Effect of Temperature on Cathode Ray Interference

By

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Abstract

Part I. Cathode ray interference due to a single crystal pyrite at the temperatures of 25°C and 220°C was investigated. It was found that the interference maxima were somewhat diffused at the higher temperature.

Part II. The Debye-Scherrer photographs of gold foils were taken at the temperatures of 28°C, 150°C and 320°C. They showed that the effect of temperature on their intensities increased with the angle of reflection and with the temperature.

Part I The Diffusion-Effect

Introduction

In the new theory of metals, especially in the theory of electric conductivity, the effect of the thermal vibrations of a crystal lattice upon metal electrons has become very important. In this work an attempt is made, in connection with this fact, to investigate the phenomena of the reflection of cathode rays by a crystal at high temperatures. Experiments of this kind have been carried out with slow electrons by several investigators[1], and the results of them show a slight broadening of the interference maxima with the rise in the temperature.

The present experiment was made with fast electrons, and the result obtained was quite similar to those of the preceding workers, that is, broadening or diffusion of the Kikuchi lines.[2] The temperature factor in the broadening has not yet been determined.

1. E. Rupp, „Ergeb. d. exakt. Naturwiss." IX, 117, 1930.
2. S. Nishikawa and S. Kikuchi, Proc. Imp. Acad. Japan, 475, 1928.

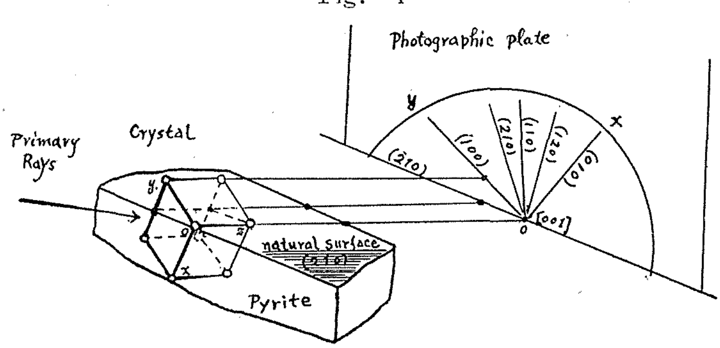
Experiment

For the purpose of the present investigation the writer used a natural single crystal of pyrite (FeS_2, cubic), because it gave very sharp P-, L- and parabolic patterns by reflection (See Photos. 1 and 2.), so that it was favourable for the comparison of the interference lines at each temperature. Beside this, the pyrite crystal has a relatively high characteristic temperature ($\Theta=645$) and consequently has a high frequency of lattice vibrations ($\nu_m=k\Theta/h$), and moreover it is a semi-conductor, with its minimum specific resistance at 20°C. These properties of the pyrite are interesting from the theoretical side in connection with the present subject.

The cathode ray tube employed was of the hot cathode type and was excited with an induction coil of 2 kilowatts. The beams of cathode rays were sharply defined by a magnetic field and by a system of three pin-hole slits, two of them being 0.3 mm. in diameter and the last about 0.1 mm. The whole length of the slit system was 12 cms. The distance between the crystal and the photographic plate was 22 cms.

The crystal surface used was a natural plane (210) of a pentagonal dodecahedron pyrite. Of the annexed photographs, Photo. 1 was taken with the crystal oriented as shown in figure 1, that is, the principal axis of the crystal was made perpendicular to the photographic plate and consequently nearly parallel to the primary beam (vacuum wave), while Photo 2a and 2b were taken with the crystal rotated a certain number of degrees about its surface normal from the above orientation.

Fig. 1



The crystal was heated in a small electric furnace, care being taken not to disturb the rays by its heating current. The furnace was equipped with a thermoelectric pyrometer for the purpose of reading the temperatures of the crystal directly.

Results

Photo 2a is a reflection photograph of the crystal taken at the room-temperature of 25°C, while Photo 2b is one taken at the temperature of 220°C. Those photographs were taken on one photographic plate succesively, with all possible care taken to maintain the same conditions in both cases, except their temperatures. But effects due to the thermal expansion of the crystal, if any, would be unavoidable.

By comparing the two photographs of the heated and unheated crystal the following facts were found:

1° The Kikuchi lines become somewhat broad or diffuse with temperature.

2° No apparent changes occur in the relative positions and shapes of the Kikuchi lines.

3° The intensities of the lines of higher orders of reflection, *e. g.* those of (010) of Photo. 2, decrease more with temperature.

These results are all plausible; fact 1° coincides with that in the case of slow electrons, 2° shows that the inner potential is unaffected by temperature, and 3° follows from Debye's formula for X-rays (See also Part II.)

The wave length of the cathode rays used was 0.0480 Å (61 KV).

Theoretical notes

The author wishes to end this article with a short remark on the theory of selective reflection. The interference of waves, which is caused by a vibrating lattice, requires the following fundamental equation as a condition of interference

$$\mathfrak{k} = \mathfrak{k}_0 + 2\pi\mathfrak{h} + \mathfrak{w} \qquad (1)$$

instead of the ordinary relation

$$\mathfrak{k} = \mathfrak{k}_0 + 2\pi\mathfrak{h}, \qquad (2)$$

where $\mathfrak{k}_0$, $\mathfrak{k}$ and $\mathfrak{w}$ are respectively the wave-vectors of the incident, reflected and elastic waves, and $\mathfrak{h}$ is a position-vector of a lattice point in a reciprocal lattice (Fahrstrahl).

H. Faxén[1], starting from equation (1), concluded with the diffusion of interference maxima of X-rays caused by scattering due to the heat motion of lattice points.

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1. H. Faxén, ZS. f. Phys. **17**, 270, 1923.

K. Shinohara[1] derived the diffusion of the Kikuchi lines, following the theory given by H. Bethe[2], which assumed equation (2) and an additional term called "Anregungsfehler". A term to be added to (2) is certainly the cause of the diffusion.

It is remarkable that the fundamental and starting equation (1) of the old theory coincides with the final and resulting equation

$$\vec{a_2}=\vec{a_1}+\vec{a}+\vec{n}/d \quad {}^{3)} \qquad (3)$$
$$\nu_2 \simeq \nu_1+\nu,$$

of the new quantum theory, where impulsevectors $\vec{a}$' s are wave vectors as well as quantum numbers or wave numbers, and d is a lattice constant. This equation would theoretically change the sharp interference in the selective reflection due to a fixed lattice[4] to the diffused one of a vibrating lattice.

Part II The Intensity-Effect

According to the Debye-Waller formula regarding the intensity of X-rays, which we shall here apply to the electron waves with Houston[5], the decrease in intensity must be proportional to

$$e^{-\frac{kT}{f}(2\pi\mathfrak{h})^2}$$

for the direction of interference $2\sin\theta/\lambda=n/d=|\mathfrak{h}|$, where obviously k is the Boltzmann constant, T is the absolute temperature of the crystal, f is a quasielastic force of the crystal lattice which depends upon its characteristic temperature, and $\mathfrak{h}$ has the same meaning as in Part I.

This formula implies that the temperature effect increases with the angle of reflection and with the temperature, for any given wave length and crystal[6], and that it becomes more noticeable for the rays of shorter wave length λ and for a crystal of smaller characteristic temperature Θ. For this and other reasons, a gold foil was adopted

1. K. Shinohara, Sc. Pap. Inst. Phys-chem. Res. Tokyo, **18**, 223, 1932.
2. H. Bethe, Ann. d. Phys. **87**, 78. Equ. (11), 1928.
3. L. Brillouin, „Die Quantenstatistik," Kap. 8, Equs. (117), (132), (144), R. Peierls, „Ergeb. d. exakt. Naturwiss.", XI, 296, 1932.
4. P. M. Morse, Phys. Rew. **35**, 1311, 1930, R. L. Kronig and W. G. Penney, Proc. Roy. Soc. London, **130**, 510, 1930.
5. W. V. Houston, ZS. f. Phys. **48**, 452, 1928.
6. W. H. and W. L. Bragg, "X-rays and Crystal Structure", 208, 1924.

for investigation, because in gold, $\Theta=175$, which is relatively small, and $\lambda=0.0460$Å (66 KV) in this experiment.

The photographs of the Debye-Scherrer ring were taken by the transmission method. (See Photos 3a, 3b and 3c of the annexed plate.) The gold foil was put in the furnace described in Part I; and then was photographed on one plate cinematographically at the temperatures of 28°C, 150°C and 320°C.

Photographs 3a, 3b and 3c clearly show the above mentioned results, namely, the diminution of the intensity with rise of temperature and with increase of the angle of reflection. The extraordinary decrease of intensity in Photo 3c may, however, be partially due to tearing of the foil by heat.

Some traces of regular arrangement of microcrystals are seen in Photo 3c, especially in the ring of the index (220). [1)]

In conclusion, the author wishes to express his sincere thanks to Professor M. Ishino for his kind guidance throughout this work, and the author's thanks are also due to Mr. H. Sekine for very useful assistance.

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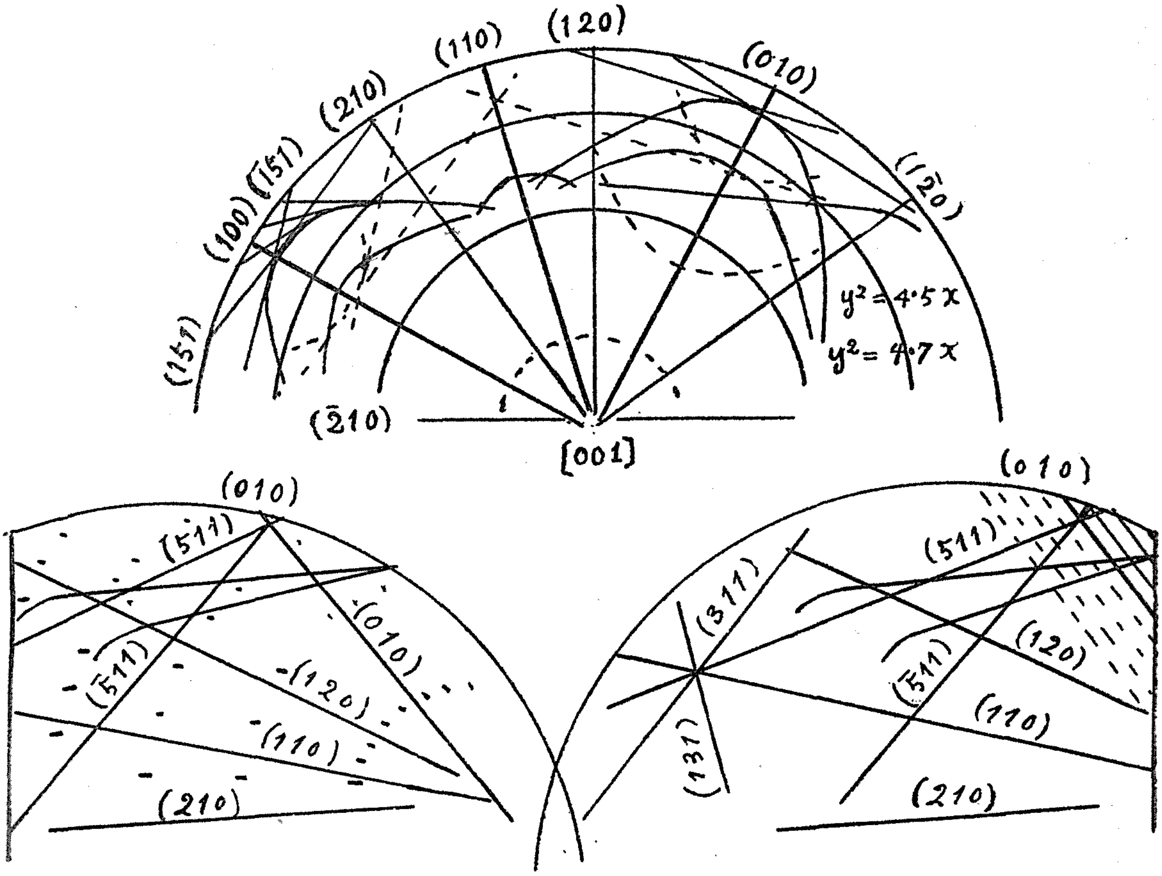
1). J. J. Trillat, Proc. Faraday Soc. XXIX, 998, 1933.



Regarding parabolic patterns, see K. Shinohara, Sc. Pap. Inst. Phys-chem. Res., Tokyo, **20**, 39, 1932, also the paper of the same author which will be soon published.

Plate I

(contact print)

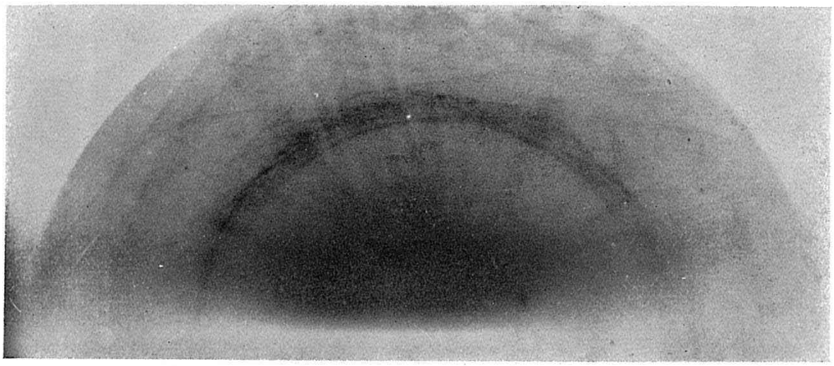


Photo. 1. Reflection by the natural surface (210) of pyrite.
$\lambda = 0.0480 \mathring{A}$ (61 KV)

Photo. 2_a.
at temp. 25°C.

Photo. 2_b.
at temp. 220°C.

Photo. 3.
Gold foil.
$\lambda = 0.0460 \mathring{A}$ (66 KV)

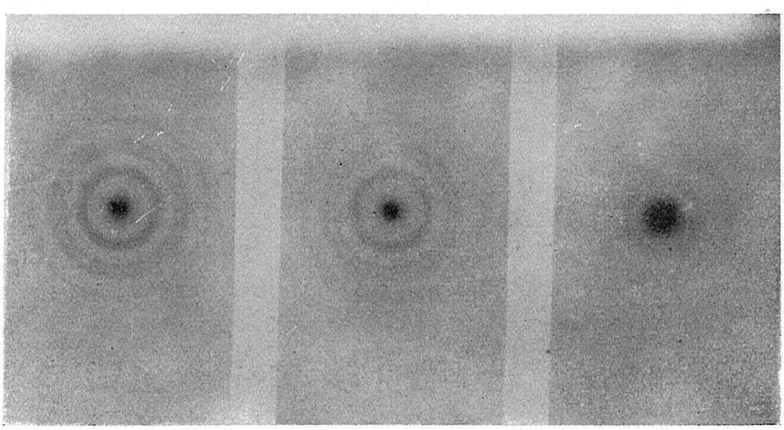


Photo. 3_a.
at 28°C.

Photo. 3_b.
at 150°C.

Photo. 3_c.
at 320°C.