On the Distribution of Bombs from a Volcanic Vent

By

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Abstract

Of the several central-cones of the world-widely known volcano Mt. Aso, Nakadake is the only one which is still active now as of old, and its fourth crater erupted many times during the years 1928–1930. On each occasion, one of the writers, Namba, examined carefully the distribution of bombs around the vent, together with various other matters and found that the area of distribution was elliptic in form and its center was extremely eccentric from the vent. The present paper aims at elucidating such peculiar distribution of bombs.

Judging from the circumstances, this seems due to the fact that the fourth crater of Aso has a cylindrical vent of remarkable depth and the pit in it usually deviates from the center of the vent's bottom.

Thus, making the depth of the vent h and the distance of the vent wall in an azimuth θ from the pit λ_θ, we calculated the maximum range, R_0, of bombs projected out with the initial velocity V on the atrio, and we obtained

$$R_0 = m\frac{h.\lambda_\theta(V^2-gh)}{g(h^2+\lambda^2_\theta)},$$

where

$$m = 1+\sqrt{1-\frac{g^2(h^2+\lambda^2_\theta)}{(V^2-gh)^2}}.$$

This equation represents generally an elliptic curve, and specially, if m is independent of azimuth θ, then it becomes an ellipse exactly.

Applying the formula to the real activity of Aso crater, we see that it proves sufficiently reliable.

I. Activity of Mount Aso during 1928—1929

§ 1. The Position and the Present State of the Recent Center of Activity of Aso.

In the group of central-cones of the great-Aso volcano, five typical cones called "Go-Gaku" in a lump have been very famous from ancient times. Their ancient names are Neko-dake, Taka-dake, Narao-dake, Ebosi-dake, and Kisima-dake, taking them in order from the east, and the whole view of them can be spectacularly commanded only from Miyazi, the chief town in this district. Narao-dake was also known as Naka-dake, and undoubtedly it included the active region of Aso which is named "Naka-dake" in the recent map produced by the Land-Survey Department, and numerous eruptions of it in historical times are recorded. It is also the only one which has maintained its volcanic activity down to the present. Naka-dake is a "somma type" volcano, and its ancient great crater is now enlarged to an elliptic form of nearly 2000^m × 1000^m in extent, and its atrio is developed mostly at the south part called "Suna-senri-ga-hama". Of the several recent craters on its central cone, the conspicuous ones are named the first, the second, the third, the fourth, the fifth, and the sixth (or New crater) in order from the north. They are arranged roughly in the direction NW—SE which is generally considered to indicate a structural weakness (Fig. 1).

Fig. 1.
Ancient and Recent Craters of Naka-dake.
(Craters 1, 2, 3, 4, 5 were named by Mr. Nôtomi. 6 is called also "New-Crater".)

Of these craters the fourth has been most energetic since 1923, and its encircling ring is roughly 400^m EW × 300^m NS in extent and is partly sunk in E, N, W and SW portions. The vent was slightly dislocated to the west from the center of crater. The whole crater is not an inverted cone but funnel-like (Fig. 2), owing to its frequent outbursts.

Numerical data of the vent measured on 20th Dec. 1928 and 8th Nov. 1929 are as follows :—

Vent's diameter ≒ 30^m (by a clinometer and tape measurement),
Depth of the bottom ≒ 160^m below the level of Senri-ga-hama (by a tape measurement),
Max. height of the somma ≒ 70^m above the level of Senri-ga-hama (by a hand level).

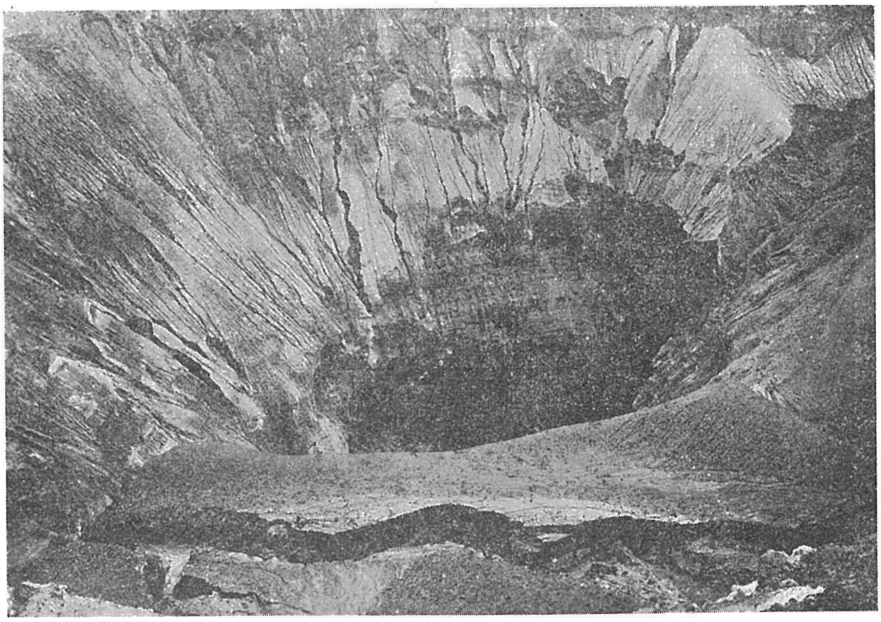


Fig. 2

The Fourth Crater and its Vent (July 1931).

On that occasion a pit filled with red-hot magma could be witnessed at the bottom of the vent, and also half-way down the vent-wall some old black magma was perceived to have adhered at a depth of about 90 meters below the level of Senri-ga-hama. The latter was inferred from its state and situation to be perhaps the residual head of the magma reservoir at the initial stage of recent activity. It seems indeed to the writers to be very probable that in the Aso crater also some rise and fall of the magma-head may take place at intervals.

The vent can be regarded as forming practically a vertical cylinder up to the level of Senri-ga-hama, though its upper rim has been slightly enlarged by collapsing. Since we are concerned only with the maximum range of projected bombs, it is scarcely necessary to take into account the effect of the somma, since through its sunken portion the ejecta are easily dispersed on the atrio.

§ 2. Sketch of the Eruptions of the Fourth Crater.

The dates and types of the eruptions of Mt. Aso from Sept. 1928 to Sept. 1929 are tabulated below.

Type I	Type II	Type III
Great eruption after a long repose	Activity after a short repose in rainy season	Activity after a short repose in dry season
1. 1928 Sept. 6–	1. 1929 May 9–	1. 1928 Dec. 19–
2. 1930 Sept. 5–	2. 1929 June 22–	2. 1929 Jan. 22–
	July 11–	Jan. 30–
	July 25–	Feb. 20–
	3. 1929 Aug. 24–	Mar. 6–
		„ 18–
		„ 29–
		3. 1929 Sept. 14–
		Oct. 6–
		„ 22–
		4. 1929 Nov. 4–

M. Namba observed all these eruptions, except that on Sept. 6, 1928, and the general aspect and characteristics of each type will be sketched below.

[I] *Great eruption after a long repose.*

Let us describe here the eruption on 5–10 Sept. 1930 as a model of the first type.

The fourth crater had ceased being active since 8th Nov. 1929, and in January 1930 a hot water pool filling the vent became visible. The bottom of the vent was constantly being buried by the ashes poured into, especially during the "Tsuyu" (most rainy season in Japan), till at last on 26th Augast 1930 the water head rose up to 60 meters below the level of Senri-ga-hama and the crater-wall was thoroughly eroded to the naked rocks by the rains. An active point existed at the west part of the vent pool. On the 29th the pool became so tumultuous that people began to fear an eruption, and on 2nd Sept. white smoke began to rise from the pool, accompanied by rumblings. At 17 o'clock 4th Sept., the white smoke abruptly began to blow out much more quickly and was gradually mingled with ash.

At last at about half past eleven o'clock on the 5th, the first great explosion took place. A great amount of so-called "Ku-sui" (苦水, hot water in the vent) was effused out in all directions in an extremely muddy state. Vast clouds of volcanic dust, "Yona" (霾) as it is called in this district, were thrown out into the sky, and bent to SW by the NE wind blowing at the time, and showered ashes on the ground in that direction. Thus at the hot spring sanatorium of Tochinoki roughly 9 kilometers SW of the crater, the ash accumulation was more than 100 grams per square meter in one day (18^h 5th–18^h 6th Sept.), and at the SW part of somma it was about 60 cms. thick on 8th Sept.

The bombs ejected during this eruption were distributed in an elliptical region whose extent was longest NNW—SSE and whose center was extremely deviated from the vent in the NE direction in spite of the NE wind as shown in Fig. 3.

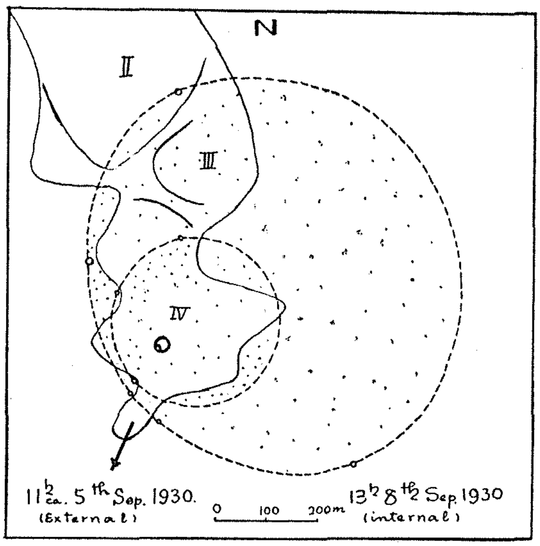


Fig. 3

Vent and Pit. Wind Direction (Smoke).
Remarkable Bombs.

Bombs weighing about 200 kgs. ejected in the north were carried even to the second crater.

As the bombs afterwards would often be buried out of sight by the accumulation of Yona, it must be borne in mind that as time passed the real distribution of the bombs might possibly be misjudged.

From the sun-set hour on the 7th, the percentage of fragments of plastic magma among the ejecta increased.

On 10th Sept. the outburst ceased, and only rumbling was heard from time to time—about once every 30 minutes. During several hours immediately after the outburst had ceased, the inside of the vent remained vacant and clear so that there was an excellent opportunity to observe the condition of the vent-bottom.

After that period the vent was filled with white smoke caused by water juveniel or vados, and then a hot water pool gradually re-appeared at the basin as time passed.

We shall add here two more facts which are very important for our present purpose.

After the eruption on 5th Sept., a mass of old ash and debris could be observed still left at the east side of the vent, showing that the outburst occurred at a point on the west wall-side and that it was to be called a "wall-side eruption". It was this indeed, together with the fact that in spite of the NE wind the distribution of ejected bombs was in an elliptical form as in the outer curve in Fig. 3, that caused us to write the present paper.

Moreover, since by the successive outbursts almost all the pre-existing old ash and debris in the vent were swept away and also an abundant amount of new magma was ejected out as bombs or other materials, the depth of the vent gradually increased and the distribution area of bombs at the later outburst became more and more circular as is shown by the inner curve in Fig. 3. This fact too greatly attracted our attention.

In short, a great eruption after a long repose advances as follows :—

1. In the time of repose, the vent bottom gradually ascends owing to both the pouring in of the old ash and debris from without, and the internal action of upheaval of the magmatic head. When the accumulation of old ash and debris has sufficiently advanced, liquid water begins to stay on it and gradually forms a mysterious pool of hot water.

2. At the earliest stage of activity the white smoke generally becomes much denser and the water of the pool boils and spouts up vigorously as described frequently in the history of Aso.

3. Meanwhile the eruption begins with thick white smoke mingled with ash and gradually sweeps out the old Yona and bebris in the vent by successive outbursts.

4. About the time when the old ash is swept away down to the magma dome, it develops into a most splendid black-smoke-eruption loaded with plastic lava fragments.

5. If the old ash has been entirely swept out and the eruption

has become a phenomenon of only the magmatic reservoir, then the terrible noise ceases and the fall of Yona diminishes but the proportion of plastic lava increases. The depth of the vent gradually increases owing to the successive outbursts.

6. After several days the eruption ceases, except for a few occasional rumblings.

7. During a few hours immediately after the end of the eruption, the vent is vacant and clear with no smoke, but it then becomes in a day filled up with white vapour of steam. The whole process just described is then repeated in the same order.

[II] *Activity after a short repose in rainy season.*

This type passes through a course of activity of type I but less vigorous. Owing to severe rain during the rainy season, a great quantity of old Yona and debris is poured into the vent and the floor of the basin ascends and water accumulates to form a pool. Though in this 2nd type of eruption the store of energy may not be very great, the emitted smoke is very dark due to its being densely laden with ash and rises up very high, often over 3000^{m}, because the season is usually calm so far as wind is concerned. Hence in this case also, there is a risk of the bombs being buried by plentiful ash and therefore of misconception as to the real distribution of them. Some variation in depth of the vent during the eruption could be expected empirically as well as theoretically, but it is not so distinct as in the case of type I.

[III] *Activity after a short repose in dry season.*

In the eruption of this type, the most of the ash is very newly made from the internal magma, and the total amount of Yona is necessarily comparatively small. Consequently the emitted smoke is lighter in colour, rather brown than black, but its baneful influence upon fields and life is surprisingly serious. When heavy smoke passes over the ground, vegetation on its path is frequently blighted. A good example is afforded by the withering of trees in the prefectural plantation on Ebosi-dake by Yona in March 1929. On the contrary, in the rainy "Tsuyu" season the baneful influence of Yona on forests is negligible, though sometimes it is appreciable on the crops.

An eruption of this type is most convinient for tracing the distribution of bombs, because the quantity of ash is comparatively small so that bombs are left bare on the ground.

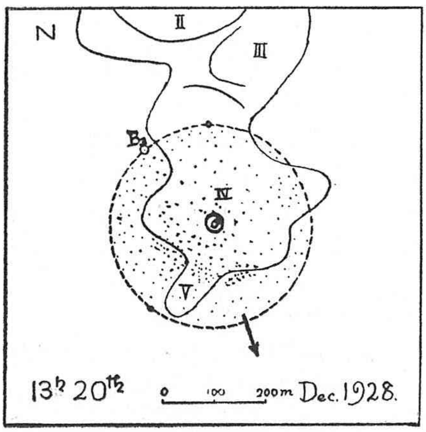
§ 3. Distribution of Bombs.

The observation of the distribution of bombs was executed as follows :—

During the eruptions, the observer only watches conspicuous ejecta and the positions where they fall are noted in a field-book. After the eruption has come to an end, their locations are verified on the ground and marked in a detailed map. In the case of the ejecta on the atrio, the positions of the most distant bombs in every direction are carefully examined with a tape and a compass. When the eruption was on such a small scale that all the bombs fell down into the crater, direct observation and ready mapping were possible.

Some examples of the distribution-area of bombs thus obtained are shown below [Fig. 4 to 8].

Fig. 4



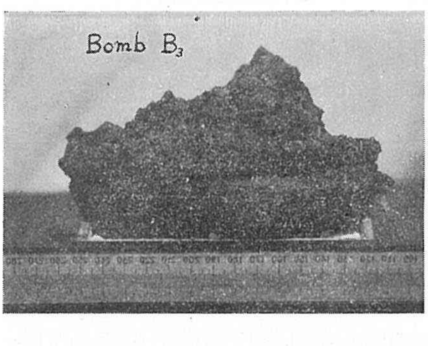
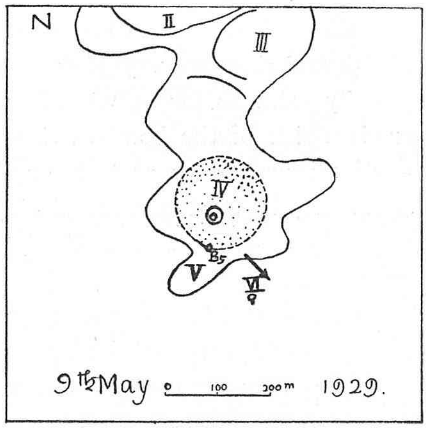


Fig. 6



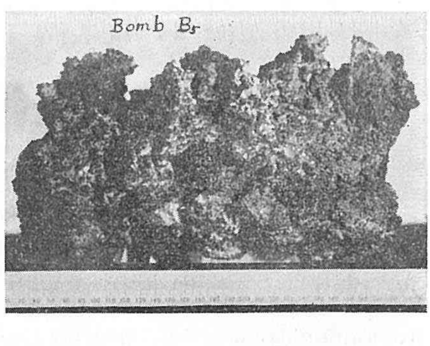


Fig. 8

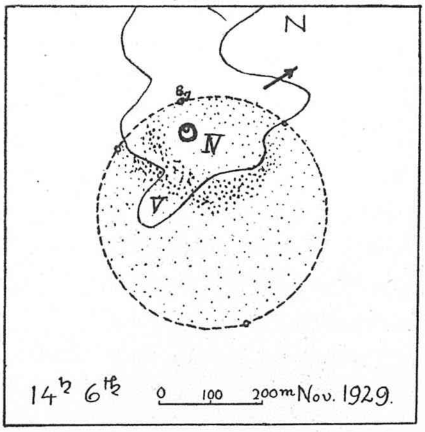
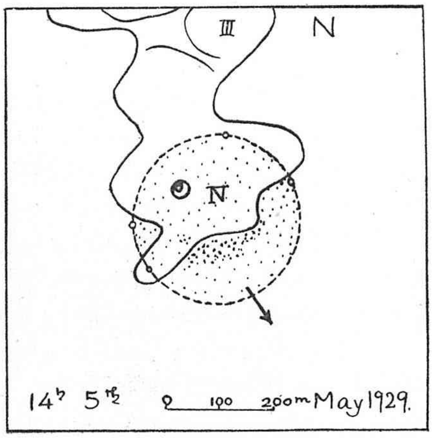


Fig. 5



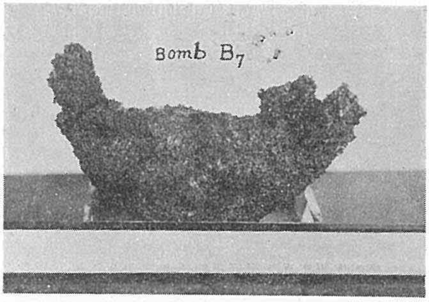
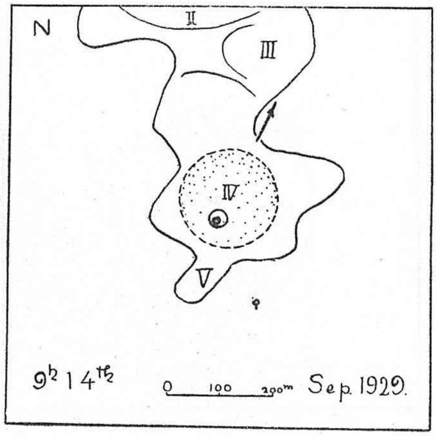


Fig. 7



The position of the pit could be inferred from the observation of the inside of the vent at the time when there was no smoke immediately after the end of the eruption, and also from many other states during the eruption, e. g., the column of smoke in the vent [Fig. 9].

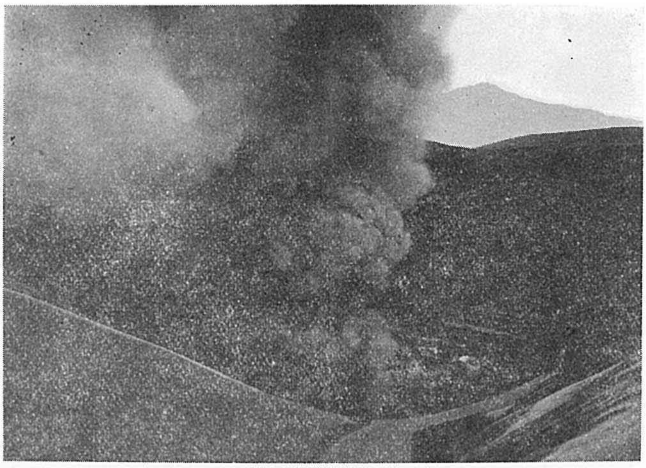
II. Law of Distribution of Bombs from a Cylindrical Vent.

§ 4. A glance at the preceding diagrams of bomb-distribution will show that although the bombs all come from the same vent, the distribution-areas of the ejected bombs are ramarkably unlike one another, and they are extremely eccentric and asymmetrical with reference to

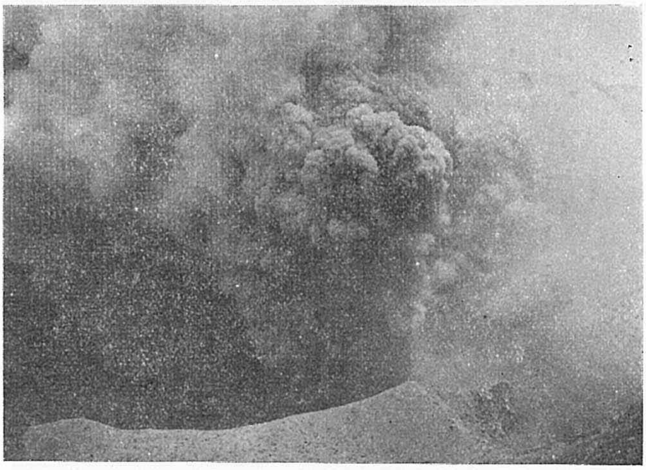
Fig. 9

Smoke revealing the pit's position deviating from the center of the vent-bottom.

(a) July 1929, from the NE side of the crater.



(b) 7th Sept. 1930, from the NW side of the crater.



the vent, and are often elliptical in form. What is the reason of this? It cannot be the effect of the wind. In the eruption on 5th Sept. 1930, in spite of the NE wind, the bombs were ejected eastward from the vent and the boundary of the distribution-area was such an ellipse as is shown in Fig. 3! Then could it be the effect of the direction of the conduit tube of the pit and consequently of the initial direction of the ejected bombs? This also is not probable. In the same eruption, the pit was obviously located near the west wall side and the mouth of the conduit tube itself was rather inclined to the west, as already described.

In our opinion, the distribution of bombs from the Aso crater seems to be chiefly affected by its tolerably deep cylindrical vent and by its pit, which generally deviates from the center of the vent-bottom. Thus if bombs are ejected with such projection-angles that they can hardly pass over the upper rim of the vent, they will reach the most distant points in each azimuth, and consequently they will limit the external boundary of the distribution-area.

From such a point of view, we will next investigate theoretically what distribution boundary must result on the atrio when bombs are ejected out with the same initial velocity from a point on the bottom of a cylindrical vent, but for the sake of simplicity, neglecting the atmospheric resistance.

§ 5. Maximum Range in one Azimuth attained by the Bombs from a Volcanic Vent.

Generally the path of a projectile in vacuo ejected with initial velocity V and projection angle α is given by

$$z = x.\tan\alpha - \frac{1}{2}\ \frac{g}{V^2}(1+\tan^2\alpha).\,x^2, \qquad (1)$$

where x and z are respectively the horizontal and the vertical distance from the point of projection.

The two values of x corresponding to $y=h$ are

$$x = \frac{V^2}{g}.\cos\alpha\left\{\sin\alpha \pm \sqrt{\sin^2\alpha - \frac{2gh}{V^2}}\right\}. \qquad (2)$$

Now let us consider bombs projected in one azimuth from a pit of a cylindrical vent. In Fig. 10 let O be the pit on the bottom of the vent and put

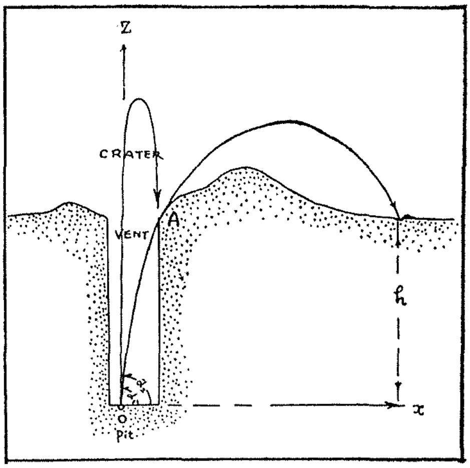
h = the depth of the vent,

λ = the distance of the vent-wall from the pit in the azimuth considered,

α = the projection angle by which the bomb can arrive on the atrio whose plane is assumed to be horizontal,

α_1, α_2 = the minimum and the maximum angle of projection by which the bomb can arrive at just the point A (λ, h), the rim of the vent.

Fig. 10



Then the possible value of α must be

$$\alpha_1 \leqq \alpha \leqq \alpha_2$$

and all the bombs projected at angles of projection without the above two limits α_1 and α_2 must return again into the vent.

Substituting $x=\lambda$ and $z=h$ in (1), we have

$$\tan\alpha = \frac{V^2}{g\lambda}\left\{1 \pm \sqrt{1 - \frac{2gh}{V^2} - \frac{g^2\lambda^2}{V^4}}\right\}. \qquad (3)$$

Hence

$$\tan\alpha_1 = \frac{V^2}{g\lambda}\left\{1 - \sqrt{1 - \frac{2gh}{V^2} - \frac{g^2\lambda^2}{V^4}}\right\} \qquad (4)$$

and

$$\tan\alpha_2 = \frac{V^2}{g\lambda}\left\{1 + \sqrt{1 - \frac{2gh}{V^2} - \frac{g^2\lambda^2}{V^4}}\right\}.$$

In order that the above equations should be possible in reality, the following relation must hold :—

$$V^2 \geqq g.h + g\sqrt{h^2+\lambda^2}. \qquad (5)$$

Especially when V is very great, it follows that

$$\alpha_2 \doteqdot 90^\circ, \quad \text{and} \quad \tan\alpha_1 \doteqdot \frac{h}{\lambda}.$$

Now at the Aso crater during 1928–1929

$$\lambda \doteqdot 30^{m}, \quad h \doteqdot 160^{m}, \quad \text{and} \quad \frac{h}{\lambda} > 1,$$

and consequently in our case α must be greater than 45°. But when projection angles are greater than 45°, the smaller the angle is, the greater the range becomes, so that the max. range x_m attained on the plane $z=h$ will be the value of x obtained from

$$h = x \tan \alpha_1 - \frac{1}{2} \frac{g}{V^2}(1 + \tan^2 \alpha_1) x^2$$

when eq. (4) is substituted in it.

i. e. $$x_m = \frac{h.\lambda}{g(h^2+\lambda^2)} \left\{ (V^2 - g.h) + \sqrt{(V^2 - g.h)^2 - g^2(h^2+\lambda^2)} \right\}. \quad (6)$$

And inversely the initial velocity corresponding to the known x_m is

$$V^2 = \frac{g}{2h\lambda x_m} \left\{ h^2(x_m + \lambda)^2 + \lambda^2 x_m^2 \right\} \quad (7)$$

which is very useful in the case of a bare atrio as that of Aso, since there is no other good means of estimating the initial velocity in such a case.

§ 6. The Form of the Distribution Area of Bombs from a Cylindrical Vent of any Form.

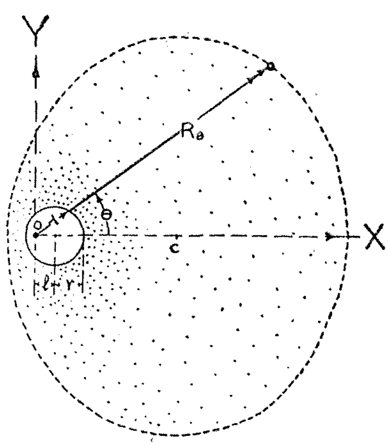
Now to derive the locus of maximum points in all azimuths, take the x-axis in the direction of the maximum value of λ, the distance of the wall-side from the pit, and the y-axis perpendicularly to it (Fig. 11). Let R_θ be the max. range in an azimuth-angle θ measured from the x-axis.

If the form of the horizontal section of the vent be

$$\lambda = f(\theta), \quad (8)$$

we substitute this in eq. (6), and get

Fig. 11



$$\left.\begin{aligned} R_0 &= m.\frac{h(V^2-g.h)f(\theta)}{g[h^2+f^2(\theta)]} \\ \text{where} \\ m &= 1+\sqrt{1-\frac{g^2[h^2+f^2(\theta)]}{(V^2-gh)^2}} \end{aligned}\right\}. \qquad (9)$$

This equation determines the form of the distribution-area of bombs.

The coefficient m is generally a function of θ, but in the following several important cases it is independent of θ and may be considered as a constant between 1 and 2.

1°). If V^2 is very great as compared with $g\sqrt{h^2+\lambda^2}$ and consequently the distribution area be very large, then $m \doteqdot 2$.

2°). If V^2 is very small and the distribution-area is so small that bombs can hardly pass over the vent rim, we have from eq. (5)

$$V^2 \doteqdot g.h + g\sqrt{h^2+\lambda^2}$$

or

$$(V^2-g.h)^2 \doteqdot g^2(h^2+\lambda^2)$$

and therefore $m \doteqdot 1$.

3°). If the depth of the vent is very great compared with its breadth, we can neglect λ in comparison with h so that

$$m = 1+\sqrt{1-\frac{g^2h^2}{(V^2-gh)^2}}$$

which is independent of θ.

4°). If the vent be a circular cylinder of radius r, and the pit situates at the center of the vent-bottom, then

$$m = 1+\sqrt{1-\frac{g^2(h^2+r^2)}{(V^2-gh)^2}}$$

which is also independent of θ.

§ 7. The Distribution Area of Bombs from a Circular Cylindrical Vent.

When the vent is a circular cylinder of radius r, and the pit is on the x-axis and distant $-l$ from the center of the vent-bottom, we have

$$\lambda = f(\theta) = l.\cos\theta + \sqrt{r^2 - l^2.\sin^2\theta}\ .$$

Substituting this in eq. (9),

$$\left.\begin{aligned} R_0 &= m.\frac{h(V^2-g.h)[l.\cos\theta+\sqrt{r^2-l^2.\sin^2\theta}]}{g\{h^2+[l.\cos\theta+\sqrt{r^2-l^2.\sin^2\theta}]^2\}} \\ \text{where} \\ m &= 1+\sqrt{1-g^2.\frac{h^2+[l.\cos\theta+\sqrt{r^2-l^2\sin^2\theta}]^2}{(V^2-g.h)^2}} \end{aligned}\right\}, \quad (10)$$

and also $1 \leqq m \leqq 2$.

In many important cases as shown in the preceding article, the coefficient m may be taken as independent of θ.

The above eq. (10) denotes generally an elliptical form, but if m is independent of θ, it becomes an exact ellipse as indicated below. Rationalizing the denominator in (10), we have

$$R_0 = M\frac{[h^2+r^2-l^2-2(r^2-l^2)].\, l\cos+(h^2+r^2-l^2)\sqrt{r^2-l^2+l^2.\cos^2\theta}}{(h^2+r^2-l^2)^2+4.h^2l^2.\cos^2\theta}$$

where

$$M = m.\frac{h(V^2-gh)}{g}. \quad (11)$$

Further, if we put

$$k = h^2+r^2-l^2,$$

then

$$R_0 = M.\frac{[k-2(r^2-l^2)].l.\cos\theta+k\sqrt{r^2-l^2+l^2.\cos^2\theta}}{k^2+4h^2l^2.\cos^2\theta}.$$

In order to transform this into rectangular coordinates, we write it as

$$\left\{R_0-\frac{M[k-2(r^2-l^2)].l.\cos\theta}{k^2+4h^2l^2\cos^2\theta}\right\}^2 = M^2\frac{k^2[(r^2-l^2)+l^2.\cos^2\theta]}{(k^2+4.h^2l^2.\cos^2\theta)^2}$$

or

$$R_0^2\{k^2.\sin^2\theta+(k^2+4h^2l^2)\cos^2\theta\} \\ -2M.[k-2(r^2-l^2)]lR_0\cos\theta-M^2(r^2-l^2)=0.$$

Putting here $x=R_0.\cos\theta$ and $y=R_0.\sin\theta$, we get

$$\frac{\left\{x-\dfrac{M[k-2(r^2-l^2)]l}{k^2+4h^2l^2}\right\}^2}{M^2\dfrac{(r^2-l^2)(k^2+4h^2l^2)+[k-2(r^2-l^2)]^2.l^2}{[k^2+4h^2l^2]^2}} + \frac{y^2}{M^2\dfrac{(r^2-l^2)(k^2+4h^2l^2)+[k^2-2(r^2-l^2)]^2l^2}{[k^2+4h^2l^2].k^2}} = 1.$$

i. e.,
$$\frac{(x-c)^2}{A^2}+\frac{y^2}{B^2}=1, \tag{12}$$

where

$$\left.\begin{aligned}
c&=M.\frac{[h^2-(r^2-l^2)]l}{[h^2+r^2-l^2]^2+4h^2l^2}\\
A&=M.\frac{\sqrt{(r^2-l^2)[(h^2+r^2-l^2)^2+4h^2l^2]+[h^2-(r^2-l^2)]^2l^2}}{(h^2+r^2-l^2)^2+4h^2l^2}\\
B&=M\frac{\sqrt{(r^2-l^2)[(h^2+r^2-l^2)^2+4h^2l^2]+[h^2-(r^2-l^2)]^2.l^2}}{(h^2+r^2-l^2)\sqrt{(h^2+r^2-l^2)^2+4h^2l^2}}\\
\frac{A}{B}&=\frac{h^2+r^2-l^2}{\sqrt{(h^2+r^2-l^2)^2+4h^2l^2}}<1
\end{aligned}\right\} \tag{13}$$

Thus, when m and consequently M is independent of θ, the area of distribution must obviously be an ellipse whose center is at (c, o), and whose major semi-axis is B, and minor A. We will call this ellipse the "Distribution Ellipse". If we denote its eccentricity by e, then

$$e=\frac{2.hl}{\sqrt{(h^2+r^2-l^2)^2+4.h^2l^2}}. \tag{14}$$

The distribution ellipse will be more eccentric as h decreases. The greater h is in comparison with r, the smaller the eccentricity becomes, and the ellipse gradually approaches a circle.

Morever at a given vent (h and r given), the eccentricity must depend only on l, the deviation of the pit from the center, and not on V, the initial velocity.

Special cases.

1). If $l=0$, that is in the case of a "central activity", eqs. (10), (12) and (13) give

$$\left.\begin{aligned}
R_0&=A=B=m.\frac{h(V^2-g.h).r}{g(h^2+r^2)}\\
c&=0
\end{aligned}\right\}. \tag{13'}$$

Therefore the distribution ellipse becomes a circle whose center coincides with that of the vent-bottom.

2). If $l=r$, that is in the case of a "wall-side activity", eq. (10) becomes

$$R_0 = M \frac{2r.\cos\theta}{h^2 + (2r)^2.\cos^2\theta}.$$

Transforming this into the rectangular coordinates,

$$\frac{(x-c)^2}{A^2} + \frac{y^2}{B^2} = 1,$$

where

$$\left.\begin{array}{ll} \text{minor radius} & A = m.\dfrac{h(V^2-gh)}{g} \cdot \dfrac{r}{h^2+4r^2} \\ \text{major radius} & B = m.\dfrac{V^2-gh}{g} \quad \dfrac{r}{\sqrt{h^2+4r^2}} \\ \text{center of the ellipse} & c = A \end{array}\right\}. \qquad (13'')$$

and

$$\left.\begin{array}{ll} \text{eccentricity} & e = \dfrac{2r}{\sqrt{h^2+4r^2}} \\ & B/A = \dfrac{\sqrt{h^2+4r^2}}{h} \end{array}\right\}. \qquad (14'')$$

Hence the distribution-area is an ellipse which touches the vent-rim above the pit and has the major axis in the y-direction.

At the Aso crater during 1928–1929, activity of this type happened very often; and especially in the great eruption after long repose, a definite change in the depth of vent-bottom was clearly perceived.

Let us consider such a case where the activity is of the "wall side type" and the depth of the vent differs at each outburst but its diameter does not change appreciably. At any two outbursts, if the depth of the vent and the eccentricity of the distribution-ellipse of bombs be h_1, e_1 and h_2, e_2 respectively, then from (14) the following relation between them must hold:—

$$\frac{h_1}{h_2} = \sqrt{\frac{\dfrac{1}{e_1^2} - 1}{\dfrac{1}{e_2^2} - 1}} \qquad (15)$$

This expression can conveniently be used to estimate the variation in depth of the vent by the observation of the distribution-ellipse only.

III. Application of our Formulae to the Activity of Mt. Aso.

§ 8. The Activity on Nov. 4—7, 1929.

As the eruption of the fourth crater of Mt. Aso on 4–7 Nov. 1929 occurred in the dry season and after a short repose, it was very favourable in all respects for our present study and we could obtain reliable data.

In the greatest outbursts at 14^h on 6th Nov. 1929, a bomb weighing about 8 kgms. fell on Senri-ga-hama, south atrio (Fig. 8). It was ascertained to have $x_m \doteqdot 400^m$ by tape-measurement after the eruption ceased.

On 8th Nov. the depth and the diameter of the vent were measured by the tape and found to be

$$h \doteqdot 160^m, \quad 2r \doteqdot 30^m.$$

Red hot magma in the pit was actually witnessed at the north wall-side of the bottom, and therefore this eruption could certainly be regarded as a " wall-side activity ". The distribution of bombs is shown in Fig. 8. In the figure we notice that the area of distribution lay entirely southward from the vent. This can be said indeed to harmonise well with the position of the pit stated just above.

If we take $x_m = 400^m$ as the maximum range, we get from (7)

$$V = 115 \ m/\text{sec.},$$

i. e., the maximum initial velocity during the last activity of 1929 was perhaps in the order of about 120 m/sec.

At any rate, V^2 on this day was very great, and the activity was perhaps a case where we can take $m = 2$.

During this eruption the minimum initial velocity by which the bombs could hardly fly over the upper rim of the vent would be

$$V \doteqdot 58 \ m/\text{sec.},$$

assuming $x_m \doteqdot 30^m$ and $h \doteqdot 160^m$ in eq. (7).

Finally in this wall-side activity, eq. (14″) gives

$$e = \frac{2r}{\sqrt{h^2 + (2r)^2}} \doteqdot \frac{1}{6}, \qquad \frac{B}{A} = \frac{\sqrt{h^2 + 4r^2}}{h} \doteqdot \frac{33}{32}.$$

These numbers show that the eccentricity must be so small that in a reduced diagram of the distribution-area it will be difficult to perceive whether the boundary is a circle or an ellipse, and it may only be determined on the real ground by the surrounding states or actual measurements. The sketch in Fig. 8 agrees very well with these statements.

§ 9. The Eruption on 5—10 September 1930.

This was the greatest eruption of those which the writer happened to witness, and is a suitable example which exhibited most clearly the distribution-ellipse of bombs. The distributions of bombs projected out by two outbursts on the 5th and the 8th are separately shown in Fig. 3, in which we find :—

Dates	Minor radius	Major radius
About 11^h, 5th Sept. 1930 (external)	$A_1 \doteqdot 360^m$	$B_1 \doteqdot 400^m$
About 13^h, 8th „ „ (internal)	$A_2 \doteqdot 160^m$	$B_2 \doteqdot 170^m$

As already stated, a mass of old ash and debris was still left at the east side of the vent bottom after the eruption on the 5th, and so the pit must have been located on the west wall-side, deviating a little to the south. Thus, this eruption also undoubtedly belongs to the wall-side type. Moreover the bottom of the vent on the 5th was obviously somewhat shallow, while on the 8th it was seen to be tolerably deep. These points agree with the fact that the distribution-area on the 5th was very eccentric while on the 8th it became very nearly a circle.

Applying eq. (15), we have

$$\frac{1}{c_1^2}-1=\frac{A_1^2}{B_1^2-A_1^2}=\frac{360^2}{400^2-360^2}$$

$$\frac{1}{c_2^2}-1=\frac{A_2^2}{B_2^2-A_2^2}=\frac{160^2}{170^2-160^2}$$

$$\frac{h_1}{h_2}=\sqrt{\frac{360^2}{400^2-360^2}\cdot\frac{170^2-160^2}{160^2}}\doteqdot\frac{51}{61}\doteqdot\frac{100}{120}$$

These figures show that the outburst occurred at first at rather the upper part of the vent, but the bottom descended day by day till at last at about 13 o'clock on the 8th it became 1.2 times as deep as on the 5th.