On the Density Current in the Ocean

I. The Case of No Bottom-Current

By

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Abstract

The writer defines a "density current" as such a current as would be produced by a local difference in density alone and not attended by any sloping of the sea-surface. He first finds the density-current in a uniform solenoidal field and shows that the current produced by the sudden generation of a local difference in density will attain its steady value in only a few hours, or days at most. But the density of the actual water in the real sea does not change sensibly during a few days and shows only the yearly variation, so that the density current may be considered as always having a steady value corresponding to the gradient of density from time to time.

Next the "convection current" composed of a density and a slope current influenced by land is discussed. The convection current of Ekman's type corresponds to the case of a long straight coast perpendicular to the density gradient. The writer corrects some errors in Ekman's result, and discusses several other cases of convection current.

1. Introduction

When the sea water locally differs in density, there must be produced a current, which is generally called the "convection current". A steady current corresponding to a constant and uniform gradient of density in one direction i. e., a current in what is called a uniform solenoidal field is solved by Ekman[1]; but in the present writer's opinion, the case of Ekman combines a density current with a slope current, and as such a composite current, it is merely a special case and not the only possible one. Moreover Ekman assumed that the current vanishes in the horizontal isobaric layer, but the present writer can not agree with this.

1. Ark. f. Mat. Astr. och Fys., Bd. 2, Nr. 11, 1905.

The writer proposes to define the "density current" as such as would be produced by a local difference in density alone, and not accompanied by any sloping of the sea surface, analogously to the "drift current" by wind. He will here first deal with such a density-current and then discuss the convection current of Ekman's and other types. The mathematical method used to solve the problem of the density current will be just as in his previous papers on the rising stage of drift current[2] and on the development of slope current[3], all the notations used being the same as before.

Let the water-density of a boundless sea differ linearly in the horizontal direction of y only, then the pressure-gradient in the water is

$$-\frac{\partial p}{\partial x}=0, \qquad -\frac{\partial p}{\partial y}=-g\frac{\partial \rho}{\partial y}z, \tag{1}$$

and the equation of motion becomes

$$\left.\begin{aligned}&\frac{\partial w}{\partial t}=\nu\frac{\partial^2 w}{\partial z^2}-i2\varpi w+ibz,\\ &\text{where}\\ &b=-\frac{g}{\rho}\frac{\partial \rho}{\partial y}.\end{aligned}\right\} \tag{2}$$

As the boundary and the initial conditions, let us at present assume the following:

$$\partial w/\partial z=0 \qquad \text{at} \quad z=0, \tag{3}$$

$$w=0 \qquad \text{at} \quad z=H, \tag{4}$$

and

$$w=0 \qquad \text{when} \quad t=0. \tag{5}$$

If ρ and consequently b do not vary with time, the current w may be separated into two parts such that

$$w=w_1-w_2, \tag{6}$$

where w_1 is a function of z only and satisfies

$$0=\nu\frac{d^2 w_1}{dz^2}-i2\varpi w_1+ibz, \tag{2a}$$

$$dw_1/dz=0 \qquad \text{at} \quad z=0, \tag{3a}$$

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1. These Memoirs, A, **16**, 161 (1933).
2. Ditto. **16**, 203 (1933).

$$w_1 = 0 \qquad \text{at } z = H, \tag{4a}$$

and w_2 is a function of both z and t, and

$$\frac{\partial w_2}{\partial t} = \nu \frac{\partial^2 w_2}{\partial z^2} - i2\varpi w_2, \tag{2b}$$

$$\partial w_2/\partial z = 0 \qquad \text{at } z = 0, \tag{3b}$$

$$w_2 = 0 \qquad \text{at } z = H, \tag{4b}$$

$$w_2 = w_1 \qquad \text{when } t = 0. \tag{5b}$$

2. The Steady Current w_1

For convenience, put

$$a = \frac{b}{\nu} = -\frac{g}{\mu}\frac{\partial \rho}{\partial y}, \qquad k = \sqrt{\frac{\varpi}{\nu}}, \tag{7}$$

then the general solution of (2_a) is obviously

$$w_1 = \frac{a}{4k^3}\left[Ke^{(1+i)kz} + K'e^{-(1+i)kz} + 2kz\right], \tag{8}$$

where K and K' are arbitrary constants.

The current $\dfrac{a}{2k^2}z = \dfrac{b}{2\varpi}z$ may be called the "pure density current", as it would be produced if the viscosity had no effect on the motion of water; and this part of a density-current just corresponds to the "gradient current" for a slope-current.

From the surface condition (3_a), we get

$$K - K' = \frac{-2}{1+i} = -(1-i).$$

Hence eq. (8) bebomes

$$w_1 = \frac{a}{4k^3}\left[2K\cosh(1+i)kz + (1-i)e^{-(1+i)kz} + 2kz\right], \tag{9}$$

and from the bottom condition (4_a)

$$-2K = \frac{(1-i)e^{-(1+i)kH} + 2kH}{\cosh(1+i)kH}. \tag{10}$$

Thus, separating w_1 in the x and the y direction, we obtain

$$\left.\begin{aligned} u_1 &= \frac{a}{4k^3}\Big[A\cosh kz\cos kz + B\sinh kz\sin kz \\ &\qquad + e^{-kz}(\cos kz - \sin kz) + 2kz\Big], \\ v_1 &= \frac{a}{4k^3}\Big[A\sinh kz\sin kz - B\cosh kz\cos kz \\ &\qquad - e^{-kz}(\cos kz + \sin kz)\Big], \end{aligned}\right\} \quad (11)$$

where

$$\left.\begin{aligned} -A &= 1 - \frac{\sinh 2kH + \sin 2kH}{\cosh 2kH + \cos 2kH} + \frac{4kH\cosh kH\cos kH}{\cosh 2kH + \cos 2kH}, \\ -B &= 1 - \frac{\sinh 2kH - \sin 2kH}{\cosh 2kH + \cos 2kH} + \frac{4kH\sinh kH\sin kH}{\cosh 2kH + \cos 2kH}. \end{aligned}\right\} \quad (12)$$

Specially, the surface value is

$$\left.\begin{aligned} \bar{u}_1 &= \frac{a}{4k^3}(A+1) \\ &= \frac{a}{4k^3}\cdot\frac{(\sinh 2kH + \sin 2kH) - 4kH\cosh kH\cos kH}{\cosh 2kH + \cos 2kH}, \\ \bar{v}_1 &= \frac{a}{4k^3}(-B-1) \\ &= \frac{a}{4k^3}\cdot\frac{4kH\sinh kH\sin kH - (\sinh 2kH - \sin 2kH)}{\cosh 2kH + \cos 2kH}. \end{aligned}\right\} \quad (13)$$

From the above equations (11), (12) and (13), Tables 1 and 2, and Figs. 1 and 2 were prepared, representing the vertical distribution of current for several seas and also a locus of the surface value according to the depth of the sea. In all of them except Fig. 1, $\frac{a}{4k^3}$ is taken as unity for convenience. In Fig. 1, in order to represent the results in one diagram, we took unit velocity as $\frac{a}{4k^3}\cdot\frac{H}{D}$, i. e., we plotted the values of Table 1 multiplied by D/H. Hence the unit in Fig. 1 is different for each curve, and the real current must be conceived as magnified in the proportion of H/D.

Table 1

Vertical distribution of density current (steady state).

	$H=D/4$		$H=D/2$		$H=D$		$H=1.25D$		$H=2D$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1	u_1	v_1	u_1	v_1
0	0.143	0.250	1.090	0.275	1.538	−0.996	+1.220	−1.218	0.953	−1.000
0.1	0.142	0.251	1.083	0.299	1.634	−0.864	1.405	−1.065	1.329	−0.763
0.2	0.137	0.251	1.062	0.361	1.897	−0.528	1.911	−0.713	2.303	−0.431
0.3	0.130	0.250	1.022	0.445	2.263	−0.067	2.635	−0.252	3.628	−0.242
0.4	0.118	0.243	0.960	0.532	2.641	+0.474	3.436	+0.298	5.152	−0.153
0.5	0.105	0.231	0.868	0.605	2.925	+1.031	4.142	+0.936	6.789	+0.043
0.6	0.089	0.210	0.755	0.642	3.007	+1.546	4.557	+1.633	8.367	+0.638
0.7	0.069	0.178	0.605	0.624	2.783	+1.905	4.470	+2.269	9.400	+1.845
0.8	0.047	0.135	0.418	0.529	2.195	+1.939	3.693	+2.570	8.947	+3.436
0.9	0.024	0.076	0.208	0.329	1.229	+1.409	2.144	+2.052	5.843	+3.964

Table 2

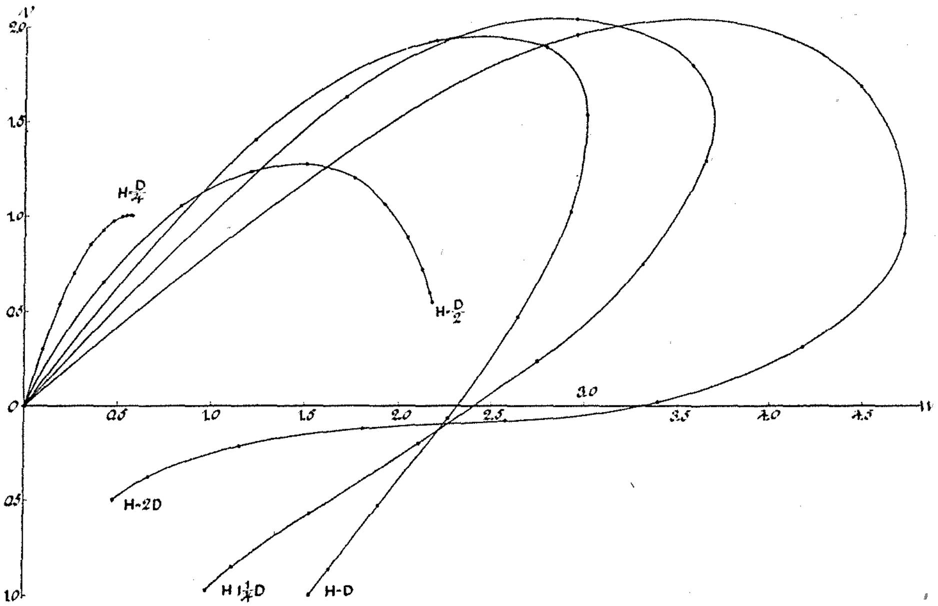
Surface values of the density current (steady state)

kH	$\bar{u}_1$	$\bar{v}_1$
0	0	0
0.1	0.0000	0.0008
0.2	0.0000	0.0054
0.3	0.0014	0.0212
0.4	0.0062	0.0419
0.5	0.0182	0.0796
0.6	0.0429	0.1311
0.7	0.0715	0.1943
0.8	0.1544	0.2600
0.9	0.2432	0.3219
1.0	0.3588	0.3698
1.1	0.4864	0.3992
1.2	0.6211	0.4071
1.3	0.7558	0.3942
1.4	0.8864	0.3627
1.5	1.0094	0.3127
1.6	1.1313	0.2559
1.7	1.2250	0.1868
1.8	1.3160	0.1096
1.9	1.3963	0.0258
2.0	1.4644	−0.0626
2.2	1.5690	−0.2467
2.4	1.6183	−0.4321
2.6	1.6475	−0.6112
2.8	1.6302	−0.7764
3.0	1.5842	−0.9126
3.5	1.3940	−1.1454
4.0	1.1920	−1.2208
4.5	1.0414	−1.1962
5.0	0.9620	−1.1300
6.0	0.9612	−1.0085
∞	1.0000	−1.0000

Fig. 1

Vertical distribution of density current in steady state.

$(u, v)\frac{D}{H}$ are plotted.



For later use, let us here calculate also the total flow from surface to bottom, namely

$$S=S_x+iS_y=\int_0^H w_1 dz.$$

Using the value of (9), we obtain

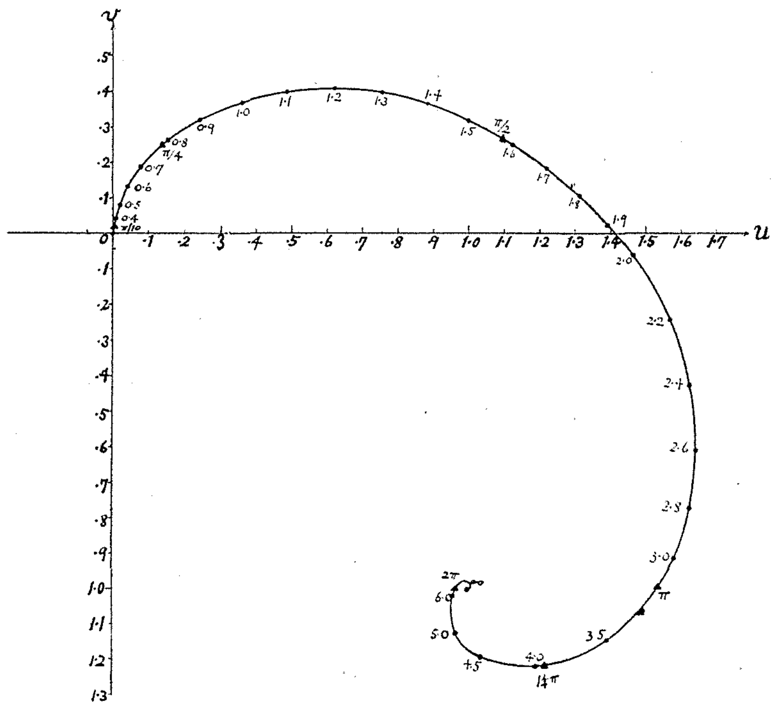
$$S=\frac{a}{4k^4}\left[(1-i)K\sinh(1+i)kH-i(1-e^{-(1+i)kH})+k^2H^2\right], \quad (14)$$

or

$$\left.\begin{aligned} S_x&=\frac{a}{4k^4}\cdot\frac{1}{\cosh 2kH+\cos 2kH}\times \\ &\quad\left[\begin{aligned}&2\sinh kH\sin kH+k^2H^2(\cosh 2kH+\cos 2kH)\\ &-kH(\sinh 2kH+\sin 2kH)\end{aligned}\right], \\ S_y&=\frac{a}{4k^4}\cdot\frac{1}{\cosh 2kH+\cos 2kH}\times \\ &\quad\left[\begin{aligned}&2\cosh kH\cos kH+kH(\sinh 2kH-\sin 2kH)\\ &-(\cosh 2kH+\cos 2kH)\end{aligned}\right]. \end{aligned}\right\} \quad (15)$$

Fig. 2

Surface values of density current.



3. Decaying of the Steady Current

When the local difference in density suddenly disappears, the mode of decaying of the steady current will be given by the current w_2'.

Now, all the equations (2_b), (3_b) and (4_b) are the same for similar parts of the drift current or of the slope current, so that we get as before

$$\left.\begin{aligned} & w_2' = \sum_{n=0}^{\infty} A_n \cos \beta_n z . e^{-(\nu\beta_n^2 + i2\omega)t} \\ \text{where} \quad & \beta_n = \left(n + \frac{1}{2}\right)\frac{\pi}{H}, \quad n = 0, 1, 2, 3 \ldots\ldots, \end{aligned}\right\} \qquad (16)$$

The coefficient A_n must be determined by the initial condition (5_b), i. e.,

$$\left.\begin{aligned} A_n &= \frac{2}{H}\int_0^H w_1.\cos\beta_n z.dz \\ &= \frac{a}{2k^4 H}\left[\frac{\{1+(-1)^n\beta_n H\}\left(\frac{\beta_n^2}{2k^2}-i\right)}{1+\left(\frac{\beta_n^2}{2k^2}\right)^2} - k^2H^2\right] \\ &= aA'_n\,(\text{say}). \end{aligned}\right\} \quad (17)$$

Therefore

$$w_2 = -\frac{a}{2k^3}\cdot\frac{D}{\pi H}(\cos 2\varpi t - i\sin 2\varpi t)$$
$$\times\sum_{n=0}^{\infty}\left[-\frac{\{1+(-1)^n\beta_n H\}(i-\beta_n^2/2k^2)}{1+(\beta_n^2/2k^2)^2}+k^2H^2\right]\cos\beta_n z.e^{-\nu\beta_n^2 t},$$

$$\left.\begin{aligned} u_2 &= -\frac{a}{2k^3}\cdot\frac{D}{\pi H}\Bigg\{\cos 2\varpi t\times \\ &\sum_{n=0}^{\infty}\left[k^2H^2-\frac{\left\{1+(-1)^n\left(n+\frac{1}{2}\right)\pi\right\}\frac{\beta_n^2}{2k^2}}{1+(\beta_n^2/2k^2)^2}\right]\cos\beta_n z.e^{-\nu\beta_n^2 t} \\ &+\sin 2\varpi t\sum_{n=0}^{\infty}\frac{1+(-1)^n\left(n+\frac{1}{2}\right)\pi}{1+(\beta_n^2/2k^2)^2}\cos\beta_n z.e^{-\nu\beta_n^2 t}\Bigg\}, \\ v_2 &= -\frac{a}{2k^3}\cdot\frac{D}{\pi H}\Bigg\{\cos 2\varpi t\times \\ &\sum_{n=0}^{\infty}\frac{1+(-1)^n\left(n+\frac{1}{2}\right)\pi}{1+(\beta_n^2/2k^2)^2}\cos\beta_n z.e^{-\nu\beta_n^2 t} \\ &+\sin 2\varpi t\sum_{n=0}^{\infty}\left[\frac{\left\{1+(-1)^n\left(n+\frac{1}{2}\right)\pi\right\}\frac{\beta_n^2}{2k^2}}{1+(\beta_n^2/2k^2)^2}-k^2H^2\right] \\ &\qquad\times\cos\beta_n z.e^{-\nu\beta_n^2 t}\Bigg\} \end{aligned}\right\} \quad (17')$$

Thus we see that in this case also the part w_2 oscillates with a period of 12 pendulum hours, and vanishes at most in a few days on account of the damping factor $e^{-\nu\beta_n^2 t}$.

4. Development of the Density Current

Using the values of w_1 and w_2 obtained above, we can calculate the current in an invariant solenoidal field

$$w = w_1 - w_2,$$

which may be written also

$$\begin{aligned} w &= \sum_{n=0}^{\infty} A_n \cos\beta_n z\left\{1 - e^{-(\nu\beta_n^2 + i2\varpi)t}\right\} \\ &= a\sum_{n=0}^{\infty} A_n' \cos\beta_n z(\nu\beta_n^2 + 2i\varpi)\int_0^t e^{-(\nu\beta_n^2 + i2\varpi)t} dt, \end{aligned} \tag{18}$$

where A_n or A_n' is given by eq. (17).

Thus we see that the current will develop in this case also, oscillating around the steady value with a period of 12 pendulum hours, and becoming almost steady in a few hours or days at most on account of the damping factor $e^{-\nu\beta_n^2 t}$.

If the density and consequently the quantity a vary with time as $a = a(t)$, we shall have

$$w = \sum_{n=0}^{\infty} A_n' \cos \beta_n z\int_0^t d\tau . a(\tau)e^{-(\nu\beta_n^2 + i2\varpi)(t-\tau)}. \tag{19}$$

We shall, however, not calculate numerically the current w in eq. (18), because such a current may never actually be seen. In a real sea, indeed, the density of the water shows a seasonal variation, but almost no daily variation, and it keeps nearly constant during a few days, so that the current may be considered as always steady practically, and eq. (19) will be reduced to eq. (9), only putting the quantity a as a function of t, i. e.,

$$w \doteqdot \frac{a(t)}{4k^3}\left[2K\cosh(1+i)kz + (1-i)e^{-(1+i)kz} + 2kz\right]. \tag{9'}$$

5. Density Current influenced by Land —— Convection Current

If a density current is checked by land, the water will accumulate somewhere and a surface slope and consequently a slope-current should be produced. The composite current made up of these two may be called a "convection current". In treating the convection current, we shall confine ourselves to the stationary state for reasons explained in the preceding paragraph.

1) *Convection current of Ekman's type.* If a long straight coast lays perpendicular to the gradient of water-density $\left(-\frac{\partial\rho}{\partial y}\right)$, or if the ocean were extended over the whole surface of the earth and the density of water varied according to latitude only, it would correspond to the case of convection current treated by Ekman[1].

Let a uniform gradient of density be in the negative direction of y, then a balancing slope will be generated in the positive direction of y, so that the horizontal isobaric layer will sink to a depth d from the surface.

Now, in a steady state, the condition of continuity requires that the total flow in the y-direction (perpendicular to the coast) must be zero, i.e.,

$$-S_y + S'_y = 0,$$

where we denote by S_y the total flow of the density-current and by S'_y that of the slope-current.

Substituting (15) for S_y and the corresponding formula for S'_y from the Ekman's paper in the above condition, we have[2]

$$0 = \frac{a}{4k^3}\Big[-2\cosh kH\cos kH - kH(\sinh 2kH - \sin 2kH) + (\cosh 2kH + \cos 2kH)\Big] + \frac{g\gamma_y}{4k\bar{\omega}}\Big[\sinh 2kH - \sin 2kH\Big],$$

or

$$\frac{H-d}{H} = \frac{\cosh 2kH + \cos 2kH - 2\cosh kH\cos kH}{kH(\sinh 2kH - \sin 2kH)}, \qquad (20)$$

where $$d = \frac{k^2 g\gamma_y}{a\bar{\omega}} = \frac{g\gamma_y}{b}, \quad \text{or} \quad \gamma_y = \frac{1}{\rho}\frac{\partial\rho}{\partial y}d. \qquad (21)$$

These formulae give the depth of the horizontal isobaric layer and the surface slope which should be produced to balance a given density gradient. Thus the current composed of the primary density-current and the secondary slope-current will be

1. loc. cit.

2. c. f. T. Nomitsu and T. Takegami, On the Convection Current and the Surface Level of a Two-layer Ocean. These Memoirs, A, **15**, 131 (1932).

$$w=\frac{a}{4k^3}\left[\frac{(1-i)e^{-(1+i)kH}+2kH}{\cosh(1+i)kH}\cosh(1+i)kz-(1-i)e^{-(1+i)kz}-2kz\right]$$

$$+\frac{g\gamma_y}{2\bar{\omega}}\left[1-\frac{\cosh(1+i)kz}{\cosh(1+i)kH}\right]$$

$$=\frac{a}{4k^3}\left[\frac{(1-i)e^{-(1+i)kH}+2k(H-d)}{\cosh(1+i)kH}\cosh(1+i)kz-(1-i)e^{-(1+i)kz}\right.$$

$$\left.+2k(d-z)\right], \tag{22}$$

or

$$\left.\begin{aligned} u&=\frac{a}{4k^3}\Big[C_1\cosh kz\cos kz+C_2\sinh kz\sin kz\\ &\qquad+e^{-kz}(\sin kz-\cos kz)+2k(d-z)\Big],\\ v&=\frac{a}{4k^3}\Big[C_1\sinh kz\sin kz-C_2\cosh kz\cos kz\\ &\qquad+e^{-kz}(\sin kz+\cos kz)\Big],\end{aligned}\right\} \tag{23}$$

where

$$\left.\begin{aligned} C_1&=1-\frac{\sinh 2kH+\sin 2kH}{\cosh 2kH+\cos 2kH}+\frac{4k(H-d)\cosh kH\cos kH}{\cosh 2kH+\cos 2kH},\\ C_2&=1-\frac{\sinh 2kH-\sin 2kH}{\cosh 2kH+\cos 2kH}-\frac{4kH(H-d)\sinh kH\sin kH}{\cosh 2kH+\cos 2kH}.\end{aligned}\right\} \tag{24}$$

Let us here compare these results with that of Ekman.

Since, in addition to the density gradient, there exists the surface slope represented by eq. (21), the pressure gradient in the sea is now

$$-\frac{\partial p}{\partial y}=g\gamma_y-bz=b(d-z), \tag{25}$$

and consequently the equation of motion becomes

$$0=\frac{d^2w}{dz^2}-2ik^2w+\frac{ib}{\mu}(d-z), \tag{26}$$

which is nothing but the equation used by Ekman.

Ekman's formulae for the convection current, however, differ a little from our formulae. The terms corresponding to the third terms of the right-hand side of eq. (24), indeed, are absent in the formulae of Ekman[1]. He constructed his formulae on the assumption that there is no current at the horizontal isobaric layer $z=d$, but our result shows that that assumption is erroneous and the current at the layer $z=d$ has the value as shown by the letter d in Fig. 3.

Moreover, Ekman assumed[2] $d=H/2$, which also is not correct, the true value of d being that given by our eq. (20) and Table 3.

Table 3.

Depth of the horizontal isobaric layer

H/D	1/10	1/4	1/2	1	$1\frac{1}{4}$	2	4	6	8	10	20	∞
d/H	3/8	0.378	0.417	0.653	0.742	0.841	0.920	0.947	0.960	0.968	0.984	1

Thus Ekman's diagram[3] for convection current can not be used rigorously, and must be replaced by our Table 4 or Fig. 3. Our diagram is, but Ekman's is not, in harmony with the current diagram in a two-layer ocean[4].

Table 4

Vertical distribution of convection current of Ekman type

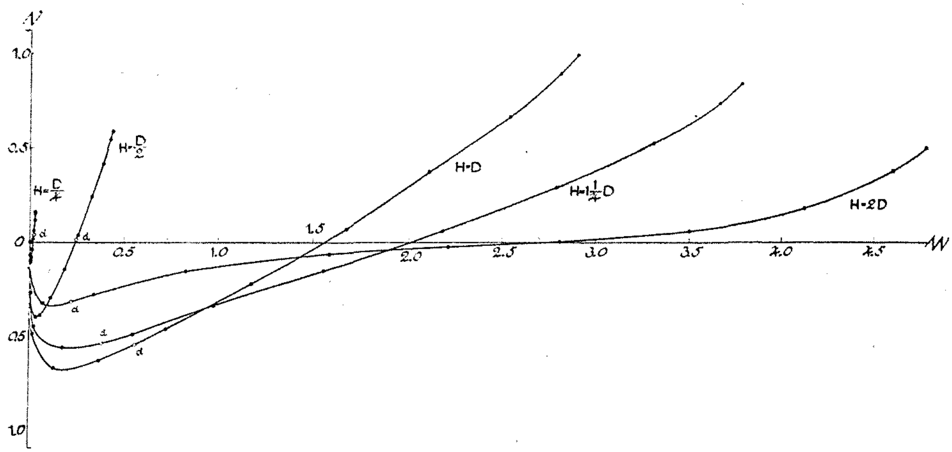
	$H=D/4$		$H=D/2$		$H=D$		$H=1.25D$		$H=2D$	
z/H	u	v	u	v	u	v	u	v	u	v
0	0.008	0.040	0.220	0.294	2.919	0.996	4.739	1.056	9.576	1
0.1	0.007	0.037	0.213	0.270	2.822	0.898	4.580	0.929	9.201	0.748
0.2	0.006	0.028	0.192	0.207	2.551	0.669	4.137	0.662	8.242	0.370
0.3	0.005	0.016	0.162	0.120	2.148	0.379	3.491	0.363	6.990	0.121
0.4	0.004	+0.003	0.126	+0.022	1.670	+0.073	2.733	+0.076	5.617	+0.010
0.5	0.003	−0.010	0.091	−0.072	1.178	−0.216	1.951	−0.188	4.329	−0.043
0.6	0.002	−0.021	0.054	−0.148	0.728	−0.462	1.225	−0.425	3.034	−0.132
0.7	0.001	−0.026	0.028	−0.194	0.368	−0.629	0.632	−0.615	1.668	−0.309
0.8	0.000	−0.027	0.015	−0.195	0.127	−0.662	0.222	−0.697	0.686	−0.555
0.9	0.000	−0.018	0.002	−0.135	0.018	−0.487	0.023	−0.556	0.132	−0.638

1. loc. cit. Eq. (20).
2. Krümmel, Handuch der Ozeanographie, Bd. II (1911), S. 504.
3. loc. cit. Fig. 8; c. f. Krümmel's Handbuch, II, Fig. 123; Defant, A., Dynamische Ozeanographie (1929), S. 127, Fig. 50.
4. These Memoirs, A, **15**, 139 (1932), or Krümmel's Handbuch, II, Fig. 143.

Fig. 3

Vertical distribution of the convection current of Ekman's type.

$(u, v)\frac{D}{H}$ are plotted.



2) *Convection current of other types.* As a convection current in a uniform solenoidal field, the case of Ekman is not the only possible one.

If a *long straight coast lays parallel to the y-direction,* the surface slope γ_x must arise in the direction of x, and the equation of motion will become

$$0=\frac{d^2w}{dz^2}-2ik^2w-iaz+g\frac{\sin\gamma_x}{\nu},$$

which is different from Ekman's equation.

The equation of continuity must be

$$-S_x+S_x'=0.$$

Using S_x of eq. (15) and the formula of S_x' for the slope current in Ekman's paper, we get the surface slope γ_x to satisfy the above conditions:

$$0=\frac{a}{4k^3}\Big[-2\sinh kH\sin kH-k^2H^2(\cosh 2kH+\cos 2kH)$$

$$+kH(\sinh 2kH+\sin 2kH)\Big]$$

$$+\frac{g}{4k\varpi}\Big[\sinh 2kH-\sin 2kH\Big],$$

$$\therefore \gamma_x = \frac{b}{g} \cdot \frac{1}{\sinh 2kH - \sin 2kH} \times$$

$$\left[2\sinh kH \sin kH + k^2H^2(\cosh 2kH + \cos 2kH) - kH(\sinh 2kH + \sin 2kH) \right].$$

The value of slope γ_x being known, the current composed of the density- and the slope-current can be easily calculated.

If *the coast makes an angle of* $45°$ *to the y-direction*, the equation of continuity must be

o = total flow perpendicular to the land,

from which we can determine the surface slope required to balance the local difference of density.

If *an enclosed sea* is in a steady state, the total flow must be zero in all directions. Hence, putting the total flows in the direction of x as well as y equal to zero, and as in the writer's previous paper[1] on the convection current in a two-layer ocean, we get the following relations:

$$\left. \begin{aligned} k(H-d) &= \frac{FQ - GP}{F^2 + G^2}, \\ \gamma_x &= -\frac{b}{g\rho k} \cdot \frac{FP + GQ}{F^2 + G^2}, \end{aligned} \right\} \qquad (27)$$

where

$$\left. \begin{aligned} F &= \sinh 2kH - \sin 2kH, \\ G &= \sinh 2kH + \sin 2kH - 2kH(\cosh 2kH + \cos 2kH), \\ P &= -2\sinh kH \sin kH + k^2H^2(\cosh 2kH + \cos 2kH), \\ Q &= \cosh 2kH + \cos 2kH - 2\cosh kH \cos kH. \end{aligned} \right\} \qquad (28)$$

Having determined the slope in both directions, we can calculate the composite current due to the density and the slope.

This current corresponds to

$$0 = \frac{d^2w}{dz^2} - 2ik^2w + ia(d-z) + \frac{g}{\nu}\sin\gamma_x,$$

which is also different from the equation of motion treated by Ekman.

For the convection current, the writer is at present contended only in making clear the nature of the current of Ekman type; and the numerical calculations for the other cases are not here carried out and will appear in another paper in which the writer is intending to study the effect of land on the ocean current of various kinds.

1. These Memoirs, A, **15**, 131 (1932).