The Irregular Lichtenberg Figures due to the Presence of Resistance and Inductance

By

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Abstract

(1) A peculiar positive figure is produced by the presence of a great resistance in series with the connecting wire to the electrode, and its formation is nearly independent of the length of the spark gap and the value of the capacity in the oscillation circuit.

(2) An irregular positive figure is obtained with a suitable branched circuit consisting of a capacity and an inductance, which circuit is inserted in series with the lead to the electrode.

(3) Analogous negative figures can not be obtained under similar conditions.

(4) Both the peculiar and the irregular figures mentioned above seem to be due to successive discharges at the electrode.

Chapter I

Introduction

An electric impulse is produced by an electric spark occurring at a gap in a circuit in obtaining ordinary Lichtenberg figures. When the electric impulse produced in such a manner reaches an electrode laid on a photographic plate which is placed on an earthed metal plate, and its amplitude is sufficient, a beautiful discharge-figure—well known as a photographic Lichtenberg figure—is impressed on the plate after development.

The figure obtained depends upon the sign of the charge at the electrode, the amplitude of the electric impulse, the length of the lead wire, the composition and the pressure of the gas, and the thickness and dielectric constant of the insulating plate. These relations were investigated by W. V. Bezold[1], S. Mikola[2], K. Przibram[3], P. O. Pedersen[4] and others. Recently, Prof. P. O. Pedersen[5] has published his recent experimental results and a theory as to the formation of the positive figures. According to him, the formation of the positive figure is due to protons and the negative figure to electrons.

The normal figures, as shown in Figs. 3 and 5 in Plate I, are formed under suitable experimental conditions. If the potential is still higher or the connecting wire to the electrode is too long, irregular figures are produced: the negative figure consists of a regular or normal negative figure and some fan shaped figures extending outside the former, as shown in Fig. 6, Plate I; and the positive figure consists of a regular or normal positive figure and some bright long spark tracks, from which new irregular branches start, covering the former as shown in Fig. 2, Plate I. These figures have been investigated by M. Toepler[6] and P. O. Pedersen[7].

The writer has investigated the character of the irregular figures obtained owing to the presence of a great resistance or an inductance inserted in series with the lead to the electrode.

Chapter II

Influence of a Great Resistance inserted in Series with the Lead Wire

(1) *Positive Peculiar Figure*

In Fig. 1 I is an influence machine whose two terminals are connected with the great resistances R_1 and R_2 respectively, R is another great resistance or shunt resistance, C a condenser, g a spark gap whose length is l, P a photographic plate laid on an earthed metal plate B. A is a point electrode which rests on the sensitive film of the photographic plate P.

In order to get a spark having a definite potential and to cause

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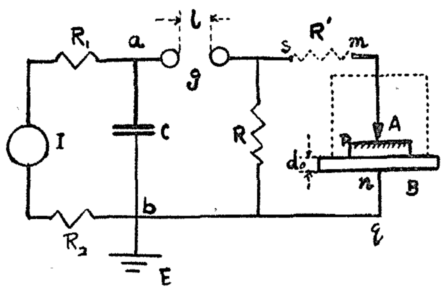
1. W. V. Bezold: Pogg. Ann., **144**, 337, 526, (1871)
2. S. Mikola: Phys. Zeitsch., **18**, 158, (1917)
3. K. Przibram: Phys. Zeitsch., **20**, 299, (1919)
4. P. O. Pedersen: Vidensk. Selsk., **I**, **11**, 3, (1919)
5. " " " " **VIII**, **10**, 3, (1929)
6. M. Toepler: Ann. d. Phys., (4), **53**, 217, (1917)
7. P. O. Pedersen: loc. cit. (1929)

only one spark each time, the influence machine must be rotated very slowly, especially when working with a short spark gap g or a small capacity C. At first, the electric capacity of the condenser C and the shunt resistance R were made to be about 1600 cms. and 10 megohms respectively, and the other experimental conditions were kept constant as far as possilble, excepting the spark length l. Then we produced many positive and negative figures with various lengths of spark gap. Generally speaking, it seems to the writer that the irregular positive figure occurs with a slightly lower potential than the irregular negative one. Fig. 2, Plate I is an irregular positive figure obtained with a spark length of 5 mm., and Fig. 5 in Plate I is a regular negative one obtained by the same length of the spark gap. If the spark length is increased to about 6 mm. or more, the irregular figure occurs in the negative as well as in the positive figure in most cases. Fig. 6, Plate I, which was taken with a spark length of 6.5 mm. is an irregular negative figure.

Fig. 1

This figure shows the electrical connection which is used for most of the experiments described below.



Next, a concentrated high resistance R'—silit—was inserted between the points s and m in Fig. 1 in series with the electrode A. Fig. 1, Plate I is a positive figure obtained under the same conditions as those of Fig. 2 in the same plate, excepting that a high resistance $R'=31{,}500$ ohms was inserted in the former case. It will be seen from these figures that the regular or normal positive figure disappears gradually as the resistance R' increases, and the irregular positive figure alone remains. Figs. 3 and 4 in Plate I were obtained under the same conditions as those of Figs. 2 and 1 in the same plate respectively, excepting that the spark length was 3 mms. in the case of Figs. 3 and 4 in Plate I and was 5 mms. in the case of Figs, 1 and 2 in Plate I. It will be seen from Figs. 3 and 4 in Plate I that the positive branches change and become irregular when a high resistance of the order of $R'=31{,}500$ ohms is inserted.

In order to make clearer the features of such irregular branches the photographs were next taken under low air pressures. The lower the pressure of the air the broader the figure-branches and the greater the range of the figure become. The electrode A, the photographic plate P and the earthed metal plate B in Fig. 1 were enclosed in a bell jar provided with an exhaust arrangement, and the lead wire was made to pass by an airtight connection through the top of the jar. Fig. 3, Plate II is a typical positive figure obtained when the pressure and the shunt resistance R are reduced to 100 mm. Hg. and 100,000 ohms respectively and the series resistance R' is increased to 167,500 ohms. Clearly the figure consists of two parts: (1) a diffuse circular part at the centre, from which many short brushes radiate outwards, and (2) some intense branches starting from the center of the former, from whose ends the irregular branches grow outwards and fit themselves in proper order into the spaces between the brushes of (1). From the feature of this figure, it seems that part (1) corresponds to the regular part of the irregular positive figure and is formed first, and part (2) to the irregular branches of the irregular positive figure, and is formed afterwards. The figure under consideration is, unlike the ordinary irregular one at atmospheric pressure, always obtained when we use a high resistance R', even if the connecting wire to the electrode is not long and the amplitude of the electric impulse is not sufficient. We will hereafter call this figure "the peculiar figure", for the sake of convenience, to distinguish it from the ordinary irregular one.

In order to investigate the formation of such a peculiar positive figure, other experiments were undertaken, some of which are described below.

(a) **The lead wire and the shunt resistance.** The formation of the peculiar positive figure is almost independent of the form of the electrode A, the material of the high series resistance R' and the length of the line $C\ a\ g\ s$ in Fig. 1. But, if the connecting wire $m\ A$ was increased to more than 4 metres, or a condenser having a capacity of about 5 cms. or more was placed in parallel to the Lichtenberg discharge gap $A\ B$ in Fig. 1, the peculiar figure never occurred with $l=1.4$ mm. as before, and the spark length l must be increased to about 2 mm. to get it.

If the shunt resistance R is reduced, the range of the Lichtenberg figure becomes smaller. The writer altered the shunt resistance from 10 megohms to 900 ohms, keeping the series resistance R' at 100,000 ohms and also keeping all the other circumstances as far as possible constant. All the figures thus obtained under various low pressures were the same in their main features as in Fig. 3, Plate II. But the ordinary irregular figures already mentioned could not be obtained when the shunt resistance was reduced below a certain value.

(b) The capacity in the oscillation circuit. Before entering into the details of the experimental results, which show whether or not the reduction of the capacity under consideration has some influence upon the occurrence of the peculiar positive figure, it will be convenient to consider a little the relation between the range of the figure and the capacity of the circuit.

S. Mikola[1] gives the following formulae for the range of the regular figures, but they are only approximate ones and can only be used within certain limits, as shown by Prof. P. O. Pedersen[2] :—

$$r = a_1\sqrt{V - V_0} \qquad \text{for the positive figure} \qquad (1)$$

and

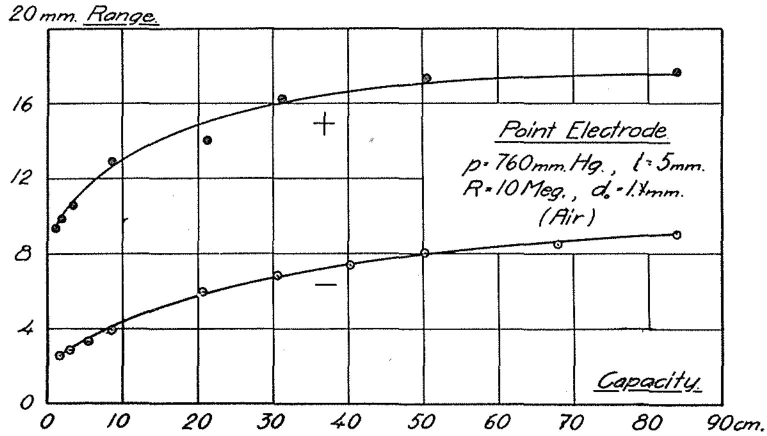
$$r = a_2(V - V_0) \qquad \text{for the negative figure} \qquad (2),$$

where r represents the linear range of the figure, V the real spark potential, and V_0 the smallest potential difference capable of forming a figure, a_1 and a_2 are constants. In these formulae the capacity in the circuit has not been considered. If the capacity of the condenser is reduced, the length of the spark gap and other conditions being kept constant, the quantity of the electric charge in the circuit is reduced. Consequently the potential difference applied by the influence machine between the electrode A and the earthed metal plate B decreases suddenly as soon as the formation of the figure commences, especially when the capacity of the circuit is small; and the growth of the figure will stop at a certain range which depends upon the supply of the electricity coming to the electrode.

In order to verify this consideration the writer carried out experiments with a variable parallel plate air condenser inserted in place of the condenser C in Fig. 1. Fig. 2 is an example showing the relation between the range of the figure and the capacity of the air condenser. From this figure it will easily be seen that the range of the figure depends upon the capacity in the circuit, and that the range-capacity curves of both the positive and the negative figures

1. S. Mikola: loc. cit. (1917)
2. P. O. Pedersen: loc. cit. (1919)

Fig. 2



are nearly parallel to each other and nearly linear for somewhat greater capacities.

Next, the experiments were carried out with various capacities and many photographs were taken with and without the series resistance R'. From these photographs, it was found that the peculiar positive figure is easily obtained under low pressures, even if $c < 2.43$ cms. and $l < 1.4$ cms., and that the ordinary irregular one does not occur with comparatively small capacities, even if the length of the spark gap is increased to 8 mm. and the lead wire $C\ a\ g\ A$ in Fig. 1 is increased to a comparatively great length.

Thus, we know that the capacity in our circuit plays an important rôle in the occurrence of the ordinary irregular figure though it is not important for the formation of the peculiar figure so far as our experiments are concerned.

(2) *Peculiar Negative Figure*

In order to try if we could also get the peculiar negative figure, experiments similar to those for the positive one were repeated. Fig. 1, plate II shows the negative figure obtained by the use of a series resistance R' of 100,000 ohms and at the air pressure of 170 mm. Hg., but we can not see any peculiar feature in this. That positive branches occur around the electrode in the regular or normal negative figures at low pressures is well known.

(3) *An Explanation of the Peculiar Figure*

The discharge of the condenser through our circuit $C\ g\ s\ m\ n\ q\ b\ C$ in Fig. 1 will generally be oscillatory and its period T is approximately determined by

$$T = 2\pi\sqrt{L_0 C} \tag{3}$$

where L_0 is the coefficient of self-induction of the circuit, and the capacity C of the circuit is roughly taken to be equal to that of the condenser C.

The stored dielectric energy E is given by

$$E = \frac{1}{2} C V^2 \tag{4}$$

just before the electric discharge commences at the spark gap g, where V is the potential difference between the spark gap. After the commencing of the electric discharge, the stored energy will gradually turn into the magnetic energy E' given by

$$E' = \frac{1}{2} L_0 I^2, \tag{5}$$

where I is the oscillating current. The magnetic energy E' pulsates, with double frequency of the voltage or the current, between zero and a maximum value equal to the total energy if there are no losses due to leakage and resistance. But it is not so simple in our case. There are the losses due to the resistance of the wire and the spark gap and the dielectric loss. Among the causes of the loss of electric energy, the losses due to the shunt resistance and the formation of the figure will be especially predominant in our case. And the damping of the oscillations will depend mostly on these two losses.

The peculiar figure can hardly be due to the electric oscillation in the circuit, for it occurs even if the shunt resistance is reduced to a certain small value, or if another great resistance of about 180,000 ohms is inserted between g and C in Fig. 1. As was stated before, a great resistance R' is connected with the lead wire at the points m and s, and there are great differences in the line constants at both sides of these points. This fact causes, of course, the reflection of the electric wave at the points m and s. We know, however, that these reflections do not play any important rôle in the formation of the figure, because (1) the feature of the peculiar figure is nearly independent of the length of the line $s\ g\ a\ C$, and (2) the peculiar figure is also obtained even when the length of the line $m\ A$ is much reduced.

As described before, the regular portion of the peculiar figure is first formed at a certain potential, and the irregular branches of it afterwards at other potentials at the electrode. Another notable character is that the range of the regular part of the figure is reduced more rapidly than that of the irregular part with increase of the series resistance R'.

Now, at a certain stage, when the potential of the electrode has a certain value, the regular figure will first be formed. The charge, which has been accumulated at the electrode before the formation of this figure, starts to spread around the electrode in forming the figure. As a result of this, the potential difference which has been produced between the electrode and the earthed metal plate, is now reduced to a certain value and the growth of the regular figure stops. The range of the regular part depends upon the quantity of the electricity supplied in such a manner. Thus the greater the series resistance becomes, the more rapidly will the potential drop, and consequently the smaller will the regular part become. Even after the growth of the primary regular part is stopped in the above way, the electric charge will still flow towards the electrode, and if the potential at the electrode again increases to a certain high value it will then cause the irregular part of the figuse. If, however, the line $m\ A$ in Fig. 1 is long and its capacity is not too small, the charge accumulated in it will be sufficient to produce a complete figure in the primary stage, and the normal figure will then be obtained.

The reason why the irregular branches in the peculiar figure fit themselves in proper order into the spaces between the branches of the regular figure, may be simply that the ends of the former have the same kind of electricity as that of the ends of the branches of the latter.

The problem still left to be explained is the reason why the peculiar negative figure did not occur in our experiments. According to Prof. P. O. Pedersen[1], the conductivity of the negative figure is greater, especially at low air pressures, than that of the positive one just after the formation of the figure. Moreover the formation of the negative figure occurs at lower potentials than that of the positive one. If that is so, the primary and the secondary stages of the discharge as in the case of the positive one, will be united in this case, resulting in the formation of only an ordinary negative figure.

If there is no great mistake in this mode of explanation of the

1. P. O. Pedersen: loc. cit. (1919, 1929)

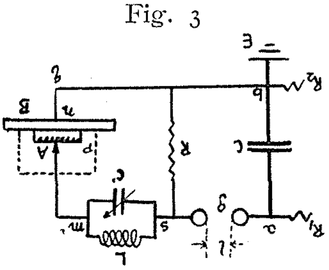
formation of the peculiar figure, we should be able to produce similar figures by any other methods, which can supply the positive potentials in rapid succession to the electrode.

Chapter III

Influence of an Inductance connected in Series with the Lead Wire

In the experiments in which we used an inductance coil instead of the series resistance R' in Fig. 1, we sometimes obtained an irregular or complex positive figure. Fig. 2, Plate II is a positive one obtained by using a coil having an inductance of 0.00019 henry. The coil used was made of insulated thin copper wire of the length of 8 metres, which had a resistance of 3 ohms in all. The coil was wound in 4 layers, and the layers were very carefully insulated from one another by interposing between them paper soaked with paraffin wax. The discharge-figure under consideration is, however, due not only to the presence of the inductance, but also to the presence of the inner capacity of the coil, for it hardly occurs with a single layer coil consisting of 31 metres of comparatively thick copper wire, which has a large inductance of 0.00022 henry and a negligible capacity. Thus, it seems to be true that the irregular figure under consideration is due to electric oscillations occurring within the coil.

Fig. 3



To verify this idea, the writer inserted a variable air condenser C' in parallel with the coil L having an inductance of 0.00022 henry (Fig. 3); and with a suitable value of the capacity, the irregular positive figure was obtained. Fig. 5, Plate II is a positive figure formed when the capacity C' was 83 cms., and the capacity C 1600 cms.. Under such circumstances the natural period T of the electric oscillation of the branched circuit $(L-C')$ is equal to about 8.9×10^{-7} secs. The figure under consideration has a feature similar to the irregular positive figures obtained with the long lead wire $s\ A$ in Fig. 1. (Compare the figure with I in Plate 28 of P. O. Pedersen's paper, Part III—1929.) If the capacity C is reduced to 80 cms., the capacity C' must be reduced to 20 cms. in order to get the irregular figure

($T = 4.3 10^{-7}$ secs.). Fig. 4, Plate II is obtained under such experimental conditions. The irregular positive figure due to the presence of the branched circuit consisting of an inductance and a capacity does not occur when the shunt resistance R shown in Fig. 3 is reduced below about 500 ohms, though the range of the figure is not small at low air pressures.

From the above experiments, it seems to the writer that the irregular figure under consideration is due to the electric oscillation occurring in the closed circuit, and consequently the irregular branches are formed by the successive positive potentials applied to the electrode by the electric oscillation after the formation of the primary regular part of figure. If the shunt resistance R is reduced below a certain value the electric oscillation in the circuit is nearly damped and the irregular figure does not occur.

The peculiar positive figure, Fig. 3 in Plate II, is quite similar in appearance to Fig. 4 in the same plate, which has been described above. From this fact, it seems to be very probable that the peculiar positive figure is, as stated in Chapter II, due to the positive potentials, ocurring in rapid succession at the electrode A by reason of the high resistance R' in Fig. 1, and that the irregular branches of the peculiar positive figure are also formed about 10^{-7} secs. after the formation of the regular part.

Finally, I desire to express my cordial thanks to Professor P. O. Pedersen of the Royal Technical College of Copenhagen for his kindness in permitting me to use his laboratory for these experiments and in kindly guiding me in these investigations; and to Mr. J. P. Christensen of the same College for his friendly help in the experiments.

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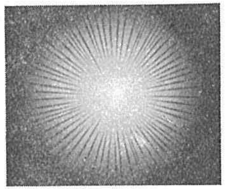
Plate I

p=760 mm.

R'=0, l=5 mm.

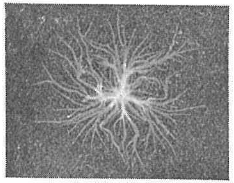
Fig. 5



p=760 mm., l=3 mm.

R'=31,500 ohms.

Fig. 4



R'=0

Fig. 3

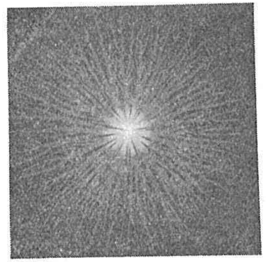
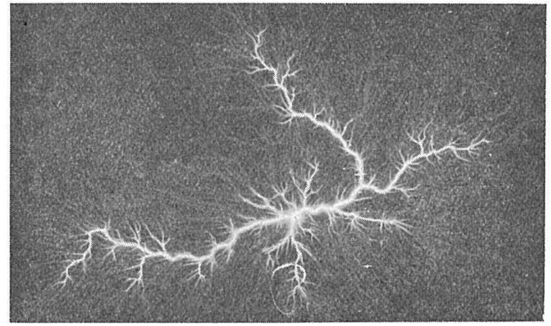


Fig. 1 p=760 mm., l=5 mm.
R'=31,500 ohms.



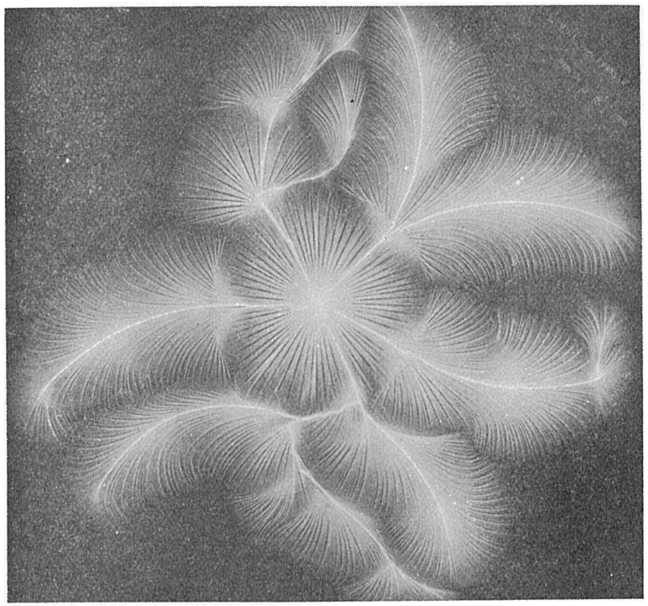


Fig. 6 p=760 mm., l=6.5 mm., R'=0

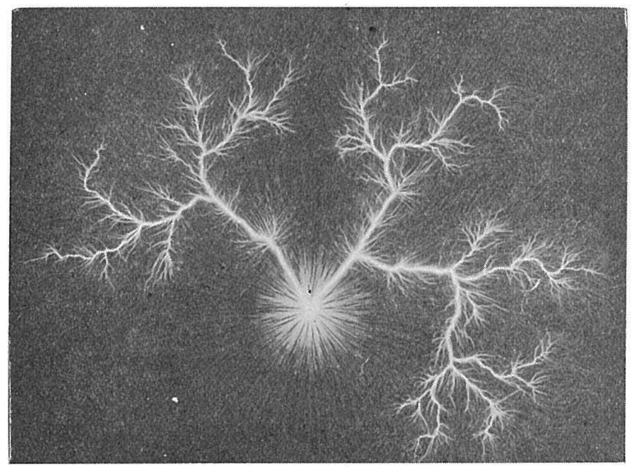
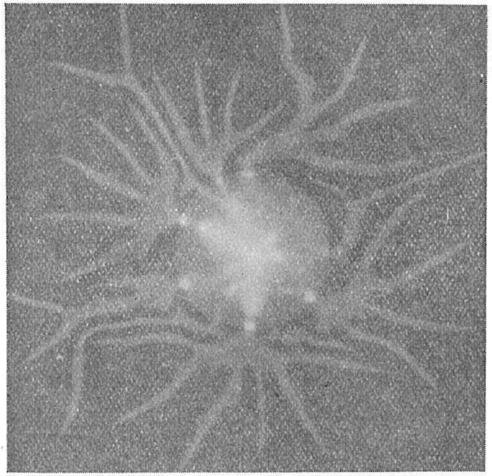


Fig. 2 p=760 mm., l=5 mm., R'=0

Fig. 4 $p=90$ mm. $l=1$ mm. $R=10$ meg. $L=22.10^{-5}$ Hen. $C=80$ cms. $C'=20$ cms.



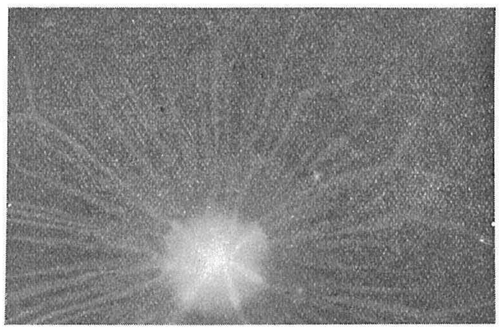


Fig. 5 $p=140$ mm. $l=1$ mm. $R=10$ meg. $L=22.10^{-5}$ Hen. $C=1600$ cms. $C'=83$ cms.

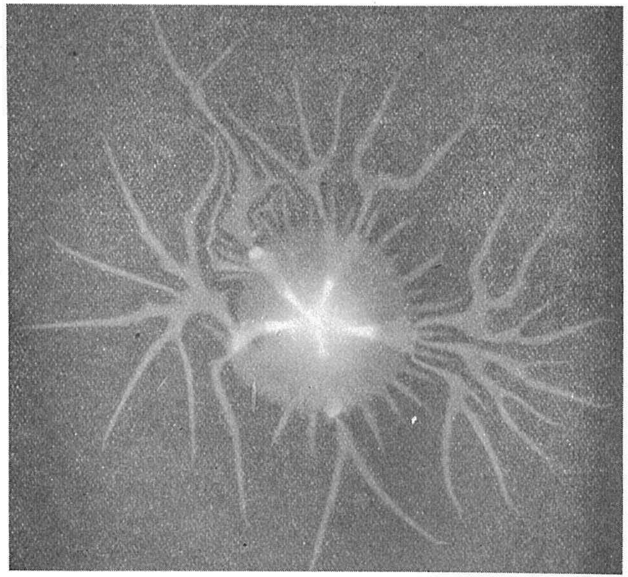
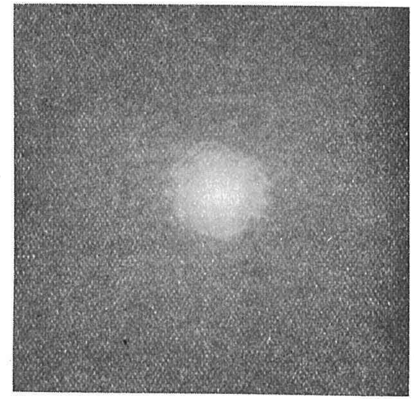


Fig. 3 $p=100$ mm. $l=1.4$ mm. $R'=167{,}500$ ohms.

Fig. 1 $p=170$ mm. $l=2$ mm. $R'=100{,}000$ ohms.



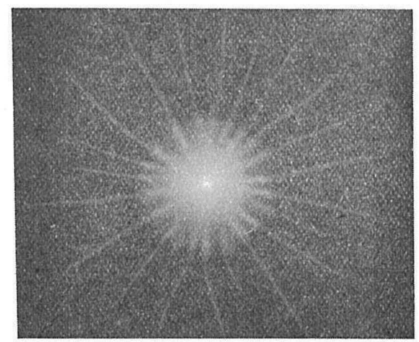


Fig. 2 $p=170$ mm. $l=1$ mm. $L=19.10^{-5}$ Hen.