Reverberation in Two Adjacent Rooms

By

Keiji Yamashita

(Received June 18, 1934)

Abstract

Several years ago the author published the formulae for reverberation in two adjacent rooms with the same acoustic properties. In that paper it was assumed that the walls of the rooms were made up of many kinds of material with so small areas that the sound-waves in each room were well diffracted. In the present paper, which is an extension of that paper, two cases are considered. The first is a case where the two rooms have the same acoustic properties and their walls are made up of a few kinds of material with large areas. Applying the method of Millington, the diffraction of sound is neglected and the number of incidences of sound-ray on each surface is treated statistically. The second case is that where the two adjacent rooms have different acoustic properties though their shapes and sizes are identical. In the latter case, assuming that the sound-waves are well diffracted, the simultaneous recurring equations for reverberation are obtained, and then by solving them the sound energies in the two rooms are found both when the source is sounding in one of the rooms and after it has been stopped.

I. Introduction

In order to obtain the formulae for reverberation in two adjacent rooms let us begin with the reverberation in a single room. Consider a room made up of ν different kinds of material whose coefficients of reflection are p_1, $p_2, \ldots\ldots p_\nu$, and whose respective surface areas are s_1, $s_2, \ldots\ldots s_\nu$, the total area being S. Let τ represent the mean value of the time-intervals between successive incidences of the sound-wave on the walls (including the floor and ceiling) of the room, and N its reciprocal, i. e. the mean number of incidences in unit time.

According to the most classical method for obtaining the reverberation equation, the total sound energy E in the room, when a source is emitting the sound energy at a constant rate ε, is given by

$$E=\frac{\varepsilon}{N(1-P)}\{1-e^{-N(1-P)t}\},\tag{1.1}$$

where t is the time mesaured from the beginning of the emission of sound and

$$P\equiv\frac{s_1p_1+s_2p_2+\cdots+s_\nu p_\nu}{S}<1.\tag{1.2}$$

And the total sound energy in the room at time t after the emission of sound has been stopped is given by

$$E=E_0e^{-N(1-P)t},\tag{1.3}$$

where E_0 is the initial sound energy in the room. Equations (1.1) and (1.3) are the formulae obtained by W. C. Sabine[1], W. S. Franklin[2] and G. Jäger[3]. A few years ago, it was discussed in detail by the present author[4] that these equations are valid only when the room is reverberant and the sound-waves are well diffracted between successive incidences. It is easy to see that equation (1.3) does not hold for absorbent rooms, i. e. for the limit $P\to 0$.

Assuming that every part of the sound energy existing in the room at any time t falls upon the wall once and only once before time $t+\tau$, and that the incident amount of energy is divided proportionally among the respective surface areas of the walls, the author[5] has obtained, instead of (1.1), (1.3), the following formulae:

$$E=\frac{\varepsilon\tau}{1-P}\{1-P^m),\tag{1.4}$$

$$E=E_0P^n.\tag{1.5}$$

Equation (1.4) gives the energy at time $t=m\tau$ in growing state and equation (1.5) the energy at time $t=n\tau$ in decaying state. Equations (1.4) and (1.5) may be used for a room where the sound-waves are well diffracted, no matter whether the room is reverberant or not, since the above assumptions which have been used to derive these equations are justifiable for such rooms. The same equations have been obtained by C. F. Eyring[6] assuming that image sources may replace walls of

1. W. C. Sabine, *Amer. Architect* (1900); Collected Papers on Acoustics, pp. 34–37.
2. W. S. Franklin, *Phys. Rev.* **16**, 372–374 (1903).
3. G. Jäger, *Wiener Sitz. Ber.* **120**, 613–634 (1911).
4. K. Yamashita, *These Memoirs*, **11**, 120–123 (1928).
5. K. Yamashita, *loc. cit.* 123–128 (1928).
6. C. F. Eyring, *J. Acous. Soc. Amer.* **1**, 217–241 (1930).

the room. If we write equations (1.4) and (1.5) as the continuous function of time t, we obtain

$$E=\frac{\varepsilon\tau}{1-P}\{1-P^{\frac{t}{\tau}}\}, \tag{1.6}$$

$$E=E_0P^{\frac{t}{\tau}}; \tag{1.7}$$

or

$$E=\frac{\varepsilon}{N(1-P)}\{1-e^{Nt\log P}\}, \tag{1.8}$$

$$E=E_0e^{Nt\log P}. \tag{1.9}$$

Thus we see that at the limit $P\to 1$ equations (1.8), (1.9) become identical with equations (1.1), (1.3).

If the walls of the room are made of many kinds of material with small areas, equations (1.4), (1.5) may be applied, since the sound-wave is well mixed up by diffraction between each pair of successive incidences. If, on the other hand, each of the areas composing the walls is large, these equations are not applicable. As a decay equation for the latter case, G. Millington[1] has obtained a new formula

$$\begin{aligned} E&=E_0p_1^{\frac{s_1}{S}n}\,p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n}\\ &=E_0P'^n\\ &=E_0e^{Nt\log P'}, \end{aligned} \tag{1.10}$$

where

$$P'\equiv p_1^{\frac{s_1}{S}}\,p_2^{\frac{s_2}{S}}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}}. \tag{1.11}$$

This formula can be derived on the assumptions that, neglecting the diffraction, the simple ray theory can be applied for the sound-waves in the room and that a fraction $\frac{s^i}{S}n$ of n incidences will take place on the surface s_i. In Millington's formula each of the fractions $\frac{s_1}{S}n$, $\frac{s_2}{S}n,\ldots\ldots\frac{s_\nu}{S}n$ must be sufficiently large since the number of incidences on each surface is treated statistically.

The three methods described above are the most prominent ones

1. G. Millington, *J. Acous. Soc. Amer.* **4**, 69-82 (1932).

for obtaining the reverberation equations in a single room. Applying the first method, i. e. the method of Sabine and others, A. H, Davis[1] has obtained the formulae for reverberation in two adjacent rooms. By the second method the author[2] already obtained the formulae for reverberation in two adjacent rooms with the same acoustic properties. One of the objects of the present paper is to obtain the decay equations for two adjacent rooms with the same acoustic properties by applying the third method of Millington and compare the results with those already obtained by the other methods. And the other object is to obtain the reverberation equations for two adjacent rooms which have different acoustic properties though they are identical in shape and size.

II. Reverberation in Two Adjacent Rooms when Diffraction is Negligible

Applying Millington's method for a single room, let us obtain the decay equations for two adjacent rooms. Consider two adjacent rooms I and II which are in acoustic communication only through an incompletely sound-proof partition W. Let the two rooms be symmetrical about the partition W not only in the shape and size but also in the distribution of absorbing materials. For instance, if there is in room I a surface of area s_i with coefficient of reflection p_i, then there is also in room II a surface of area s_i with the same coefficient of reflection p_i.

Let S represent the total area of the walls in room I (including the partition area); $s_1, s_2, \ldots\ldots s_\nu$ the areas of the elements of S; $p_1, p_2, \ldots\ldots p_\nu$ the respective coefficients of reflection; q the coefficient of transmission of the partition whose area is s_ν; τ the mean value of the time-intervals between successive incidences of the sound-waves in room I. Since the two rooms have the same acoustic properties, the symbols defined above may be used for room II.

Let E_{I0}, E_{II0} represent the initial values of the total sound energies in rooms I and II respectively and E_I, E_{II} the total energies remaining in the two rooms at time $t=n\tau$ after n incidences. E_I is composed of many elements of sound-ray which take different courses in the time-interval from $t=0$ to $t=n\tau$. Let $E_I^{(0)}$ be an element of E_I which

1. A. H. Davis, *Phil. Mag.* **50**, 75-80 (1925).
2. K. Yamashita, *loc. cit.* 128-133 (1928).

does not enter into room II at all and is reflected only in room I. Of the n reflections a fraction $\frac{s_1}{S}n$ takes place on surface s_1, another fraction $\frac{s_2}{S}n$ on s_2 and so on. Therefore we have

$$E_I^{(0)}=E_{I0}p_1^{\frac{s_1}{S}n}p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n}.$$

Divide the time-interval from $t=0$ to $t=n\tau$ into n equal short intervals. Let $E_I^{(i,j)}$ be an element of E_I which enters room II in the i-th short interval from the beginning and returns to room I in the j-th short interval not to reenter into room II. This element passes through the partition s_ν twice and consequently the number of reflections on s_ν is $\frac{s_\nu}{S}n-2$, the $\frac{s_\nu}{S}-2$ reflections being the sum of the reflections in rooms I and II. The number of reflections of this element on s_1 is $\frac{s_1}{S}n$, which is the sum of the number of reflections in both rooms. Therefore $E_I^{(i,j)}$ is given by

$$E_I^{(i,j)}=E_{I0}p_1^{\frac{s_1}{S}n}p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-2}q^2.$$

There are many elements of E_I which pass through the partition twice as $E_I^{(i,j)}$ taking possibly different values for (i,j). Let $E_I^{(2)}$ represent their sum, then $E_I^{(2)}$ is given by

$$\begin{aligned}E_I^{(2)}&={}_nC_2E_I^{(i,j)}\\&={}_nC_2E_{I0}p_1^{\frac{s_1}{S}n}p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-2}q^2,\end{aligned}$$

where ${}_nC_2$ denotes the combination of 2 among n.

Similarly the sum of the elements of E_I which pass through the partition four times, six times, etc. are

$$E_I^{(4)}={}_nC_4E_{I0}p_1^{\frac{s_1}{S}n}p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-4}q^4,$$

$$E_I^{(6)}={}_nC_6E_{I0}p_1^{\frac{s_1}{S}n}p_2^{\frac{s_2}{S}n}\cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-6}q^6,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

The indices of p_ν in the above equations must be positive or zero.

Of E_I there are many other elements, besides the elements considered above, which were initially in room II. These elements can be classified by grouping the elements which pass through the partition respectively once, three times, five times, etc. Let $E_I^{(1)}$, $E_I^{(3)}$, $E_I^{(5)}$, etc. represent the sums of the elements thus grouped, then by a similar method we get

$$E_I^{(1)} = {}_nC_1 E_{II0}\, p_1^{\frac{s_1}{S}n}\, p_2^{\frac{s_2}{S}n} \cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-1}\, q,$$

$$E_I^{(3)} = {}_nC_3 E_{II0}\, p_1^{\frac{s_1}{S}n}\, p_2^{\frac{s_2}{S}n} \cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-3}\, q^3,$$

$$E_I^{(5)} = {}_nC_5 E_{II0}\, p_1^{\frac{s_1}{S}n}\, p_2^{\frac{s_2}{S}n} \cdots\cdots\cdots p_\nu^{\frac{s_\nu}{S}n-5}\, q^5,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

The indices of p_ν must be positive or zero.

The total sound energy E_I in room I at time $t = n\tau$ is given by $\Sigma E_I^{(k)}$ and the corresponding energy E_{II} in room II may be derived similarly. Therefore we obtain

$$\left.\begin{aligned} E_I &= E_{I0} P'^n \left\{ 1 + {}_nC_2\left(\frac{q}{p_\nu}\right)^2 + {}_nC_4\left(\frac{q}{p_\nu}\right)^4 + \cdots\cdots\cdots\right\} \\ &\quad + E_{II0} P'^n \left\{ {}_nC_1\left(\frac{q}{p_\nu}\right) + {}_nC_3\left(\frac{q}{p_\nu}\right)^3 + {}_nC_5\left(\frac{q}{p_\nu}\right)^5 + \cdots\right\} \\ &= E_{I0} P'^n \sum_{i=0}^{i \leqq \frac{s_\nu}{S}\frac{n}{2}} {}_nC_{2i}\left(\frac{q}{p_\nu}\right)^{2i} \\ &\quad + E_{II0} P'^n \sum_{i=0}^{i \leqq \frac{s_\nu}{S}\frac{n}{2}-\frac{1}{2}} {}_nC_{2i+1}\left(\frac{q}{p_\nu}\right)^{2i+1}. \end{aligned}\right\} \tag{2.1}$$

$$\left.\begin{aligned} E_{II} &= E_{II0} P'^n \left\{ 1 + {}_nC_2\left(\frac{q}{p_\nu}\right)^2 + {}_nC_4\left(\frac{q}{p_\nu}\right)^4 + \cdots\cdots\cdots\right\} \\ &\quad + E_{I0} P'^n \left\{ {}_nC_1\left(\frac{q}{p_\nu}\right) + {}_nC_3\left(\frac{q}{p_\nu}\right)^3 + {}_nC_5\left(\frac{q}{p_\nu}\right)^5 + \cdots\right\} \\ &= E_{II0} P'^n \sum_{i=0}^{i \leqq \frac{s_\nu}{S}\frac{n}{2}} {}_nC_{2i}\left(\frac{q}{p_\nu}\right)^{2i} \\ &\quad + E_{I0} P'^n \sum_{i=0}^{i \leqq \frac{s_\nu}{S}\frac{n}{2}-\frac{1}{2}} {}_nC_{2i+1}\left(\frac{q}{p_\nu}\right)^{2i+1}, \end{aligned}\right\} \tag{2.2}$$

where

$$P' \equiv p_1^{\frac{s_1}{S}} p_2^{\frac{s_2}{S}} \cdots\cdots p_\nu^{\frac{s_\nu}{S}}. \tag{2.3}$$

These are the decay equations for two adjacent rooms for which it is justifiable to neglect the diffraction of sound.

III. Comparison of Reverberation Formulae in Two Adjacent Rooms

The reverberation equations in two adjacent rooms obtained by the author[1] by means of the second method are as follows:

$$E_I^{(\infty)} = E_{I0}^{(\infty)}\{P^n + {}_nC_2 Q^2 P^{n-2} + {}_nC_4 Q^4 P^{n-4} + \cdots\cdots\}$$
$$+ E_{II0}^{(\infty)}\{{}_nC_1 Q P^{n-1} + {}_nC_3 Q^3 P^{n-3} + {}_nC_5 Q^5 P^{n-5} + \cdots\}, \tag{3.1}$$

$$E_{II}^{(\infty)} = E_{II0}^{(\infty)}\{P^n + {}_nC_2 Q^2 P^{n-2} + {}_nC_4 Q^4 P^{n-4} + \cdots\cdots\}$$
$$+ E_{I0}^{(\infty)}\{{}_nC_1 Q P^{n-1} + {}_nC_3 Q^3 P^{n-3} + {}_nC_5 Q^5 P^{n-5} + \cdots\}, \tag{3.2}$$

$$E_{I0}^{(\infty)} = \varepsilon\tau \frac{1-P}{(1-P)^2 - Q^2}, \tag{3.3}$$

$$E_{II0}^{(\infty)} = \varepsilon\tau \frac{Q}{(1-P)^2 - Q^2}, \tag{3.4}$$

where

$$P \equiv \frac{s_1 p_1 + s_2 p_2 + \cdots + s_\nu p_\nu}{S} < 1, \tag{3.5}$$

$$Q \equiv \frac{s_\nu}{S} q, \tag{3.6}$$

and $E_{I0}^{(\infty)}$, $E_{II0}^{(\infty)}$ are the total sound energies in rooms I and II in the steady state, and $E_I^{(\infty)}$, $E_{II}^{(\infty)}$ are the energies remaining at time $t = n\tau$ later, and in (3.1), (3.2) the terms ${}_nC_\lambda Q^\lambda P^{n-\lambda}$ are to be summed up for all positive integers satisfying $n \geqq \lambda$.

In order to compare the above formulae (3.1), (3.2) with (2.1) and (2.2), the former equations may be written in the following forms:

$$\left.\begin{aligned} E_I^{(\infty)} = E_{I0}^{(\infty)} P^n\left\{1 + {}_nC_2\left(\frac{Q}{P}\right)^2 + {}_nC_4\left(\frac{Q}{P}\right)^4 + \cdots\cdots\cdots\cdots\right\} \\ + E_{II0}^{(\infty)} P^n\left\{{}_nC_1\left(\frac{Q}{P}\right) + {}_nC_3\left(\frac{Q}{P}\right)^3 + {}_nC_5\left(\frac{Q}{P}\right)^5 + \cdots\cdots\right\} \end{aligned}\right|$$

1. K. Yamashita, *loc. cit.* 131 (1928).

$$
\left.
\begin{aligned}
&= E_{I0}^{(\infty)} P^n \sum_{i=0}^{i \leqq \frac{n}{2}} {}_nC_{2i} \left(\frac{Q}{P} \right)^{2i} \\
&\quad + E_{II0}^{(\infty)} P^n \sum_{i=0}^{i \leqq \frac{n}{2} - \frac{1}{2}} {}_nC_{2i+1} \left(\frac{Q}{P} \right)^{2i+1},
\end{aligned}
\right\}
\tag{3.7}
$$

$$
\left.
\begin{aligned}
E_{II}^{(\infty)} &= E_{II0}^{(\infty)} P^n \left\{ 1 + {}_nC_2 \left(\frac{Q}{P} \right)^2 + {}_nC_4 \left(\frac{Q}{P} \right)^4 + \dots\dots\dots\dots\dots \right\} \\
&\quad + E_{I0}^{(\infty)} P^n \left\{ {}_nC_1 \left(\frac{Q}{P} \right) + {}_nC_3 \left(\frac{Q}{P} \right)^3 + C_5 \left(\frac{Q}{P} \right)^5 + \dots \right\} \\
&= E_{II0}^{(\infty)} P^n \sum_{i=0}^{i \leqq \frac{n}{2}} {}_nC_{2i} \left(\frac{Q}{P} \right)^{2i} \\
&\quad + E_{I0}^{(\infty)} P^n \sum_{i=0}^{i \leqq \frac{n}{2} - \frac{1}{2}} {}_nC_{2i+1} \left(\frac{Q}{P} \right)^{2i+1}.
\end{aligned}
\right\}
\tag{3.8}
$$

Comparing (2.1) with (3.7) we see that, though the two equations are similar in form, the number of the terms in (2.1) is by far smaller than that in (3.7), but that the terms in (2.1) are greater than the corresponding terms in (3.7), because we have

$$
\frac{Q}{P} = \frac{s_\nu q}{s_1 p_1 + s_2 p_2 + \dots + s_\nu p_\nu} < \frac{q}{p_\nu}.
$$

The magnitudes of P and P' have been compared by G. Millington[1] in the reverberation equation for a single room. It has been proved that under all conditions $P' < P$, except when $p_1 = p_2 = \dots\dots = p_\nu$ in which $P' = P$.

By the first method A. H. Davis[2] has obtained the following decay equations for two adjacent roomes with volumes V_I, V_{II} and total areas S_I, S_{II}:

$$
E_I = E_{I0} \left\{ \frac{1 - \dfrac{4V_{II}\lambda_1}{c(1 - P_{II}) S_{II}}}{1 - \dfrac{\lambda_1}{\lambda_2}} e^{-\lambda_1 t} + \frac{1 - \dfrac{4V_{II}\lambda_2}{c(1 - P_{II}) S_{II}}}{1 - \dfrac{\lambda_2}{\lambda_1}} e^{-\lambda_2 t} \right\},
\tag{3.9}
$$

1. G. Millington, *loc. cit.* 77 (1932).
2. A. H. Davis, *loc. cit.* 77–78 (1927).

$$E_{II}=E_{II0}\left\{\frac{1}{1-\frac{\lambda_1}{\lambda_2}}e^{-\lambda_1 t}-\frac{1}{\frac{\lambda_2}{\lambda_1}-1}e^{-\lambda_2 t}\right\}, \qquad (3.10)$$

where

$$\lambda_1=\frac{c}{8}\left\{\frac{(1-P_I)S_I}{V_I}+\frac{(1-P_{II})S_{II}}{V_{II}}\right\}$$
$$-\frac{c}{8}\sqrt{\left\{\frac{(1-P_{II})S_{II}}{V_{II}}-\frac{(1-P_I)S_I}{V_I}\right\}^2+\frac{4q^2s_v^2}{V_IV_{II}}} \qquad (3.11)$$

$$\lambda_2=\frac{c}{8}\left\{\frac{(1-P_I)S_I}{V_I}+\frac{(1-P_{II})S_{II}}{V_{II}}\right\}$$
$$+\frac{c}{8}\sqrt{\left\{\frac{(1-P_{II})S_{II}}{V_{II}}-\frac{(1-P_I)S_I}{V_I}\right\}^2+\frac{4q^2s_v^2}{V_IV_{II}}}, \qquad (3.12)$$

where P_I, P_{II} denote the value of P for the two rooms and c the propagation-speed of sound in air. These equations are valid only when the two rooms are reverberant, since they were obtained by the first method.

When the two rooms have the same acoustic properties the above equations may be reduced, by putting

$$S_I=S_{II}=S, \quad V_I=V_{II}=V, \quad P_I=P_{II}=P,$$

to the following forms:

$$E_I=\frac{E_{I0}}{2}\left[\left\{1+\frac{qs_v}{(1-P)S}\right\}e^{-\lambda_1 t}+\left\{1-\frac{qs_v}{(1-P)S}\right\}e^{-\lambda_2 t}\right]. \qquad (3.13)$$

$$E_{II}=\frac{E_{II0}}{2}\left[\left\{\frac{(1-P)S}{qs_v}+1\right\}e^{-\lambda_1 t}-\left\{\frac{(1-P)S}{qs_v}-1\right\}e^{-\lambda_2 t}\right]. \qquad (3.14)$$

where

$$\lambda_1=\frac{c(1-P)S}{4V}\left\{1-\frac{qs_v}{(1-P)S}\right\}. \qquad (3.15)$$

$$\lambda_2=\frac{c(1-P)S}{4V}\left\{1+\frac{qs_v}{(1-P)S}\right\}. \qquad (3.16)$$

In order to compare equations (3.13), (3.14) with equations (3.1), (3.2), we shall transform (3.1), (3.2) into exponential forms:

$$\left.\begin{aligned}E_I^{(\infty)} &= \frac{E_{I0}^{(\infty)}}{2}\{(P+Q)^n+(P-Q)^n\} \\ &\quad+\frac{E_{II0}^{(\infty)}}{2}\{(P+Q)^n-(P-Q)^n\} \\ &= \frac{E_{I0}^{(\infty)}}{2}\{e^{n\log(P+Q)}+e^{n\log(P-Q)}\} \\ &\quad+\frac{E_{II0}^{(\infty)}}{2}\{e^{n\log(P+Q)}-e^{n\log(P-Q)}\},\end{aligned}\right\} \quad (3.17)$$

$$\left.\begin{aligned}E_{II}^{(\infty)} &= \frac{E_{II0}^{(\infty)}}{2}\{(P+Q)^n+(P-Q)^n\} \\ &\quad+\frac{E_{I0}^{(\infty)}}{2}\{(P+Q)^n-(P-Q)^n\} \\ &= \frac{E_{II0}^{(\infty)}}{2}\{e^{n\log(P+Q)}+e^{n\log(P-Q)}\} \\ &\quad+\frac{E_{I0}^{(\infty)}}{2}\{e^{n\log(P+Q)}-e^{n\log(P-Q)}\}.\end{aligned}\right\} \quad (3.18)$$

By equations (3.3), (3.4)

$$\frac{E_{I0}^{(\infty)}}{1-P}=\frac{E_{II0}^{(\infty)}}{Q}$$

and therefore equations (3.17), (3.18) may be reduced to

$$E_I^{(\infty)}=\frac{E_{I0}^{(\infty)}}{2}\left\{\frac{1-P+Q}{1-P}e^{n\log(P+Q)}+\frac{1-P-Q}{1-P}e^{n\log(P-Q)}\right\}, \quad (3.19)$$

$$E_{II}^{(\infty)}=\frac{E_{II0}^{(\infty)}}{2}\left\{\frac{1-P+Q}{Q}e^{n\log(P+Q)}-\frac{1-P-Q}{Q}e^{n\log(P-Q)}\right\}. \quad (3.20)$$

When each of the surfaces reflects sound-waves very well, Q is very small and P takes a value near to unity and consequently $P+Q$, $P-Q$ are nearly equal to unity. Therefore we have

$$\log(P+Q)\fallingdotseq-(1-P-Q),$$
$$\log(P-Q)\fallingdotseq-(1-P+Q),$$

and equations (3.19), (3.20) become

$$E^{(\infty)}=\frac{E_{I0}^{(\infty)}}{2}\left\{\frac{1-P+Q}{1-P}e^{-n(1-P-Q)}+\frac{1-P-Q}{1-P}e^{-n(1-P+Q)}\right\}, \quad (3.21)$$

$$E_{II}^{(\infty)}=\frac{E_{II0}^{(\infty)}}{2}\left\{\frac{1-P+Q}{Q}e^{-n(1-P-Q)}-\frac{1-P-Q}{Q}e^{-n(1-P+Q)}\right\}. \tag{3.22}$$

Hence we see that these equations are identical with (3.13), (3.14) by the two relations

$$n=\frac{t}{\tau}, \quad \frac{1}{\tau}=\frac{cS}{4V}.$$

Equations (3.1), (3.2) are the general solutions for decaying when the waves are well diffracted, equations (3.13), (3.14) being a special case of them, and therefore they may be used not only for reverberant rooms but also for absorbent rooms. It seems impossible to obtain the decay equations by similar methods when the two rooms are not equal in size. By Davis' method, however, the reverberation equations for two rooms with different sizes are obtained as equations (3.9), (3.10) show, although these equations are valid only for reverberant rooms.

IV. Remarks on a Paper by Carl F. Eyring

Carl F. Eyring[1] in his paper "Reverberation Time Measurements in Couple Rooms," modified A. H. Davis' formulae, i. e., (3.9), (3.10), (3.11), (3.12) as follows:

$$\rho_I=\frac{1}{m_2-m_1}\{m_2(\rho_{I0}+m_1\rho_{II0})e^{-(b_I-m_1\beta_{II})t} - m_1(\rho_{I0}+m_2\rho_{II0})e^{-(b_I-m_2\beta_{II})t}\}. \tag{4.1}$$

$$\rho_{II}=\frac{1}{m_1-m_2}\{(\rho_{I0}+m_1\rho_{II0})e^{-(b_I-m_1\beta_{II})t} - (\rho_{I0}+m_2\rho_{II0})e^{-(b_I-m_2\beta_{II})t}\}. \tag{4.2}$$

$$\frac{\rho_{I0}}{A_{II}+W}=\frac{\rho_{II0}}{W}; \tag{4.3}$$

where

$$m_1\equiv\frac{(b_I-b_{II})+\sqrt{(b_{II}-b_I)^2+4\beta_I\beta_{II}}}{2\beta_{II}},$$

$$m_2\equiv\frac{(b_I-b_{II})-\sqrt{(b_{II}-b_I)^2+4\beta_I\beta_{II}}}{2\beta_{II}}.$$

1. C. F. Eyring, *J. Acous. Soc. Amer.* **3**, 181–206 (1931).

$$\left.\begin{aligned} b_I &\equiv \frac{ck_I(A_I+W)}{4V_I}, \\ \beta_I &\equiv \frac{ck_{II}W}{4V_I}, \\ k_I &\equiv \frac{-\log(1-a_I)}{a_I}; \end{aligned}\right\} \tag{4.4}$$

and ρ_I, ρ_{II} are the sound energy densities in rooms I, II at time t; ρ_{I0}, ρ_{II0} their initial values; W the area of an open window through which the sound energies are transmitted; A_I the absorbing power of the surface of room I not including the open window; a_I the average coefficient of absorption of room I.

In the case where a_I, a_{II} are very small, k_I and k_{II} are approximately equal to unity and we get

$$\begin{aligned} & b_I - m_1\beta_{II} \\ & \fallingdotseq \frac{c}{8}\left\{\frac{A_{II}+W}{V_{II}} + \frac{A_I+W}{V_I}\right\} \\ & \qquad - \frac{c}{8}\sqrt{\left\{\frac{A_{II}+W}{V_{II}} - \frac{A_I+W}{V_I}\right\}^2 + \frac{W^2}{V_IV_{II}}} \\ & = \lambda_1, \\ & b_I - m_2\beta_{II} \\ & \fallingdotseq \frac{c}{8}\left\{\frac{A_{II}+W}{V_{II}} + \frac{A_I+W}{V_I}\right\} \\ & \qquad + \frac{c}{8}\sqrt{\left\{\frac{A_{II}+W}{V_{II}} - \frac{A_I+W}{V_I}\right\}^2 + \frac{W^2}{V_IV_{II}}} \\ & = \lambda_2, \\ & \lambda_1 - \lambda_2 = \beta_{II}(m_2 - m_1), \\ & m_1m_2 = -\frac{\beta_I}{\beta_{II}} \fallingdotseq -\frac{V_{II}}{V_I}. \end{aligned}$$

By using these relations we can see easily that, when the rooms are reverberant, Eyring's results (4.1), (4.2) become identical with Davis' results (3.9), (3.10). Eyring has noticed that equations (4.1), (4.2) may be used for absorbent rooms too.

Let us compare equations (4.1), (4.2) with the author's results (3.17), (3.18). From (3.17), (3.18) we have

$$E_I^{(\infty)} = \frac{1}{2}\left\{(E_{I0}^{(\infty)} + E_{II0}^{(\infty)})e^{\frac{t}{\tau}\log(P+Q)}\right.$$

$$+(E_{I0}^{(\infty)}-E_{II0}^{(\infty)})e^{\frac{t}{\tau}\log(P-Q)}\Big\}, \qquad (4.5)$$

$$E_{II}^{(\infty)}=\frac{1}{2}\Big\{(E_{I0}^{(\infty)}+E_{II0}^{(\infty)})e^{\frac{t}{\tau}\log(P+Q)}$$

$$-(E_{I0}^{(\infty)}-E_{II0}^{(\infty)})e^{\frac{t}{\tau}\log(P-Q)}\Big\}. \qquad (4.6)$$

In Eyring's equations (4.1), (4.2), if the rooms have the same acoustic properties, we get

$$k_I=k_{II} \quad [=k \quad \text{say}],$$
$$b_I=b_{II} \quad [=b \quad \text{say}],$$
$$\beta_I=\beta_{II} \quad [=\beta \quad \text{say}],$$
$$m_1=1, \quad m_2=-1,$$

and then the energy densities ρ_I, ρ_{II} are given by

$$\rho_I=\frac{1}{2}\Big\{(\rho_{I0}+\rho_{II0})e^{-(b-\beta)t}+(\rho_{I0}-\rho_{II0})e^{-(b+\beta)t}\Big\}, \qquad (4.7)$$

$$\rho_{II}=\frac{1}{2}\Big\{(\rho_{I0}+\rho_{II0})e^{-(b-\beta)t}-(\rho_{I0}-\rho_{II0})e^{-(b+\beta)t}\Big\}. \qquad (4.8)$$

By the relations

$$\left.\begin{aligned} W&=QS, \quad A+M=(1-P)S, \\ \frac{1}{\tau}&=\frac{cS}{4V}, \end{aligned}\right\} \qquad (4.9)$$

$b-\beta$ and $b+\beta$ may be expressed as

$$\left.\begin{aligned} b-\beta&=\frac{1}{\tau}\,\frac{-\log P}{1-P}(1-P-Q), \\ b+\beta&=\frac{1}{\tau}\,\frac{-\log P}{1-P}(1-P+Q). \end{aligned}\right\} \qquad (4.10)$$

These are the indices in equations (4.7), (4.8) expressed by P and Q, and the corresponding values in (4.5), (4.6) are

$$-\frac{1}{\tau}\log(P+Q), \quad -\frac{1}{\tau}\log(P-Q). \qquad (4.11)$$

Therefore we see that equations (4.5), (4.6) coincide with equations (4.7), (4.8) either when the rooms are so reverberant that we may take approximately

$$-\log P \doteqdot 1-P,$$

$$-\log(P\pm Q)\doteqdot 1-P\mp Q,$$

or when Q is so small that the approximations

$$1-P+Q\doteqdot 1-P,$$

$$\log(P\pm Q)\doteqdot\log P$$

may be permissible. But for the other cases they do not coincide generally.

Let us examine the assumption on which equations (4.1), (4.2) have been derived. Eyring has considered the reverberation in a single room first, and he has said that the rate at which sound energy falls on a unit area of wall surface is given by

$$\sigma=\frac{c\rho}{4} \tag{4.12}$$

for a steady state, and that an equation of the form

$$\sigma=\frac{kc\rho}{4}, \tag{4.13}$$

where

$$k\equiv\frac{-\log(1-\alpha)}{\alpha}, \tag{4.14}$$

must be used for the case of decaying. By using equation (4.13) and a differential equation

$$V\frac{d\rho}{dt}=-\sigma\alpha S, \tag{4.15}$$

or

$$\frac{d\rho}{dt}=\frac{cS\log(1-\alpha)}{4V}\rho, \tag{4.16}$$

the correct decay equation for a single room can be obtained as

$$\rho=\rho_0 e^{-bt} \tag{4.17}$$

where

$$b\equiv\frac{-cS\log(1-\alpha)}{4V}. \tag{4.18}$$

Thus equation (4.13) gives the rate at which sound energy falls on a unit area of wall surface of a single room when no sound is being supplied to the room, but it is questionable whether equation (4.13) may be used for a room to which the sound energy is being supplied. Eyring has, however, used equation (4.13) for coupled rooms. In case of coupled rooms sound energies are always supplied to each of the rooms from the other both in growing state and in decaying state. Therefore equation (4.13) must be examined if we want to use it for coupled rooms.

For the sake of simplicity let us consider a case where the sound energy is supplied at a constant rate. The total sound energy in a single rooms when a source is sounding is given by

$$E=\varepsilon\tau\frac{1-P^m}{1-P}, \tag{4.19}$$

where ε is the rate of emission from the source. Equation (4.19) has been obtained by the author[1] and by Eyring[2]. If equation (4.19) is expressed in a continuous function of time t, we have

$$E=\varepsilon\tau\frac{1-P^{\frac{t}{\tau}}}{1-P}=\frac{\varepsilon\tau}{1-P}\left\{1-e^{\frac{t}{\tau}\log P}\right\},$$

and the rate of change of the sound energy is

$$\frac{dE}{dt}=\frac{\log P}{\tau}E-\frac{\log P}{1-P}\varepsilon. \tag{4.20}$$

In equation (4.19) the initial sound energy is zero, but, if the initial value is E_0, an equation of the form

$$E=E_0P^m+\varepsilon\tau\frac{1-P^m}{1-P} \tag{4.21}$$

may be used instead of (4.19), and consequently the rate of change of the sound energy is given by

$$\frac{dE}{dt}=\frac{\log P}{\tau}E-\frac{\log P}{1-P}\varepsilon. \tag{4.22}$$

Equations (4.20) and (4.22) are identical in form.

Thus, if the source is emitting sound energy at a constant rate ε, the rate of change of the total sound energy in the room is given by

$$\begin{aligned}V\frac{d\rho}{dt}&=\frac{cS\log(1-\alpha)}{4}\rho-\frac{\log(1-\alpha)}{\alpha}\varepsilon\\&=\varepsilon-\left\{\frac{-cS\log(1-\alpha)}{4}\rho+\frac{\alpha+\log(1-\alpha)}{\alpha}\varepsilon\right\},\end{aligned} \tag{4.23}$$

1. K. Yamashita, *loc. cit.* 125. (1928).
2. C. F. Eyring, *J. Acous. Soc. Amer.* 1, 228 (1930).

and consequently the rate at which sound energy falls on a unit area of wall surface is given by

$$\sigma=\left\{\frac{-cS\log(1-a)}{4}\rho+\frac{a+\log(1-a)}{a}\varepsilon\right\}\Big/aS$$

$$=\frac{kc\rho}{4}+\frac{a+\log(1-a)}{a^2S}\varepsilon. \qquad (4.24)$$

Equations (4.13), (4,24) differ by a term $\varepsilon\{a+\log(1-a)\}/a^2S$. From this it follows that equation (4.13) may be used as an approximation when the supply of sound is very small.

The foregoing discussion concerns the case where the sound energy is continuously supplied at a constant rate ε, but it is not difficult to assume that equation (4.13) is applicable as an approximation also when the supply ε is changing with the time, provided its value remains small.

From this we may say that Eyring's formulae (4.1), (4.2) are not the exact equations, but approximate ones which are valid only when the communication of sound through the window is small.

V. Reverberation in Two Adjacent Rooms with Different Acoustic Properties

Equations (3.1), (3.2) or (3.17), (3.18) are the decay equations for two adjacent rooms which, besides being equal in shape and size, have the same properties with regard to the reflection and transmission of sound. Let us now consider a case where the two rooms have different properties with regard to the reflection and transmission of sound, though they are equal in shape and size. Assume that the sound-waves are well diffracted in each of the two rooms, and consequently that the second method may be used in estimating the absorption of sound energy by the walls.

Let q_I, q_{II} represent the coefficients of transmission for sound-waves, q_I being the coefficient when the sound-waves pass through the partition from room I to room II and q_{II} the coefficient for the reverse direction. Let P_I represent the arithmetic mean of the coefficients of reflection for the exposed surface areas in room I and P_{II} the corresponding value for room II, the coefficients of reflection of the partition W being included in P_I and P_{II}. Since the two rooms are equal in shape and size, the mean values of the time-intervals between successive incidences for the two rooms are equal. Let τ represent these mean values.

If the initial values of the total sound energies in the two rooms are E_{I0} and E_{II0} respectively, the fraction of E_{I0} remaining in room I at time $t=\tau$ after one reflection is $P_I E_{I0}$ and the energy transmitted to room I from room II in the interval from $t=0$ to $t=\tau$ is $Q_{II}E_{II0}$, where Q_{II} is given by

$$Q_{II}=\frac{s_v}{S_{II}}q_{II},$$

S_{II} being the total area of exposed surface in room II and s_v the area of the partition W. Consequently the total sound energy E_{I1} in room I at time $t=\tau$ is given by the sum of $P_I E_{I0}$ and $Q_{II}E_{II0}$. Similarly the total sound energy in room II at time $t=\tau$ is the sum of $P_{II}E_{II0}$ and $Q_I E_{I0}$, where

$$Q_I=\frac{s_v}{S_I}q_I,$$

and S_I is the total area of exposed surface in room I. Therefore we have

$$\left.\begin{aligned}E_{I1}&=P_I E_{I0}+Q_{II}E_{II0},\\ E_{II1}&=P_{II}E_{II0}+Q_I E_{I0}.\end{aligned}\right\}\qquad(5.1)$$

Similarly the total sound energies in rooms I and II at times $t=2\tau,\ 3\tau,\ldots\ldots n\tau$ are given by the following equations:

$$\left.\begin{aligned}E_{I2}&=P_I E_{I1}+Q_{II}E_{II1},\\ E_{II2}&=P_{II}E_{II1}+Q_I E_{I1};\end{aligned}\right\}\qquad(5.2)$$

$$\left.\begin{aligned}E_{I3}&=P_I E_{I2}+Q_{II}E_{II2},\\ E_{II3}&=P_{II}E_{II2}+Q_I E_{I2};\end{aligned}\right\}\qquad(5.3)$$

....

$$\left.\begin{aligned}E_{In}&=P_I E_{In-1}+Q_{II}E_{IIn-1},\\ E_{IIn}&=P_{II}E_{IIn-1}+Q_I E_{In-1}.\end{aligned}\right\}\qquad(5.4)$$

Therefore we get

$$\left.\begin{aligned}E_{I2}&=(P_I^2+Q_IQ_{II})E_{I0}+(P_IQ_{II}+P_{II}Q_{II})E_{II0},\\ E_{II2}&=(P_{II}^2+Q_{II}Q_I)E_{II0}+(P_{II}Q_I+P_IQ_I)E_{I0};\end{aligned}\right\}\qquad(5.5)$$

$$\left.\begin{aligned}E_{I3}&=(P_I^3+2P_IQ_IQ_{II}+P_{II}Q_IQ_{II})E_{I0}\\ &\quad+(P_I^2Q_{II}+P_IP_{II}Q_{II}+P_{II}^2Q_{II}+Q_IQ_{II}^2)E_{II0},\\ E_{II3}&=(P_{II}^3+2P_{II}Q_{II}Q_I+P_IQ_{II}Q_I)E_{II0}\\ &\quad+(P_{II}^2Q_I+P_{II}P_IQ_I+P_I^2Q_I+Q_{II}Q_I^2)E_{I0}.\end{aligned}\right\}\qquad(5.6)$$

In order to get the sound energy at time $t=n\tau$, let us obtain from (5.4) an equation which does not involve E_{II} but E_I only.

$$E_{In}=(P_I+P_{II})E_{In-1}-(P_IP_{II}-Q_IQ_{II})E_{In-2}. \qquad (5.7)$$

Equation (5.7) is the recurring formula for E_{In} and a similar equation holds for E_{IIn}.

Therefore to get the sound energies E_{In} and E_{IIn}, it is necessary to obtain the solution of a recurring formula of the form :

$$E_n=aE_{n-1}+bE_{n-2} \quad [n=2,\ 3,\ 4,\ldots\ldots\ldots]. \qquad (5.8)$$

Since the coefficients a and b are constants, that is, are the same for all values of n, the solution of (5.8) may be obtained by considering the power series

$$f(x)\equiv E_0+E_1x+E_2x^2+\ldots\ldots+E_nx^n+\ldots\ldots. \qquad (5.9)$$

Multiplying both sides of (5.9) by $ax+bx^2$ and using the relations (5.8), we have

$$(ax+bx^2)f(x)=f(x)-E_0-E_1x+aE_0x,$$

or

$$f(x)=\frac{E_0+(E_1-aE_0)x}{1-ax-bx^2}. \qquad (5.10)$$

As equation (5.9) shows, E_n is the coefficient of x^n and therefore E_n is given by the coefficient of x^n in the expansion of the right-hand side of (5.10), namely

$$E_n=\frac{E_1-\frac{1}{2}(a-\sqrt{a^2+4b}\ )E_0}{\sqrt{a^2+4b}}\ \frac{(a+\sqrt{a^2+4b}\ )^n}{2^n}$$
$$-\frac{E_1-\frac{1}{2}(a+\sqrt{a^2+4b}\ )E_0}{\sqrt{a^2+4b}}\ \frac{(a-\sqrt{a^2+4b}\ )^n}{2^n}. \qquad (5.11)$$

Consequently we get

$$E_{In}=\frac{E_{I1}-\frac{1}{2}(P_I+P_{II}-R)E_{I0}}{R}\ \frac{(P_I+P_{II}+R)^n}{2^n}$$
$$-\frac{E_{I1}-\frac{1}{2}(P_I+P_{II}+R)E_{I0}}{R}\ \frac{(P_I+P_{II}-R)^n}{2^n},$$

where

$$R\equiv\sqrt{(P_I-P_{II})^2+4Q_IQ_{II}}.$$

If the value of E_{I1} given by (5.1) is substituted in the above equation,

E_{I_n} is expressed by an equation involving E_{I0} and E_{II0}, and E_{II_n} may also be written in a similar form. Thus we obtain

$$E_{I_n}=E_{I0}\left\{\frac{P_I-P_{II}+R}{2R}\frac{(P_I+P_{II}+R)^n}{2^n}\right.$$
$$\left.-\frac{P_I-P_{II}-R}{2R}\frac{(P_I+P_{II}-R)^n}{2^n}\right\}$$
$$+E_{II0}\frac{Q_{II}}{R}\left\{\frac{(P_I+P_{II}+R)^n}{2^n}-\frac{(P_I+P_{II}-R)^n}{2^n}\right\}, \quad (5.12)$$

$$E_{II_n}=E_{II0}\left\{\frac{P_{II}-P_I+R}{2R}\frac{(P_{II}+P_I+R)^n}{2^n}\right.$$
$$\left.-\frac{P_{II}-P_I-R}{2R}\frac{(P_{II}+P_I-R)^n}{2^n}\right\}$$
$$+E_{I0}\frac{Q_I}{R}\left\{\frac{(P_{II}+P_I+R)^n}{2^n}-\frac{(P_{II}+P_I-R)^n}{2^n}\right\}, \quad (5.13)$$

where

$$R\equiv\sqrt{(P_I-P_{II})^2+4Q_IQ_{II}}. \quad (5.14)$$

Equations (5.12), (5.13) give the decay of sound in the two adjacent rooms which have equal values for τ, but take different values for P and Q. The second method is used in the above calculation and so the results are valid only to the extent to which the method is justifiable.

Let the sound energy be emitted continuously at a constant rate ε from a source in room I until a steady state is established in both rooms, and let $E_{I0}^{(\infty)}$, $E_{II0}^{(\infty)}$ be respectively the total sound energies in the state. Then the sound energy lost from room I in an interval τ is

$$E_{I0}^{(\infty)}(1-P_I),$$

and this must be equal to the sum of the sound energy $\varepsilon\tau$ emitted from the source and the energy $E_{II0}^{(\infty)}Q_{II}$ received in that interval from room II through the partition. Therefore we have

$$E_{I0}^{(\infty)}(1-P_I)=\varepsilon\tau+E_{II0}^{(\infty)}Q_{II}. \quad (5.15)$$

Similarly for room II we have

$$E_{II0}^{(\infty)}(1-P_{II})=E_{I0}^{(\infty)}Q_I. \quad (5.16)$$

Solving the above two equations, we get

$$E_{I0}^{(\infty)}=\frac{\varepsilon\tau(1-P_{II})}{(1-P_I)(1-P_{II})-Q_IQ_{II}}. \quad (5.17)$$

$$E_{II0}^{(\infty)}=\frac{\varepsilon\tau Q_I}{(1-P_I)(1-P_{II})-Q_IQ_{II}}, \qquad (5.18)$$

which give the total sound energies in the steady state.

If the source is stopped to emit sound after the steady state has been established, the decay equations are obtained by using $E_{I0}^{(\infty)}$ and $E_{II0}^{(\infty)}$ given by (5.17), (5.18) as the values of E_{I0} and E_{II0} in equations (5.12), (5.13).

Consider the growing state of sound in both rooms when the sound energy is emitted continuously at a constant rate ε from a source in room I. Let $E_{I0}^{(m)}$, $E_{II0}^{(m)}$ represent the total sound energies in rooms I and II at time $t=m\tau$ and assume that there is initially no sound in both rooms. Divide the time-interval from $t=0$ to $t=m\tau$ into m equal short intervals and consider the sound energy $\varepsilon\tau$ emitted in the i-th short interval from the beginning. Let $e_I^{(i)}$, $e_{II}^{(i)}$ represent the fractions of that energy $\varepsilon\tau$, which are remaining in rooms I and II at time $t=m\tau$ after $m-i$ incidences. The energies $e_I^{(i)}$, $e_{II}^{(i)}$ can be obtained by equations (5.12) and (5.13). It is assumed that the energy $\varepsilon\tau$ emitted from the source in the i-th short interval does not fall upon any surface before $t=i\tau$, so that at time $t=i\tau$ the energy $\varepsilon\tau$ is left wholly in room I and no fraction is transmitted to room II. Therefore substituting

$$E_{I0}=\varepsilon\tau,\quad E_{II0}=0,\quad n=m-i$$

in equations (5.12) and (5.13), the energies $e_I^{(i)}$, $e_{II}^{(i)}$ are obtained:

$$e_I^{(i)}=\varepsilon\tau\left\{\frac{P_I-P_{II}+R}{2R}\,\frac{(P_I+P_{II}+R)^{m-i}}{2^{m-i}}-\frac{P_I-P_{II}-R}{2R}\,\frac{(P_I+P_{II}-R)^{m-i}}{2^{m-i}}\right\},$$

$$e_{II}^{(i)}=\varepsilon\tau\frac{Q_I}{R}\left\{\frac{(P_{II}+P_I+R)^{m-i}}{2^{m-i}}-\frac{(P_{II}+P_I-R)^{m-i}}{2^{m-i}}\right\}.$$

By summing up $e_I^{(i)}$, $e_{II}^{(i)}$ from $i=1$ to $i=m$, the energies $E_{I0}^{(m)}$, $E_{II0}^{(m)}$ are given by

$$E_{I0}^{(m)}=\varepsilon\tau\left\{\frac{P_I-P_{II}+R}{2R}\,\frac{1-X^m}{1-X}-\frac{P_I-P_{II}-R}{2R}\,\frac{1-Y^m}{1-Y}\right\}. \qquad (5.19)$$

$$E_{II0}^{(m)}=\varepsilon\tau\frac{Q_I}{R}\left\{\frac{1-X^m}{1-X}-\frac{1-Y^m}{1-Y}\right\}, \qquad (5.20)$$

where

$$\left.\begin{aligned} R &\equiv \sqrt{(P_I-P_{II})^2+4Q_IQ_{II}}. \\ X &\equiv \frac{1}{2}\left\{P_I+P_{II}+\sqrt{(P_I-P_{II})^2+4Q_IQ_{II}}\right\}. \\ Y &\equiv \frac{1}{2}\left\{P_I+P_{II}-\sqrt{(P_I-P_{II})^2+4Q_IQ_{II}}\right\}. \end{aligned}\right\} \quad (5.21)$$

Equations (5.19), (5.20) are the growth equations.

In a limiting case where m tends to infinity, equations (5.19), (5.20) become

$$\lim_{m\to\infty} E_{I0}^{(m)} = \varepsilon\tau \frac{1-P_{II}}{(1-P_I)(1-P_{II})-Q_IQ_{II}},$$

$$\lim_{m\to\infty} E_{II0}^{(m)} = \varepsilon\tau \frac{Q_I}{(1-P_I)(1-P_{II})-Q_IQ_{II}},$$

which are identical with equations (5.17), (5.18) in the steady state.

In the special case where the two rooms have the same acoustic properties and P_I, Q_I are equal to P_{II}, Q_{II} respectively, equations (5.12), (5.13), (5.17), (5.18), (5.19) and (5.20) may be written in the following forms:

$$E_{In} = \frac{1}{2}E_{I0}\{(P+Q)^n+(P-Q)^n\} + \frac{1}{2}E_{II0}\{(P+Q)^n-(P-Q)^n\}, \quad (5.22)$$

$$E_{IIn} = \frac{1}{2}E_{II0}\{(P+Q)^n+(P-Q)^n\} + \frac{1}{2}E_{I0}\{(P+Q)^n-(P-Q)^n\}; \quad (5.23)$$

$$E_{I0}^{(\infty)} = \frac{\varepsilon\tau(1-P)}{(1-P)^2-Q^2}, \quad (5.24)$$

$$E_{II0}^{(\infty)} = \frac{\varepsilon\tau Q}{(1-P)^2-Q^2}; \quad (5.25)$$

$$E_{I0}^{(m)} = \frac{\varepsilon\tau}{2}\left\{\frac{1-(P+Q)^m}{1-(P+Q)} + \frac{1-(P-Q)^m}{1-(P-Q)}\right\}, \quad (5.26)$$

$$E_{II0}^{(m)} = \frac{\varepsilon\tau}{2}\left\{\frac{1-(P+Q)^m}{1-(P+Q)} - \frac{1-(P-Q)^m}{1-(P-Q)}\right\}. \quad (5.27)$$

If $E_{I0}^{(\infty)}$ and $E_{II0}^{(\infty)}$ are used as the initial values instead of E_{I0} and E_{II0}, equations (5.22), (5.23) become identical with equations (3.17), (3.18) obtained already by the author. Expanding (5.26), (5.27) in power series of Q, we get

$$E_{I0}^{(m)}=\varepsilon\tau\left\{\frac{1-P^m}{1-P}+Q^2\sum_{j=2}^{m-1}{}_jC_2P^{j-2}+Q^4\sum_{j=4}^{m-1}{}_jC_4P^{j-4}+\dots\right\}, \quad (5.28)$$

$$E_{II0}^{(m)}=\varepsilon\tau\left\{Q\sum_{j=1}^{m-1}{}_jC_1P^{j-1}+Q^3\sum_{j=3}^{m-1}{}_jC_3P^{j-3}+\dots\dots\right\}. \quad (5.29)$$

These are the decay equations, expressed in power series of Q, for two adjacent rooms with the same acoustic properties and they are the same as the equations already obtained by the author[1].

The writer wishes to express his sincere thanks to Professor K. Tamaki for the interest he has taken in the investigation, and for his valuable advice.

1. K. Yamashita, *loc. cit.* 130–131 (1928).