# Studies in Photographic Sensitivity, Part II. On the Sensitivity of Photographic Plates at Various Temperatures

By

**Osamu Masaki**

(Received October 29, 1928)


### Abstract

By raising the temperature of photographic plates the sensitivity of panchromatic and other slow emulsion plates was found to increase while that of rapid plates was found to behave in the opposite way. But their contrasts became always steeper. The corresponding change in the inertia of characteristic curves was slower up to the temperature of about 30°C. and then became greater as the temperature was raised to about 70°C. and again became slower at higher temperatures. The change in the contrast was smaller up to the temperature of about 60°C. and became more marked at higher temperatures. $\gamma$ was found to be proportional to $e^{t^2}$, $t$ being the temperature of the plate in °C. The tangents at inflexion points of the characteristic curves taken at various temperatures seem to meet at one point. From these facts the following formula expressing the density as a function of temperature is obtained

$$D = D_m e^{at^2}\left\{1 - e^{-(bt+c)E}\right\},$$

where $a$, $b$, and $c$ are constants and $D_m$ is the maximum density at 0°C. This formula is valid from 10°C. up to 80°C.


## Experiments

In my first paper[1] it was shown that by raising the temperature of photographic plates the contrasts of their characteristic curves always become steeper and the sensitivity of such plates as panchromatic and those of slow emulsion is increased, but that of such plates as very rapid emulsion is decreased. In the present experiment it is intended

1 These Memoirs **12**, 1, (1928)

to find how the inertia and contrast $\gamma$ of the plate depend on its tempeature.

The experimental apparatus and methods were the same as those given in previous paper. Several pieces were cut out from a plate carefully preserved in a desiccator, and they were heated before the exposure at a desired temperature for one hour in a steam bath. The

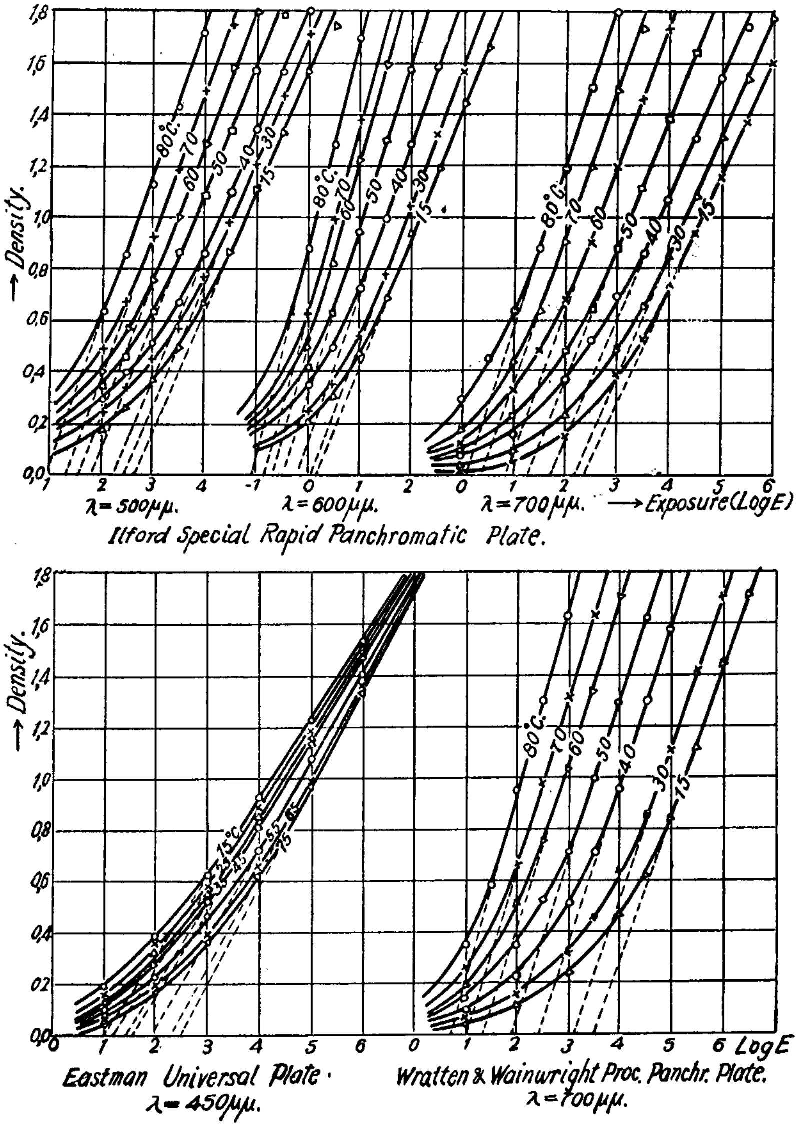


*Fig. 1. Characteristic Curves taken at Various Temperatures.*

temperature was read by a thermometer inserted into the plate holder, the temperature being kept as constant as possible during the exposure, but it varied within $\pm 1$°C.

## Analysis of the Characteristic Curves

The characteristic curves of certain photographic plates taken at various temperatures for lights of special wave lengths are represented in fig. 1. From these curves the inertia and the contrast of such plates are plotted against the temperatures as shown in Fig. 2.

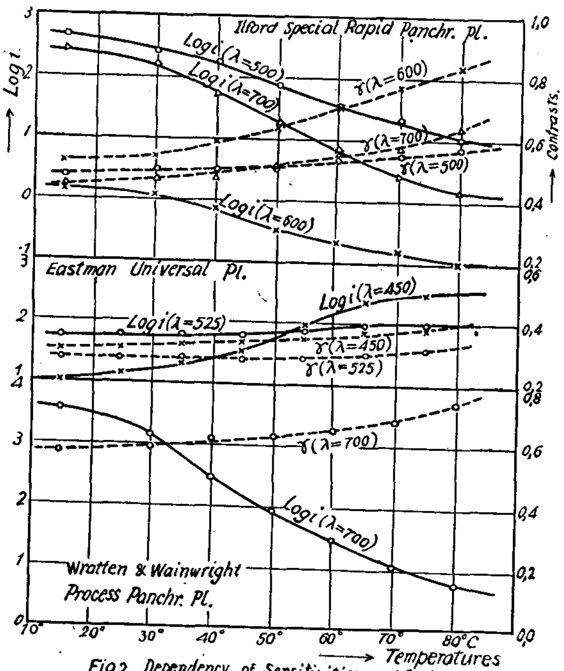


Fig. 2. Dependency of Sensitivities and Contrasts on the Temperature.

The values of inertia log i and contrast $\gamma$ determined from the characteristic curves are given in the following tables.

As is seen from the diagrams, though the variation of the inertia is generally slow up to the temperature of about 30°C., it becomes very marked after that temperature is reached and again becomes smaller at temperatures higher than 70°C.. On the other hand, the increasing of the contrast is not marked at lower temperatures until the temperature of 50°C. or 60°C. is reached, but becomes gradually steeper at higher

Table I

| t | Ilford Special Rapid Panchromatic Plate. | | | | | |
|---|---|---|---|---|---|---|
| | λ=500 μμ. | | λ=600 μμ. | | λ=700 μμ. | |
| | Log i | γ | Log i | γ | Log i | γ |
| 15°C. | 2.65 | 0.466 | 2.17 | 0.520 | 2.40 | 0.440 |
| 30°C. | 2.48 | 0.482 | 2.06 | 0.536 | 2.20 | 0.462 |
| 40°C. | 2.25 | 0.488 | 1.82 | 0.590 | 1.70 | 0.468 |
| 50°C. | 1.85 | 0.510 | 1.50 | 0.636 | 1.28 | 0.508 |
| 60°C. | 1.53 | 0.524 | 1.30 | 0.700 | 0.86 | 0.554 |
| 70°C. | 1.32 | 0.550 | 1.20 | 0.770 | 0.39 | 0.562 |
| 80°C. | 0.97 | 0.566 | 0.96 | 0.840 | 0.14 | 0.634 |

| t | Wratten & Wainwright Process Panchr. Pl. | | t | Eastman Universal Plate. | | | |
|---|---|---|---|---|---|---|---|
| | λ=700 μμ. | | | λ=450 μμ. | | λ=525 μμ. | |
| | Log i | γ | | Log i | γ | Log i | γ |
| 15°C. | 3.54 | 0.575 | 15°C. | 1.00 | 0.304 | 1.76 | 0.270 |
| 30°C. | 3.13 | 0.588 | 25°C. | 1.16 | 0.306 | 1.75 | 0.270 |
| 40°C. | 2.45 | 0.613 | 35°C. | 1.40 | 0.328 | 1.79 | 0.280 |
| 50°C. | 1.90 | 0.618 | 45°C. | 1.54 | 0.332 | 1.76 | 0.274 |
| 60°C. | 1.45 | 0.645 | 55°C. | 1.94 | 0.346 | 1.84 | 0.286 |
| 70°C. | 1.06 | 0.675 | 65°C. | 2.37 | 0.374 | 1.95 | 0.296 |
| 80°C. | 0.78 | 0.740 | 75°C. | 2.44 | 0.376 | 1.95 | 0.310 |

temperatures. It may be assumed from the curves that the contrast changes exponentially with the temperature.

Now if we plot the values of $\log \gamma$ against $t^2$, the straight lines shown in Fig. 4 are obtained. Thus the contrast $\gamma$ is represented by the expression:

$$\gamma = e^{at^2+k} \quad \text{..............................(1)},$$

where $a$ and $k$ are constants depending on the wave length of the light used for the exposure, the time of the development and the properties of the emulsion, and $t$ is the temperature expressed in °C..

The constants $a$ and $k$ of the equation (1) are calculated from the straight lines given in Fig. 4 as follows;—

Table II

| | Ilford Spec. Rap. Panchr. Pl. | | Eastman Universal Pl. |
|---|---|---|---|
| | $\lambda=500$ μμ. | $\lambda=700$ μμ. | $\lambda=450$ μμ. |
| $a$ | 0.0000300 | 0.000057 | 0.000044 |
| $k$ | $-0.74$ | $-0.822$ | $-1.172$ |

It may also be seen that the tangents at the point of inflexion of each characteristic curve given in Fig. 1 seem to meet at one point. If this is correct for every temperature the following relation must hold:

$$\frac{-\beta}{\log_2 i-\alpha}=\gamma \quad \text{.....................................}(2),$$

where $\alpha$ and $\beta$ are the coordinates of the point of intersection.

That is, there must be linear relation between $\frac{1}{\gamma}$ and $\log_2 i$.

A graph is therefore drawn taking $1/\gamma$ as abscissa and $\log i$ as ordinate and a straight line is thus also obtained. The examples of such diagrams are represented in Fig. 3, in which points almost lie on the straight lines showing that equation (2) holds in the present case, *i. e.*, the tangents at the point of inflexion to each characteristic curve taken at various temperatures intersect at a point.

The constants $\alpha$, $\beta$ are calculated from these lines as follows.

Table III

| | Ilford Spec. Rap. Panchr. Pl. | | Eastman Universal Pl. |
|---|---|---|---|
| | $\lambda=500$ μμ. | $\lambda=700$ μμ. | $\lambda=450$ μμ. |
| $\alpha$ | $-8.5$ | $-6.0$ | 2.4 |
| $\beta$ | $-5.3$ | $-3.8$ | 8.8 |

## Empirical Formula

The equation connecting the relation between the density of a developed photographic plate and the exposure is given by the following equation by Elder,

$$D=D_m(1-e^{-KE}), \quad \text{.................................}(3),$$

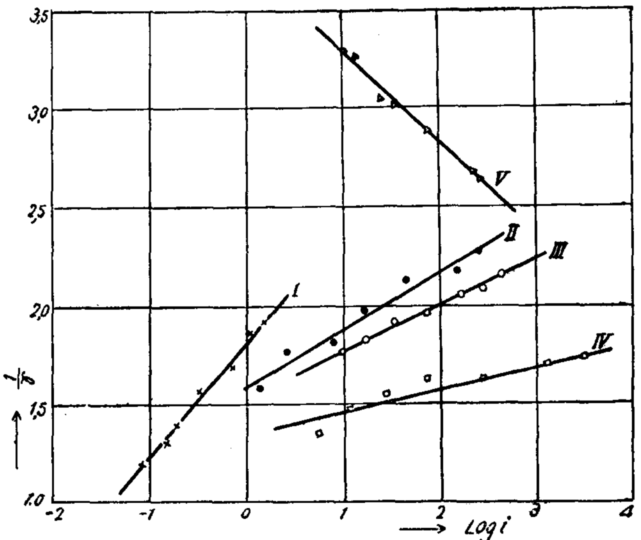


*Fig. 3. The Relations between the Inertias and the Contrasts.*

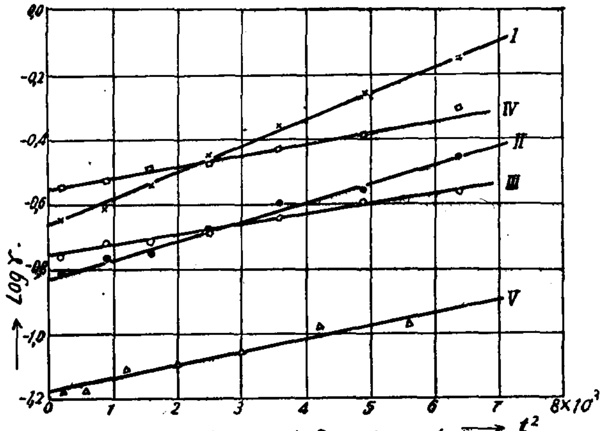


*Fig. 4. The Relations between the Contrasts and the Temperatures.*

I Ilford Special Rapid Panchr. Pl. ($\lambda$=600 $\mu\mu$.)
II " " " ($\lambda$=700 $\mu\mu$.)
III " " " ($\lambda$=500 $\mu\mu$.)
IV Wratten & Wainwright Process Panchr. Pl. ($\lambda$=700 $\mu\mu$.)
V Eastman Universal Plate. ($\lambda$=450 $\mu\mu$.)

temperature of the plate being supposed to be constant. But if the temperature of the dry plate is changed, $D_m$ and $K$ should also vary as a certain function of the temperature, consequently, in considering characteristic curves, $x=\log E$ is to be taken as abscissa, and then we get

$$D=D_m(t)\left\{1-e^{-K(t)e^{x}}\right\} \qquad \text{.........................(4)},$$

by differentiating this equation we obtain

$$\frac{dD}{dx}=D_m(t)K(t)e^{x-K(t)e^x} \quad \dots\dots\dots\dots\dots\dots\dots(5),$$

$$\frac{d^2D}{dx^2}=D_m(t)e^{x-K(t)e^x}\{1-K(t)e^x\}.$$

At the inflexion point this equatin must be equal to zero, or

$$1-K(t)e^x=0$$

this is,

$$K(t)=e^{-x} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(6).$$

Substituting the last value in equation (5) we obtain

$$\frac{dD}{dx}=D_m(t)e^{-1}$$

This is the slope of the tangent at the inflexion point of the characteristic curve, and it is not anything else than the contrast of the characteristic curve. But in an ordinary characteristic curve $E=2^\xi$ is taken as the exposure inteat of $E=e^x$, thus the abscissa axis in the accompanying curves representing the value of $\xi$, consequently, $2^\xi=e^x$ or $\xi\log 2=x$

then

$$\frac{d\xi}{dx}=\frac{1}{\log 2}$$

therefore

$$\frac{dD}{dx}=\frac{dD}{d\xi}\frac{d\xi}{dx}=\frac{dD}{d\xi}\frac{1}{\log 2}$$

$$\frac{dD}{d\xi}=D_m(t)\log 2\cdot e^{-1}$$

that is

$$\gamma=D_m(t)e^{-1.366} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(7),$$

because Log $2=e^{-0.366}$.

As shown above the contrast may be given by the equation

$$\gamma=e^{at^2+k}$$

therefore

$$D_m(t)e^{-1.366}=e^{at^2+k}$$

or

$$D_m(t)=e.^{at^2+k+1.366}$$

putting

$$k+1.366=k'$$

$$D_m(t)=e^{at^2}+k' \quad \text{.....................................(8)}$$

The last equation shows how the maximum density changes with the temperature. Now, let the value of $D_m(t)$ when $t=0$ be represented by $D_m$, then $D_m=e^{K'}$, and equation (4) becomes

$$D=D_m e^{at^2}\{1-e^{-K(t)E}\} \quad \text{...........................(9)}$$

In order to see how the relation (8) holds in the present experiment, densities calculated from equation (8) and those calculated from observed characteristic curve are compared in the following table, in which the data are taken from Ilford Special Rapid Panchlomatic Plates for the ray of $\lambda=500\ \mu\mu$..

Table IV

| Temp. | 15°C. | 30°C. | 40°C. | 50°C. | 60°C. | 70°C. | 80°C. |
|---|---|---|---|---|---|---|---|
| $D_m$ cal. from (8). | 1.90 | 1.94 | 1.98 | 2.04 | 2.10 | 2.18 | 2.29 |
| $D_m$ cal. from real curve. | 1.93 | 1.97 | 2.02 | 2.04 | 2.13 | 2.18 | 2.25 |
| Difference. | 0.03 | 0.03 | 0.04 | 0.00 | 0.03 | 0.00 | −0.04 |

The maximum densities given in the above table are calculated according to a method similar to that given by Nietz in his book "Theory of development" on p. 89. Thus it will be observed that our equation (9) gives correct results as the differences lie within the limits of experimental errors.

Next, the functional form of $K(t)$ in the equation (9) must be determined. The equation (9) is now transformed into the following form:

$$\text{Log}\frac{D_m e^{at^2}}{D_m e^{at^2}-D}=K(t)E \quad \text{.....................(10).}$$

Thus, the value of the left hand side of the equation (10) is proportional to $K(t)$ when the exposure is constant. If we draw curves representing Log $D_m e^{at^2}/(D_m e^{at^2}-D)$ as a function of $K(t)$, a straight line is obtained only when $t^2$ is taken as abscissa.

Fig. 5 represents such relations for some kinds of plates for several values of $E$.

Therefore the function $K(t)$ is to be expressed as follows

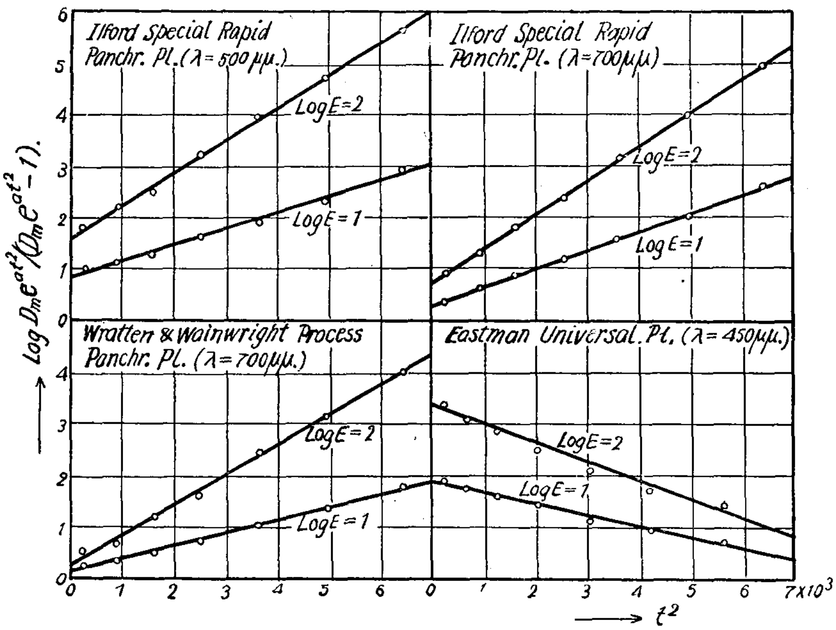


*Fig. 5.*

$$K(t)=bt^2+c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11),$$

where $b$ and $c$ are constants, their values being calculated from Fig. 5 and are given in the following table.

Table V

| Kind of plates | Wave length of light | b | c | a |
|---|---|---|---|---|
| Ilford Spec. Rap. Panchr. Plate | 500 μμ. | 0.0000090 | 0.022 | 0.000031 |
| Ilford Spec. Rap. Panchr. Plate | 700 | 0.000019 | 0.019 | 0.000057 |
| Eastman Universal Plate | 450 | −0.0000046 | 0.046 | 0.000038 |
| Wratten & Wainwright Process Panchr. Plate | 700 | 0.000016 | 0.0058 | 0.000034 |

Thus, as the result of these analyses of the characteristic curves the following empirical formula of density growth expressed as a function of the temperature is obtained:

$$D=D_m e^{at^2}\left\{1-e^{-(bt^2+c)E}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(12),$$

where $D_m$ is the maximum density at 0°C., $E$ is the exposure, $t$ is the temperature of the dry plate in °C., and $a$, $b$ and $c$ are constants depending on the wave length of the exposure light, time of development and properties of emulsion, where $a$ decreases as the time of development increases, and $b$ is negative for some rapid plates and positive for other plates.

The following table gives the values of the observed and calculated densities using equation (12) for Ilford Special Rapid Panchromatic Plate for light of $\lambda = 500\ \mu\mu$.

Table VI

| Temp. / Exp. | 15°C. | | 30°C. | | 40°C. | | 50°C. | | 60°C. | | 70°C. | | 80°C. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ob. | cal. | ob. | cal. | ob. | cal. | ob. | cal. | ob. | cal. | ob. | cal. | ob. | cal. |
| $2^2$ | 0.18 | 0.18 | 0.23 | 0.23 | 0.29 | 0.27 | 0.34 | 0.34 | 0.41 | 0.42 | 0.48 | 0.52 | 0.63 | 0.64 |
| $2^3$ | 0.36 | 0.34 | 0.43 | 0.42 | 0.51 | 0.51 | 0.63 | 0.63 | 0.75 | 0.74 | 0.92 | 0.93 | 1.15 | 1.12 |
| $2^4$ | 0.66 | 0.62 | 0.76 | 0.76 | 0.85 | 0.89 | 1.08 | 1.07 | 1.28 | 1.26 | 1.48 | 1.48 | 1.71 | 1.70 |
| $2^5$ | 1.10 | 1.06 | 1.20 | 1.23 | 1.34 | 1.39 | 1.53 | 1.59 | 1.82 | 1.82 | | | | |

The coincidence is pretty good, showing that this formula holds from 10°C. to 80°C..

In conclusion, the author wishes to express his thanks to Prof. M. Kimura for his kind advice and the great interest he has taken in the present investigation, and the author's thanks are also due to President T. Maruyama of the Konan College for his good offices.