On the Development of the Slope Current and the Barometric Current in the Ocean.

II. Different Bottom-Conditions Assumed.

By

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(Received July 10, 1933)

Abstract.

For the same reasons as those given in the writer's second and third reports upon the drift current in the ocean, the slope current is here treated again on two different assumptions with regard to the bottom condition: (1) that there is "no bottom-friction" and (2) that there acts a finite resistance depending on the bottom-velocity.

In the case of "no bottom-friction", the whole water from surface to bottom must have the same velocity and it coincides with the motion of the surface water of an infinitely deep ocean with "no bottom-current"

In the case of a "finite bottom-friction" due to the bottom-velocity, the mode of generation and distribution of the current will differ very much according to the magnitude of the surface slope and the viscosity of water. For a large slope and a small viscosity, the current approaches that seen in the case of "no bottom-current"; and for a small slope and a large viscosity it approaches that seen in the case of "no bottom-friction." Further, the current possesses a character similar in many other respects to that of the drift current found in the case of similar bottom-conditions, explained in the previous papers.

To put it briefly, the effect of a bottom-friction is to add to the current found in the case of "no bottom-friction" such a "no bottom-friction" drift current as would be produced by a wind traction equal to the given bottom-friction and acting there.

I. Introduction

One[1] of the writers has already studied the development of the slope current under the condition that no current exists at the sea bottom. But such a condition will not generally be quite appropriate

1. Nomitsu, These Memoirs, A, **16**, 203 (1933).

for the real sea, as stated in his other papers[1] on the drift current.

As in those papers, therefore, the writers will here treat the slope current again on two assumptions with regard to the bottom condition, (1) that no resistance acts there, and (2) that there acts a finite resistance depending on the bottom-velocity.

II. Case of "no bottom-friction"

1. The steady current w_1 under constant slope.

Using the same notations as before, we must solve the diff. equation

$$0=\nu\frac{d^2w_1}{dz^2}-2i\bar{\omega}w_1+ig\sin\gamma,$$

with the conditions

$$0=dw_1/dz \qquad \text{for} \quad z=0 \text{ and } z=H.$$

The general solution is obviously

$$\left.\begin{aligned} w_1&=\frac{g\sin\gamma}{2\bar{\omega}}+A\sinh az+B\cosh az, \\ \text{where} \quad a&=(1+i)k, \ k=\sqrt{\bar{\omega}/\nu}\,. \end{aligned}\right\} \tag{1}$$

But the surface condition requires $A=0$, and the bottom condition gives $B=0$, so that

$$w_1=\frac{g\sin\gamma}{2\omega\sin\lambda}=V_g, \tag{2}$$

where V_g means the so-called "gradient current".

Thus the steady current in this case is independent of z and H, so that the whole water in any sea from surface to bottom moves with the same velocity and in the direction perpendicular to the slope *cum sole*.

2. The development of current.

Let w denote the current to be produced in an initially motionless sea by a constant slope γ acting from $t=0$, then it can be written

$$w=w_1-w_2, \tag{3}$$

where w_2 is such that

1. Nomitsu, These Memoirs, A, **16**, 275 and 309 (1933).

$$\frac{\partial w_2}{\partial t}=\nu\frac{\partial^2 w_2}{\partial z^2}-i2\bar{\omega}w_2,$$

with the conditions

$$\partial w_2/\partial z=0 \qquad \text{for } z=0 \text{ and } z=H,$$

and $$w_2=w_1 \qquad \text{when} \quad t=0,$$

just as in the previous paper on the drift-current with " no bottom-friction ".

The solution which satisfies the equations up to the bottom condition, will be

$$w_2=\sum_{n=0}^{\infty}A_n \cos\beta_n z\ e^{-(\nu\beta_n^2+i2\bar{\omega})t}, \tag{4}$$

where $\beta_n=n\pi/H$, $\qquad n=0, 1, 2, 3,\ldots\ldots\ldots\ldots\ldots$

The initial condition will give

$$\sum_{n=0}^{\infty}A_n\cos\beta_n z=w_1=g\sin\gamma/2\bar{\omega}.$$

Determining the coefficients of this Fourier's cosine series, we have

$$A_n=0 \qquad \text{for all } n\neq 0,$$

and $$A_0=w_1=g\sin\gamma/2\bar{\omega} \qquad \text{for } n=0.$$

Thus we get

$$w_2=\frac{g\sin\gamma}{2\bar{\omega}}e^{-i2\bar{\omega}t}, \tag{5}$$

or

$$\left.\begin{aligned} u_2&=\frac{g\sin\gamma}{2\omega\sin\lambda}\cos 2\bar{\omega}t,\\ v_2&=\frac{g\sin\gamma}{2\omega\sin\lambda}\sin 2\bar{\omega}t,\end{aligned}\right\} \tag{6}$$

which indicates a circular motion.

Hence, finally the current in the rising state will be denoted by

$$w=w_1-w_2=\frac{g\sin\gamma}{2\bar{\omega}}\left(1-e^{-2i\bar{\omega}t}\right), \tag{7}$$

or

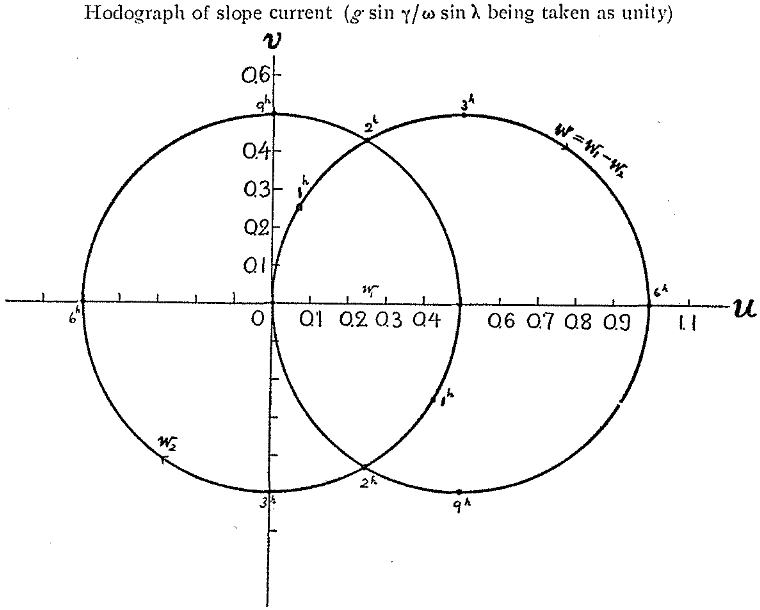
$$u=\frac{g\sin\gamma}{2\omega\sin\lambda}(1-\cos 2\bar{\omega}t),$$

$$v=\frac{g\sin\gamma}{2\omega\sin\lambda}\cdot\sin 2\bar{\omega}t, \quad \Bigg\} . \tag{8}$$

which denotes one and the same circular hodograph, Fig. 1, for the whole water from surface to bottom, and moreover it coincides with the hodograph for the surface layer of an infinitely deep sea in the case of "no bottom-current" obtained before.

Fig. 1.

Hodograph of slope current ($g\sin\gamma/\omega\sin\lambda$ being taken as unity)



If the slope γ is *variable* with time such that $\gamma=\gamma(t)$, then the current will be

$$w=ig\int_0^t d\tau\sin\gamma(\tau)e^{-i2\bar{\omega}(t-\tau)}. \tag{9}$$

III. Case of a finite bottom-friction

1. The steady current

(a) *If the bottom friction is proportional to the square of the bottom-velocity*, we must have

$$-\mu\frac{dw_1}{dz}=f\rho V_H^2 e^{i\theta} \qquad \text{for} \quad z=H, \tag{10}$$

where $\quad V_H=$the speed of the bottom current,

$\theta=$the angle between the bottom current and the x-axis,

f=the coefficient of resistance.

Then the integration constants of the general solution (1) become

$$A=0, \qquad B=-\frac{f\rho V_H^2 e^{i\theta}}{\mu a \sinh aH},$$

and therefore

$$w_1=V_g-\frac{f\rho V_H^2 e^{i\theta}}{\mu a \sinh aH}\cosh az. \tag{11}$$

The first term is equal to the current in the case of "no bottom-friction", and hence the second term may be considered as a correction term due to the bottom-friction and it is of course in the same form as the correction term for drift current in the same condition of the bottom.

Now in order to determine the unknown quantities V_H and θ, we use the obvious relation

$$w_1=V_H e^{i\theta} \qquad \text{for} \quad z=H,$$

i. e.,

$$V_H=V_g e^{-i\theta}-\frac{f\rho V_H^2 \cosh aH}{\mu a \sinh aH}, \tag{12}$$

or putting the real and the imaginay part separately

$$\left.\begin{aligned} V_H&=V_g\cos\theta-\frac{f\rho V_H^2(\sinh 2kH-\sin 2kH)}{2\mu k(\cosh 2kH-\cos 2kH)},\\ 0&=-V_g\sin\theta-\frac{f\rho V_H^2(\sinh 2kH+\sin 2kH)}{2\mu k(\cosh 2kH-\cos 2kH)}.\end{aligned}\right\} \tag{13}$$

Eliminating θ from (13), we have

$$V_g^2=\frac{f^2\rho^2(\sinh^2 2kH+\sin^2 2kH)}{2\mu^2k^2(\cosh 2kH-\cos 2kH)^2}V_H^4+\frac{f\rho(\sinh 2kH-\sin 2kH)}{\mu k(\cosh 2kH-\cos 2kH)}V_H^3 + V_H^2. \tag{14}$$

If we put

$$\left.\begin{aligned} \xi&=\sqrt{\frac{\mu k}{f\rho V_g}}, \quad \eta=V_H\sqrt{\frac{f\rho}{\mu k V_g}}=\frac{V_H}{\xi V_g},\\ a&=\frac{\cosh 4kH-\cos 4kH}{4(\cosh 2kH-\cos 2kH)^2},\\ b&=\frac{\sinh 2kH-\sin 2kH}{\cosh 2kH-\cos 2kH},\end{aligned}\right\} \tag{15}$$

the above equation becomes

$$a\eta^4+b\eta^3\xi+\eta^2\xi^2-1=0. \tag{16}$$

With this equation we can obtain the corresponding values of η for several values of ξ graphically, just as in the previous paper[1] on the drift current under the same bottom-condition. Thus we get Table 1 and the curves in Fig. 2.

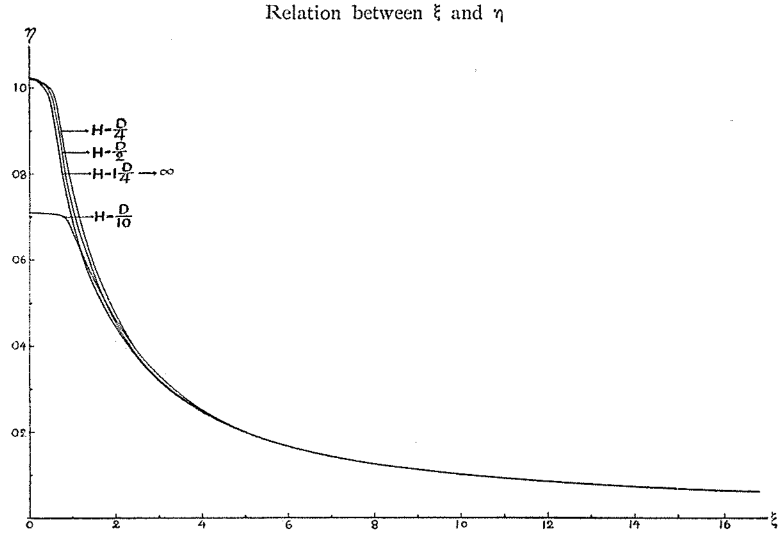
Table 1.

Relation between ξ and η.

$H=\frac{D}{10}$		$H=\frac{D}{4}$		$H=\frac{D}{2}$		$H=1\frac{1}{4}D$	
ξ	η	ξ	η	ξ	η	ξ	η
0	0.710	0	1.19	0	1.23	0	1.19
0.819	0.7	0.494	1.0	0.430	1.0	0.366	1.0
1.284	0.6	0.927	0.8	0.828	0.8	0.784	0.8
2.377	0.4	1.463	0.6	1.389	0.6	1.340	0.6
4.969	0.2	2.382	0.4	2.309	0.4	2.291	0.4
9.989	0.1	4.946	0.2	4.908	0.2	4.898	0.2
12.492	0.08	9.974	0.1	9.954	0.1	9.950	0.1
16.660	0.06	12.479	0.08	12.463	0.08	12.460	0.08
24.996	0.04	16.650	0.06	16.638	0.06	16.636	0.06
50.000	0.02	24.990	0.04	24.982	0.04	25.000	0.04
100.000	0.01	50.000	0.02	50.000	0.02	50.000	0.02

Fig. 2.

Relation between ξ and η



1. These Memoirs, A, **16**, 297 (1933).

Now from (13) we get

$$\left.\begin{aligned}\theta &= \tan^{-1}\frac{\varepsilon(\sinh 2kH+\sin 2kH)}{2(\cosh 2kH-\cos 2kH)+\varepsilon(\sinh 2kH-\sin 2kH)},\\ \text{where}\quad \varepsilon &= \eta/\xi = f\rho V_H/\mu k.\end{aligned}\right\}\quad (17)$$

Using this equation we can determine the direction of the bottom current. Table 2 and Fig. 3 show the variation of θ for various values of ε and consequently of ξ, in a sea of depth D/10, D/4, D/2, 1.25D as usual.

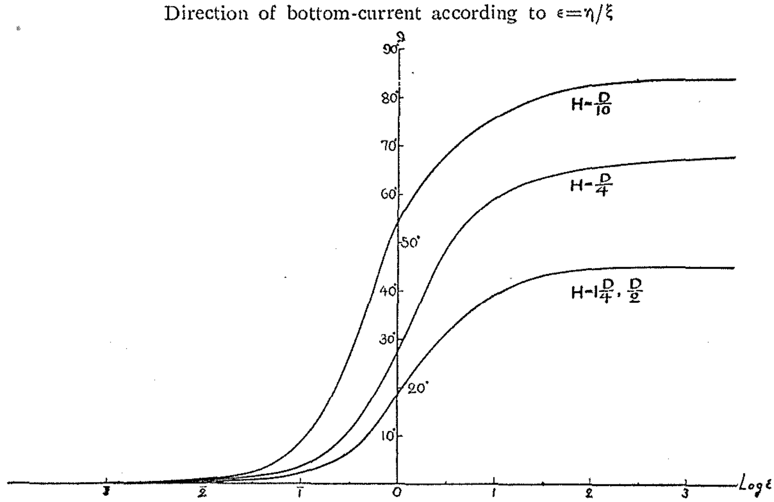
Table 2.

Direction of bottom-current according to $\varepsilon=\eta/\xi$.

$H=\frac{D}{10}$		$H=\frac{D}{4}$		$H=\frac{D}{2}$		$H=1\frac{1}{4}D$	
$\varepsilon=\eta/\xi$	θ	$\varepsilon=\eta/\xi$	θ	$\varepsilon=\eta/\xi$	θ	$\varepsilon=\eta/\xi$	θ
0.855	51.3°	2.024	41.2°	2.326	27.3°	2.732	30.0°
0.467	35.3°	0.863	24.9°	0.966	17.2°	1.020	18.7°
0.168	14.7°	0.410	13.7°	0.432	9.3°	0.448	10.3°
0.040	3.7°	0.168	6.0°	0.173	4.2°	0.175	4.7°
0.010	0.9°	0.040	1.5°	0.041	1.0°	0.041	1.2°
0.006	0.6°	0.010	0.5°	0.010	0.3°	0.010	0.3°
0.004	0.3°	0.006	0.2°	0.006	0.2°	0.006	0.2°
0.002	0.2°	0.004	0.0°	0.004	0.2°	0.004	0.2°
0.0004	0.0°	0.001	0.0°	0.002	0.1°	0.002	0.1°
0.0001	0.0°	0.0004	0.0°	0.0004	0.0°	0.0004	0.1°

Fig. 3.

Direction of bottom-current according to $\varepsilon=\eta/\xi$



With these values of θ and η we can calculate definitely the slope current, (11), or in the x and the y direction separately

$$\left.\begin{aligned} u_1 &= \frac{g \sin \gamma}{2\omega \sin \lambda}(1 - E \cosh kz \cos kz - F \sinh kz \sin kz), \\ v_1 &= -\frac{g \sin \gamma}{2\omega \sin \lambda}(E \sinh kz \sin kz - F \cosh kz \cos kz), \end{aligned}\right\} \quad (18)$$

where

$$\left.\begin{aligned} E &= \frac{\eta^2}{\cosh 2kH - \cos 2kH}\Big[(\cosh kH \sin kH + \sinh kH \cos kH)\sin\theta \\ &\qquad - (\cosh kH \sin kH - \sinh kH \cos kH)\cos\theta\Big], \\ F &= \frac{\eta^2}{\cosh 2kH - \cos 2kH}\Big[(\cosh kH \sin kH - \sinh kH \cos kH)\sin\theta \\ &\qquad + (\cosh kH \sin kH + \sinh kH \cos kH)\cos\theta\Big]. \end{aligned}\right\} \quad (19)$$

Using these formulae we constructed Table 3 and the curves in Figs. 4, 5, 6, 7, the coefficient $g \sin \gamma / \omega \sin \lambda$ being assumed as unity. It must be here noted that this unity chosen is different for different values of slope γ i. e., for different values of ξ.

Glancing at the results we see that:

1) At a given latitude, the smaller the value of ξ (or the larger the surface-slope γ and the smaller the viscosity μ), the greater is the ratio of bottom velocity V_H to ξV_g; but this ratio can not exceed a limited value even if the slope is very large and the viscosity becomes nil.

2) In this case the ratio $V_H/\xi V_g$ is nearly the same whatever the depth of the sea may be, except in the case of the small value of ξ in very shallow water.

3) The angle, which the direction of the bottom-current makes with the x-axis, is in the same quadrant for any depth and any value of ξ, and becomes nil for very large values of ξ.

4) The mode of the vertical distribution of the current is intermediate between those in the case of "no bottom-current" and of "no bottom-friction".

5) The mode of the distribution approaches that seen in the case of "no bottom-current" for the smaller values of ξ, i. e., the larger values of γ and the smaller values of μ, and that seen in the case of

"no bottom-friction" for the larger values of ξ i. e., the smaller values of γ and the larger values of μ.

Table 3.

Vertical distribution of steady current when bottom-friction ∞ (bottom-velocity)^2

	$H=\frac{D}{10}$							
	$\xi=0.819$		$\xi=1.284$		$\xi=2.377$		$\xi=4.969$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1	u_1	v_1
0	0.204	0.254	0.343	0.239	0.472	0.124	0.499	0.032
0.1	0.204	0.253	0.342	0.239	0.472	0.124	0.499	0.032
0.2	0.203	0.253	0.342	0.239	0.472	0.124	0.499	0.032
0.3	0.202	0.251	0.341	0.237	0.471	0.124	0.499	0.032
0.4	0.200	0.249	0.339	0.237	0.470	0.124	0.499	0.032
0.5	0.198	0.246	0.337	0.235	0.469	0.124	0.498	0.032
0.6	0.195	0.243	0.334	0.234	0.468	0.123	0.498	0.032
0.7	0.193	0.240	0.332	0.232	0.467	0.123	0.498	0.032
0.8	0.189	0.235	0.328	0.229	0.464	0.122	0.497	0.032
0.9	0.184	0.230	0.324	0.227	0.462	0.121	0.497	0.032
1.0	0.180	0.224	0.320	0.223	0.460	0.121	0.496	0.032

Table 3.

continued

	$H=\frac{D}{4}$					
	$\xi=0.494$		$\xi=0.927$		$\xi=1.46$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.345	0.275	0.454	0.197	0.495	0.107
0.1	0.343	0.274	0.453	0.197	0.495	0.107
0.2	0.338	0.269	0.449	0.196	0.493	0.107
0.3	0.329	0.266	0.443	0.195	0.489	0.107
0.4	0.318	0.258	0.435	0.193	0.485	0.107
0.5	0.303	0.249	0.425	0.190	0.479	0.106
0.6	0.289	0.238	0.412	0.186	0.472	0.106
0.7	0.265	0.224	0.396	0.180	0.463	0.105
0.8	0.242	0.207	0.379	0.175	0.458	0.102
0.9	0.215	0.186	0.358	0.166	0.447	0.100
1.0	0.186	0.162	0.338	0.157	0.430	0.098

Table 3.

continued

	$H=\frac{D}{2}$					
	$\xi=0.430$		$\xi=0.828$		$\xi=1.389$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.543	0.134	0.542	0.080	0.530	0.041
0.1	0.540	0.135	0.540	0.081	0.529	0.042
0.2	0.530	0.139	0.534	0.084	0.525	0.044
0.3	0.516	0.143	0.524	0.088	0.520	0.047
0.4	0.489	0.149	0.510	0.095	0.513	0.052
0.5	0.458	0.153	0.490	0.101	0.502	0.057
0.6	0.419	0.155	0.466	0.107	0.489	0.062
0.7	0.373	0.153	0.437	0.111	0.474	0.066
0.8	0.318	0.145	0.402	0.112	0.454	0.070
0.9	0.259	0.127	0.362	0.107	0.431	0.071
1.0	0.191	0.099	0.316	0.097	0.406	0.068

continued

	$H=1\frac{1}{4}D$					
	$\xi=0.366$		$\xi=0.784$		$\xi=1.340$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.507	−0.012	0.503	−0.009	0.501	−0.005
0.1	0.509	−0.011	0.505	−0.008	0.502	−0.005
0.2	0.514	−0.008	0.509	−0.007	0.504	−0.004
0.3	0.522	0.002	0.514	−0.002	0.508	−0.003
0.4	0.528	0.016	0.520	0.007	0.512	0.002
0.5	0.530	0.039	0.524	0.017	0.515	0.010
0.6	0.518	0.071	0.520	0.044	0.515	0.023
0.7	0.483	0.107	0.503	0.071	0.509	0.039
0.8	0.419	0.139	0.466	0.099	0.489	0.058
0.9	0.310	0.144	0.405	0.115	0.453	0.073
1.0	0.159	0.090	0.295	0.101	0.394	0.074

Fig 4.

Vertical distribution of steady current
when bottom-friction ∝ (bottom-velocity)²

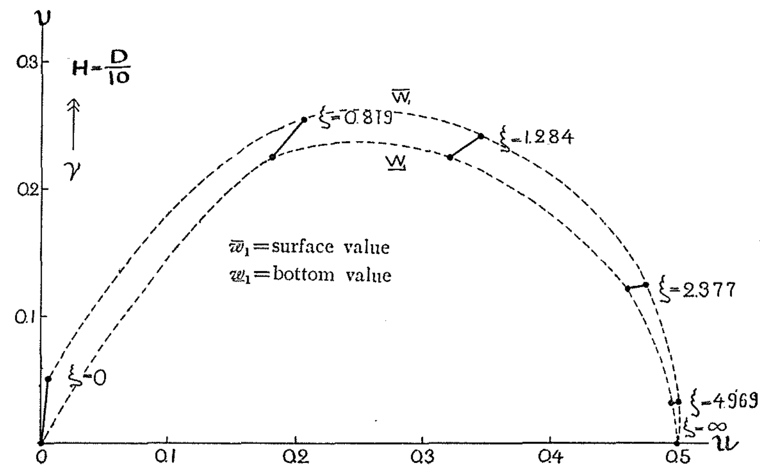


Fig. 5.

Vertical distribution of steady current
when bottom-friction ∝ (bottom-velocity)²

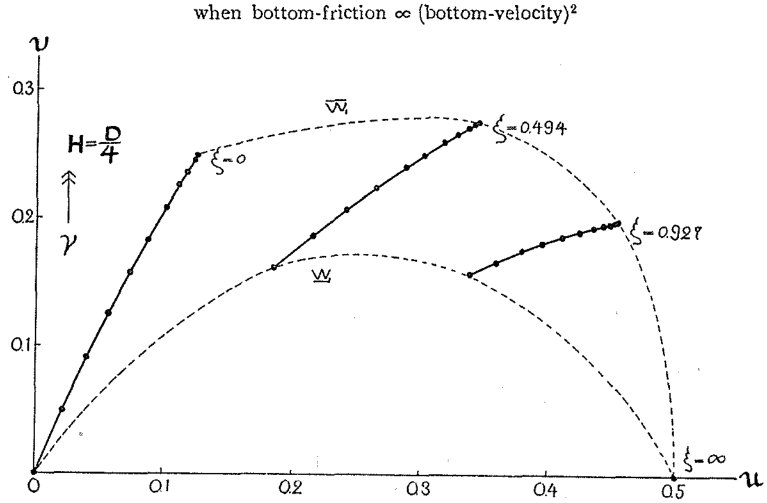


Fig. 6.

Vertical distribution of steady current when bottom-friction ∞ (bottom-velocity)²

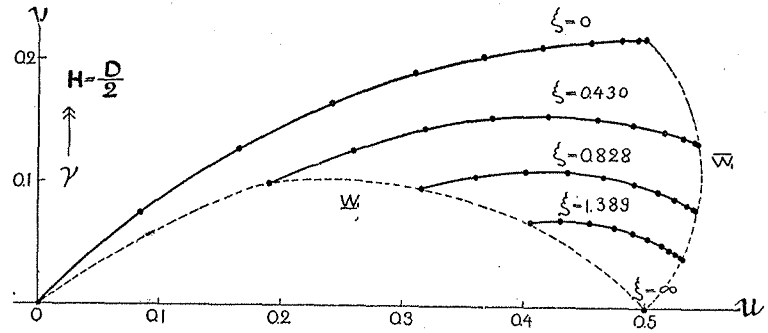
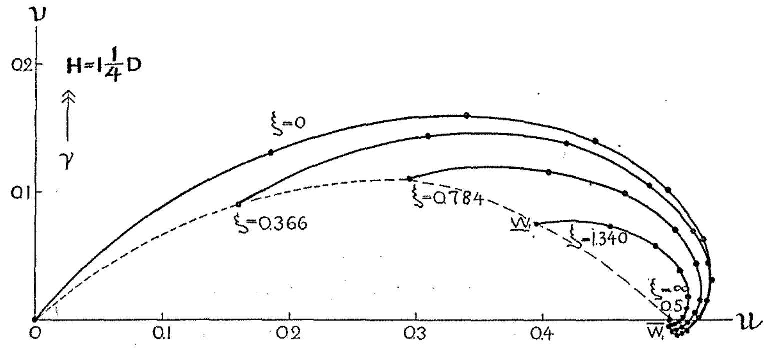


Fig. 7.

Vertical distribution of steady current when bottom-friction ∞ (bottom-velocity)²



(b) *When the bottom-friction is proportional to the bottom-velocity*, we must put

$$-\mu\frac{dw_1}{dz}=f'\rho w_1=f'\rho V_H e^{i\theta}\qquad \text{at}\qquad z=H. \tag{20}$$

Then the current will be

$$w_1=V_g-\frac{f'\rho V_H e^{i\theta}}{\mu a\sinh aH}\cosh az. \tag{21}$$

To determine the bottom-velocity, we have from equations similar to (13)

$$V_g^2=V_H^2\left[1+\frac{f'\rho}{\mu k}b+\frac{f'^2\rho^2}{\mu^2k^2}a\right],$$

or

$$\left.\begin{aligned} \eta' &\doteq \frac{V_H}{\xi' V_g} = \frac{1}{\sqrt{a+b\xi'+\xi'^2}}, \\ \text{where} \quad \xi' &= \frac{\mu k}{f'\rho}, \end{aligned}\right\} \tag{22}$$

and a, b are the same quantities as given in eq. (15).

For the direction of the bottom-velocity,

$$\left.\begin{aligned} \tan\theta &= \frac{c}{2\xi'+b}, \\ \text{where} \quad c &= \frac{\sinh 2kH + \sin 2kH}{\cosh 2kH - \cos 2kH}. \end{aligned}\right\} \tag{23}$$

Introducing the above bottom-current into (21) we can calculate at once the current numerically for any depth z. Table 4 and curves between two lines $\overline{w}_1$ and w_1 in Figs. 8, 9, 10 show some numerical examples.

Table 4.

Vertical distribution of steady current when bottom-friction ∝ bottom-velocity.

	$H=\frac{D}{10}$					
	$\xi'=0.392$		$\xi'=1.822$		$\xi'=4.635$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.049	0.156	0.375	0.293	0.454	0.153
0.1	0.049	0.155	0.374	0.293	0.454	0.153
0.2	0.048	0.154	0.374	0.292	0.454	0.153
0.3	0.047	0.154	0.372	0.292	0.453	0.152
0.4	0.046	0.149	0.370	0.291	0.451	0.152
0.5	0.045	0.144	0.367	0.290	0.450	0.152
0.6	0.043	0.140	0.365	0.288	0.449	0.150
0.7	0.042	0.135	0.361	0.287	0.447	0.150
0.8	0.039	0.127	0.356	0.284	0.445	0.149
0.9	0.037	0.119	0.351	0.282	0.442	0.148
1.0	0.034	0.111	0.346	0.280	0.439	0.148

Table 4.

continued

	$H=\frac{D}{4}$					
	$\xi'=0.494$		$\xi'=0.804$		$\xi'=1.272$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.346	0.275	0.414	0.238	0.462	0.186
0.1	0.344	0.274	0.412	0.237	0.461	0.186
0.2	0.339	0.271	0.408	0.236	0.458	0.185
0.3	0.330	0.267	0.400	0.233	0.452	0.184
0.4	0.319	0.259	0.390	0.229	0.444	0.182
0.5	0.304	0.250	0.376	0.225	0.434	0.180
0.6	0.286	0.239	0.362	0.218	0.422	0.176
0.7	0.266	0.224	0.343	0.209	0.407	0.171
0.8	0.241	0.207	0.322	0.198	0.390	0.166
0.9	0.214	0.188	0.299	0.185	0.371	0.158
1.0	0.187	0.163	0.274	0.169	0.351	0.151

continued

	$H=\frac{D}{2}$					
	$\xi'=0.430$		$\xi'=0.704$		$\xi'=1.143$	
z/H	u_1	v_1	u_1	v_1	u_1	v_1
0	0.543	0.135	0.545	0.104	0.542	0.074
0.1	0.540	0.136	0.543	0.105	0.540	0.075
0.2	0.530	0.139	0.535	0.108	0.534	0.078
0.3	0.515	0.143	0.522	0.112	0.525	0.083
0.4	0.490	0.148	0.504	0.118	0.511	0.088
0.5	0.457	0.153	0.478	0.125	0.493	0.095
0.6	0.419	0.155	0.449	0.129	0.471	0.101
0.7	0.372	0.153	0.412	0.132	0.443	0.105
0.8	0.318	0.145	0.368	0.130	0.411	0.106
0.9	0.257	0.128	0.319	0.121	0.373	0.103
1.0	0.186	0.099	0.262	0.104	0.330	0.095

2. The rising stage.

The rising stage of the current can be easily solved if the bottom-friction is proportional to the slip velocity, although it will be very complicated for the case where the friction varies with the square of the velocity.

When the bottom-friction is

$$-\mu\partial w/\partial z=f'\rho w \qquad \text{at } z=H,$$

the condition for the decaying part w_2 also takes form

$$-\mu\partial w_2/\partial z=f'\rho w_2 \qquad \text{at } z=H. \tag{24}$$

Then the decaying part will be given by

$$w_2=\sum_{n=1}^{\infty}A_n\cos\beta_n z.e^{-(\nu\beta_n^2+i2\varpi)t},$$

where β_n represents the roots of

$$\beta_n\tan\beta_n H-h=0, \qquad h=f'\rho/\mu=k/\xi', \tag{25}$$

and

$$A_n=\frac{2(\beta_n^2+h^2)}{H(\beta_n^2+h^2)+h}\int_0^H w_1\cos\beta_n z.dz.$$

Substituting the steady value w_1 with eq. (21), we have

$$A_n=V_g\frac{2(\beta_n^2+h^2)}{H(\beta_n^2+h^2)+h}\cos\beta_n H\Big[\frac{h}{\beta_n^2}-\frac{\eta' e^{i\theta}}{(a^2+\beta_n^2)(1+i)}\times(a+h\coth aH)\Big], \tag{26}$$

or separating into the real and the imaginary part

$$\left.\begin{aligned}&\text{The real part of } A_n=C\Big[\frac{k^2}{\beta_n^2}-\frac{\eta'}{4+\beta_n^4/k^4}(M\cos\theta-N\sin\theta)\Big],\\ &\text{The imagin. part of } A_n=-C\frac{\eta'}{4+\beta_n^4/k^4}(M\sin\theta+N\cos\theta),\end{aligned}\right\} \tag{26'}$$

where

$$C=V_g\frac{2\left(\dfrac{\beta_n^2}{k^2}+\dfrac{1}{\xi'^2}\right)}{kH\left(\dfrac{\beta_n^2}{k^2}+\dfrac{1}{\xi'^2}\right)+\dfrac{1}{\xi'}}\cdot\frac{\cos\beta_n H}{\xi'},$$

$$\left.\begin{aligned} M&=\frac{\beta_n^2}{k^2}\left(\xi'+\frac{b}{2}\right)-c, \qquad N=\frac{\beta_n^2}{k^2}\cdot\frac{c}{2}+b+2\xi', \\ b&=\frac{\sinh 2kH-\sin 2kH}{\cosh 2kH-\cos 2kH}, \qquad c=\frac{\sinh 2kH+\sin 2kH}{\cosh 2kH-\cos 2kH}, \end{aligned}\right\} \quad (27)$$

and ξ', η', θ are given by eq. (22) and (23) for the steady current.

We can now calculate numerically the decaying part w_2 and consequently the total current $w=w_1-w_2$. Some examples are shown in Figs. 8, 9, 10 and Table 5.

Table 5.

Decaying part w_2

	$H=D/10$					
	$\xi'=0.392$		$\xi'=1.822$		$\xi'=4.635$	
t	u_2	v_2	u_2	v_2	u_2	v_2
0ʰ	+0.049	+0.156	+0.375	+0.293	+0.454	+0.153
1	+0.023	+0.022	+0.307	+0.045	+0.391	−0.089
2	+0.006	+0.002	+0.188	−0.074	+0.259	−0.222
3	+0.001	+0.001	+0.081	−0.106	+0.091	−0.268
6	−0.000	−0.000	−0.028	−0.022	−0.159	−0.054
9			+0.006	−0.008	−0.032	+0.094
12			+0.002	+0.002	+0.056	+0.019
18			−0.000	−0.000	−0.020	−0.007
24					+0.007	+0.002
36					+0.001	+0.000
48					+0.000	+0.000

continued

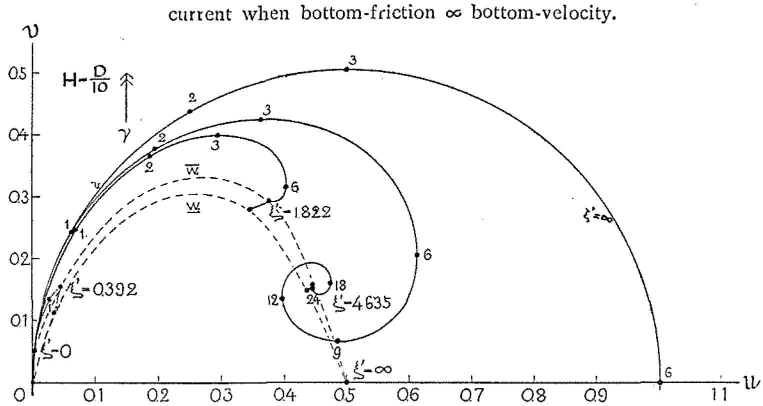
	$H=D/4$					
	$\xi'=0.494$		$\xi'=0.804$		$\xi'=1.272$	
t	u_2	v_2	u_2	v_2	u_2	v_2
0ʰ	+0.346	+0.275	+0.414	+0.238	+0.462	+0.186
1	+0.283	+0.047	+0.350	+0.004	+0.399	−0.049
2	+0.174	−0.066	+0.224	−0.125	+0.257	−0.196
3	+0.077	−0.094	+0.096	−0.160	+0.098	−0.235
6	−0.026	−0.021	−0.063	−0.038	−0.126	−0.053
9	−0.006	+0.007	−0.013	+0.024	−0.027	+0.065
12	+0.002	+0.002	+0.005	+0.009	+0.034	+0.014
18	−0.000	−0.000	−0.001	−0.002	−0.009	−0.004
24			+0.000	+0.000	+0.003	+0.001
36					+0.000	+0.000
48						

Table 5.
continued

	$H=D/2$					
	$\xi'=0.430$		$\xi'=0.704$		$\xi'=1.143$	
t	u_2	v_2	u_2	v_2	u_2	v_2
0^h	+0.543	+0.135	+0.545	+0.104	+0.542	+0.074
1	+0.478	−0.112	+0.480	−0.143	+0.477	−0.172
2	+0.303	−0.241	+0.302	−0.324	+0.295	−0.355
3	+0.104	−0.345	+0.089	−0.384	+0.067	−0.420
6	−0.227	−0.070	−0.314	−0.056	−0.435	−0.067
9	−0.043	+0.140	−0.050	+0.218	−0.055	+0.362
12	+0.087	+0.027	+0.151	+0.035	+0.274	+0.043
18	−0.033	−0.010	−0.074	−0.017	−0.168	−0.026
24	+0.012	+0.007	+0.035	+0.008	+0.103	+0.016
36	−0.002	−0.001	+0.008	+0.002	+0.039	+0.006
48	+0.000	+0.000	+0.002	+0.000	+0.015	+0.003
72			+0.000	+0.000	+0.002	+0.000
96					+0.000	+0.000

Fig. 8.

Vertical distribution of steady current and development of surface current when bottom-friction ∝ bottom-velocity.



$\bar{w}_1$=surface value of the steady current
$\underline{w}_1$=bottom value of the steady current

Thus we see that:

1) The smaller the value of ξ', the more and more the mode of the development of current approaches the case of "no bottom-current"; and the larger the value of ξ', the nearer the current development approaches that seen in the case of "no bottom-friction".

Fig. 9.

Vertical distribution of steady current and development of surface current when bottom-friction ∞ bottom-velocity.

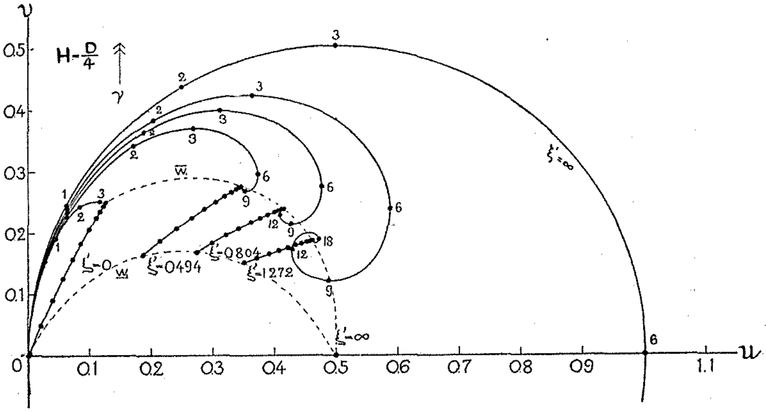
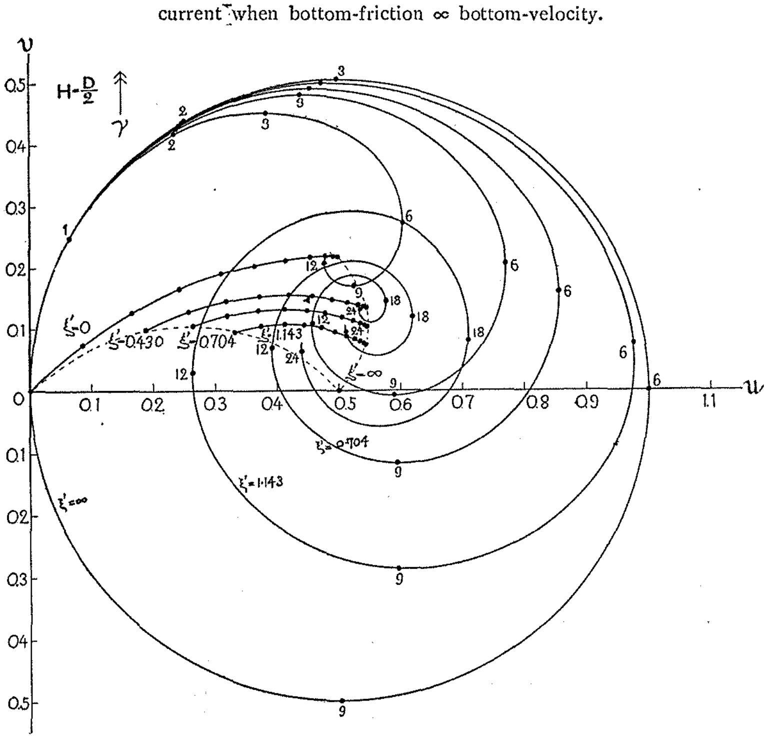


Fig. 10.

Vertical distribution of steady current and development of surface current when bottom-friction ∞ bottom-velocity.



2) The damping for a shallow sea is fairly good generally even when the steady value is quite near the current with "no bottom-friction".

In conclusion, the writers' sincere thanks are due to the Hattori Hōkōkwai for the subsidy given to one of the writers (T. N.) with which he could get an assistant to help him in the laborious numerical calculations.