On the Frequencies of the Sound emitted by Japanese Hanging Bells

By

Keiji Yamashita and **Ichiro Aoki**

(Received August 3, 1932)

Abstract

From the experimental data, of which a part is here presented, we have found that the frequencies of the partial tones of a Japanese hanging bell are nearly in the ratios

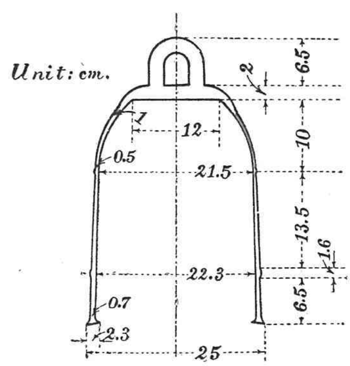
$$2^2 : 3^2 : 4^2 : 5^2 : \ldots\ldots,$$

and that the frequencies of the vibrations are inversely proportional to the linear dimensions of the bell.

The sound of a Japanese hanging fire-bell 13.5 kgms. in weight and with the dimensions shown in the figures were recorded by a Low-Hilger audiometer. Fig. 1 and Fig. 2 in the plate show the wave-forms

Fig. 1

Fig. 2



immediately after the bell has been struck by a wooden clapper, and Fig. 3 and Fig. 4 show the wave-forms 1 second and 2.5 seconds respectively after the striking. Fig. 1 and Fig. 2 were recorded by the audiometer without its horn, and Fig. 3 and Fig. 4 with it. By the method of periodogram-analysis the curves in Fig. 1, Fig. 2 and Fig. 3 were analyzed and the results are shown in the annexed diagrams.

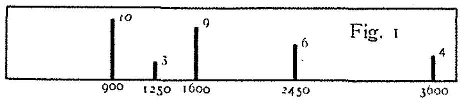


Fig. 1

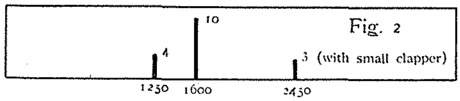


Fig. 2 (with small clapper)

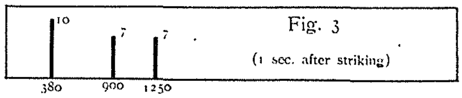


Fig. 3 (1 sec. after striking)

The periodogram-analysis shows that the frequencies of partial tones are

$$380,\ 900,\ 1250,\ 1600,\ 2450,\ 3600. \qquad \text{(I)}$$

If we divide these values, except 1250, by the numbers 2^2, 3^2, 4^2...... respectively, we get

$$380 \div 2^2 = 85,$$
$$900 \div 3^2 = 100,$$
$$1600 \div 4^2 = 100,$$
$$2450 \div 5^2 = 98,$$
$$3600 \div 6^2 = 100.$$

We see that these quotients are approximately equal to one another, and therefore this suggests that there may be a simple and regular relation in these frequencies.

To ascertain if such a relation really exisis in other Japanese hanging bells, we shall examine the frequencies of two bells already studied by one of the writers, namely :—

$$340,\ 800,\ 1100,\ 1400,\ 1800; \qquad \text{(II)}$$
$$129,\ 335,\ 550,\ 900,\ 1250. \qquad \text{(III)}$$

The frequencies (II) are of a fire-bell[1] 19 kgms. in weight and 28 cms. in diameter, and the frequencies (III) are of a hanging bell[2] " Ōjikichō " in Myōshinji-Temple, whose diameter is about 84.5 cms. From (II) and (III), we get

$$340 \div 2^2 = 85, \qquad 129 \div 2^2 = 32,$$
$$800 \div 3^2 = 89, \qquad 335 \div 3^2 = 37,$$

1. I. Aoki, Memoirs, Coll. of Sci. Kyoto, **14**, 213—218 (1931).
2. I. Aoki, loc. cit. **15**, 311—313 (1932).

$1400 \div 4^2 = 87,$ $500 \div 4^2 = 34,$

$900 \div 5^2 = 36,$

$1250 \div 6^2 = 35,$

from which we see that the same relation also holds approximately in the frequencies of these bells.

From these facts it seems most probable that the frequencies of the partial tones of Japanese hanging bells are in the ratios

$$2^2 : 3^2 : 4^2 : 5^2 : \ldots\ldots .$$

Thus the frequencies of the partial tones follow a regular sequence, but a partial tone in (I) with frequency 1250 and two partial tones in (II) with frequencies 1100 and 1800 are not included in that sequence. It is to be supposed that these partial tones are emitted by vibrations quite different from those which emit the partial tones following the regular sequence. They are perhaps due to the vibration with nodal circles while the partial tones whose frequencies are as $2^2 : 3^2 : 4^2 : \ldots\ldots$ are due to the vibrations with nodal meridians and without any nodal circle.

In the preceding paper[1] the frequencies of the vibrations of the Japanese hanging bell were calculated by one of the writers on the assumption that it is a circular cylinder with a hemispherical cap and that no line traced upon the middle surface of the shell undergoes extension when the bell is sounding. By that calculation the frequencies of the partial tones of a Japanese hanging bell, of which the Poisson's ratio is $1/3$ and the ratio of the length of the cylindrical part to its radius is $3/2$, were found to be in the ratios $359 : 981 : 1847 : \ldots\ldots$.

In order to compare these ratios with the experimental data, we divide the above values by 2^2, 3^2, 4^2 and get

$$359 \div 2^2 = 90,$$
$$981 \div 3^2 = 109,$$
$$1847 \div 4^2 = 115.$$

These quotients differ a little from one another, which is probably due to the assumption made in the theoretical investigations concerning the shape and thickness of the bell.

The diameters of the three bells (I), (II), and (III) are respectively

$$25,\ 28,\ 84.5.$$

1. K. Yamashita, Memoirs, Coll. of Sci. Kyoto, **15**, 315—322 (1932).

Multiplying these diameters by the frequencies of their respective first partial tones, we get

$$380 \times 25 = 9500,$$
$$340 \times 28 = 9520,$$
$$129 \times 84.5 = 10900.$$

These values are approximately equal to one another, and therefore we may say that the frequencies of vibrations are inversely proportional to the linear dimensions of the bell, which is a relation obtained in the theoretical investigations.

The writers wish to express their hearty thanks to Professor K. Tamaki, to whom they are indebted for his suggestions and advice.

Plate

Fig. 1

analyzed — $\frac{1}{50}$ sec.

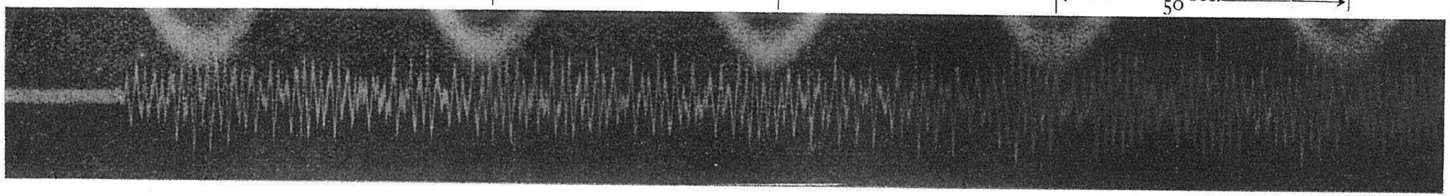


Fig. 2 (with small clapper)

analyzed

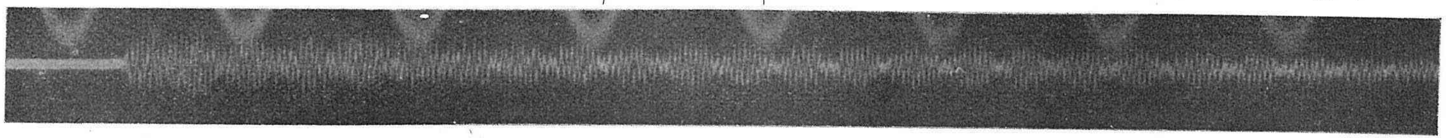


Fig. 3 (1 sec. after striking)

analyzed

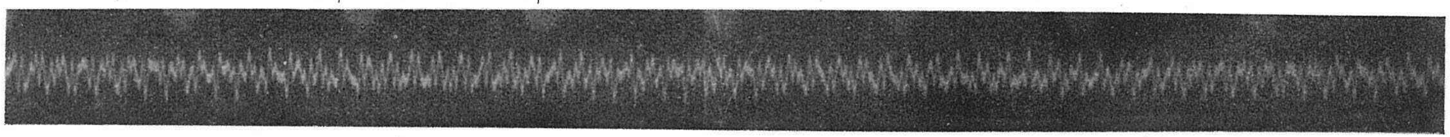
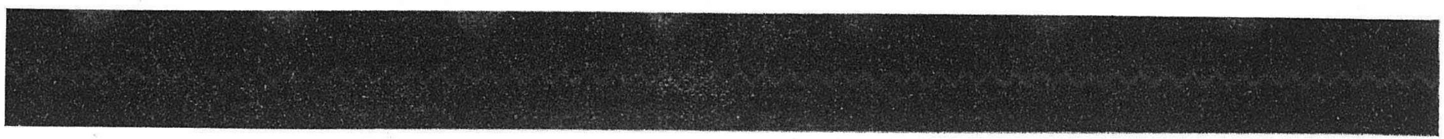


Fig. 4 (2.5 secs. after striking)



Keiji Yamashita and Ichiro Aoki