29	(211)	
									d_β	1·15	(1·16)			
									d_γ	1·12	(1·14)			
	r_{p2}	18·29		18·66	θ_{p2}	32°18′·6	3°57·′9	14°10·′35	$d_{\alpha'}$	1·45	(1·48)			$(211)_\beta$
									d_α	1·45	(1·47)			
									d_β	1·29	(1·31)	1·29	(211)	
									d_γ	1·23	(1·29)			
6	r_n	9·55		9·78	θ_n	18°20·′5								
	r_p	9·55	+0·23	9·78	θ_p	18°20·′5	0	9°10·′25	$d_{\alpha'}$	2·23	(2·28)	2·23	(110)	$(110)_\alpha$
									d_α	2·22	(2·27)	2·23	(110)	
									d_β	1·98	(2·02)			
									d_γ	1·94	(1·99)			

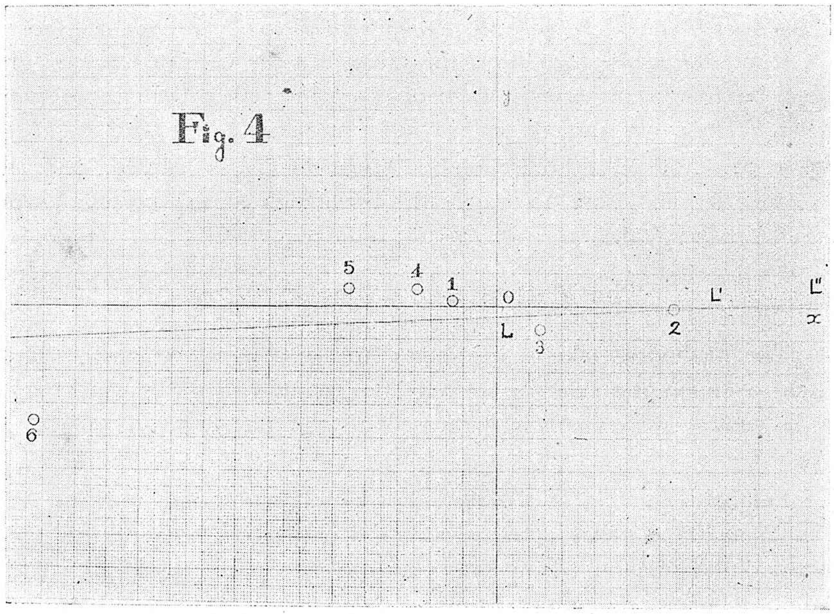
Now, assuming that the spectrum lines, 1, 2, 3, etc., are caused respectively by the crystal plane shown in the last column of Table I, the corresponding values of the glancing angle θ were calculated. When the spot contains two spectrum lines the mean of two values of θ obtained with these two lines is tabulated. For the values of the grating constants the writer has taken the Davey[1] data and for the wave lengths the same values as in the case of Table I were used.[2]

The difference between $\tan 2\theta_n$ and $\tan 2\theta_n'$ are calculated and given in the 8th column of Table II, and by using the observed values of φ' in the 9th column the writer plotted the relation expressed by equation (5) as shown in Fig. 3. Keeping the distribution of the plotted spots in mind the circle OO′ and the diameter OO″O′ which passed through the origin were drawn. The length of this diameter and its vectorial angle are denoted by $\frac{c_0}{a}$ and α_0 respectively. With these values of $\frac{c_0}{a}$ and α_0, the writer calculated the values of $y=\frac{\Delta}{\cos(\varphi'-\alpha_0)}$ and $x=\tan(\varphi'-\alpha_0)$, and plotted the relation expressed by equation (10) in Fig. 4 for the spots 1, 2, 3, etc. These data are represented in Table II, where all symbols represent the same meaning as in the case of



1 & 2, loc. cit.

Table I, and the values of $2d\theta'$ and $2\theta_n'$ in the 4^{th} and 7^{th} columns respectively are reproduced from Table I.



In Figs. 3 and 4, the spots 1, 2, 3, etc., refer respectively to those represented in Fig. 3, Plate I, and Tables I and II. In Fig. 3 one division corresponds to 10^{-4} and in Fig. 4 one division of the abscissa and one of the ordinate correspond to 10^{-2} and 10^{-4} respectively. In Fig. 4 it can easily be seen that the spots lying outside of the circle OO′ in Fig. 3, are in the 1^{st} and 2^{nd} quadrants, and those lying inside the circle in Fig. 3 are in the 3^{rd} and 4^{th} quadrants. From the position and the inclination of the line LL′ drawn in Fig. 4 the values of $-\frac{\Delta c}{a}=\mathrm{OL}$ and $\frac{c_0}{a}(\Delta a)=\tan \mathrm{L''L'}x$ were obtained, and by getting the values of Δa by calculation, the corrected values of c and a are obtained and shown at the bottom of Table II.

The corrected values of $dc=\frac{c}{a}\cos(\varphi'-a)$ and φ were calculated for all spots and are shown in Table III.

With the values of dc thus corrected the writer obtained the corrected values of r_n and r_p and calculated the corrected values of θ for all spectrum lines obtained. With these values of the glancing angle, the grating constants of the various atomic planes were recalculated and compared with the Davey data in Table IV. In this table all symbols have the same meaning as those in Table I. For comparison the uncorrected grating constants represented in Table I are reproduced in brackets in the 12th column of the same table.

It will be easily seen from the table that the corrected values of d are in better agreement with those obtained by Davey than those uncorrected. Thus we can determine more accurately the indices of the atomic plane by whose reflection the spectrum line is caused, and also we can determine more accurately the orientation of the crystallographic axes of the crystal.

The Method using Crossed Wires

The method of taking the photograph was improved by the writer so as to obtain the exact position of the centre of the intense nucleus of the central spot which is impressed by the direct beam of the X-rays.

Before the setting of the photographic plate, crossed metallic wires of high atomic number were so stretched and fixed, between the specimen and the position where the photographic plate would be placed, that the line connecting the centre of the slit and the crossed point of the wires became perpendicular to the plane of the crossed wires. Then the position of the X-ray tube was so adjusted as to project the image of the centre of the focus at the crossed point, and then the photographic plate was placed parallel to the plane containing the crossed wires behind it. For the crossed wires an unannealed tungsten wire was used, the diameter of which was about 0.07 mm. In adjusting the position of the X-ray tube a fluorescent screen was used at first, and then fine adjustment was made by taking a pin hole photograph of the focus of the target as shown in Fig. 9 of Plate II.

With this apparatus a Laue diffraction pattern of thin tungsten wire composed of a single crystal was taken and is represented in Fig. 10 of Plate II. In this photograph we can see the shadow of the crossed wires in the region outside the intense central spot. Though the intensity of the central spot is too strong and the position of the crossed point can hardly be detected in the central part of the central spot, its position is obtained exactly and easily by elongating the detectable part of the shadow-lines.

By setting initially the axis of the specimen parallel to one of the crossed wires, the direction of the axis of the specimen can be obtained exactly on the photograph.

The correction before mentioned, in addition to the method mentioned above, will give us a more exact position of the centre of the direct image of the focus of the target.

In conclusion, the writer wishes to express his sincere thanks to Professor U. Yoshida, of Kyoto Imperial University for his invaluable suggestions.

Research Laboratory,
Tokyo Electric Company,
Kawasaki, Kanagawa-Ken,
Japan.

Plate I

Fig. 1

Fig. 2

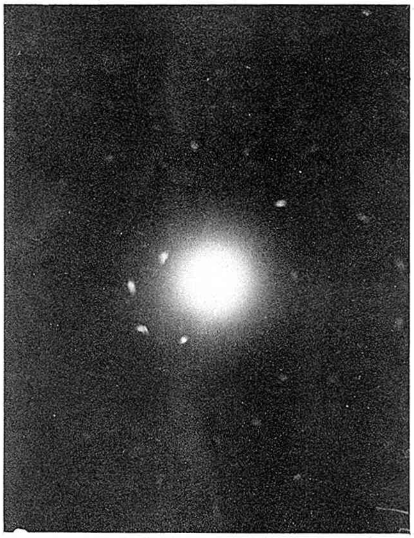


Fig. 3

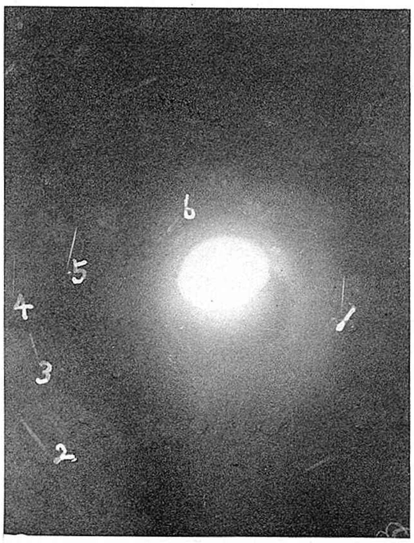


Fig. 4

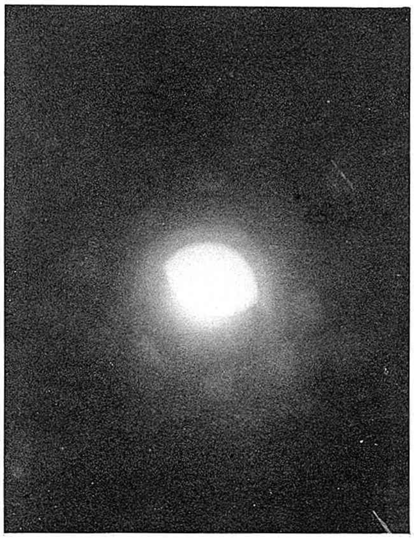


Plate II

Fig. 5

Fig. 6

Fig. 7

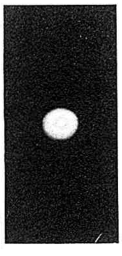


Fig. 8

Fig. 9

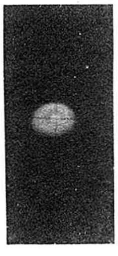


Fig. 10