Young's Modulus of Aluminium Rod composed of Large Crystal Grains

By

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Abstract

Metals composed of large crystal grains are generally very soft. The writer measured Young's modulus of aluminium rods composed of the crystal grains of various sizes by the methods of elongation-testing and of acoustical vibration. It was found that the value of Young's modulus of aluminium remained nearly the same irrespective of the sizes of the crystal grains, but that the limit of elasticity decreased considerably with the growth of the crystal grains.

Experimental Method

A cylindrical rod of the required cross-section was cut out from a commercial aluminium rod. Then it was put into an electric furnace and heated for about 80 hours at about 280°C. After the furnace was slowly cooled, the rod was taken out, and after being elongated by about 1.5% was cut to the required length.

It is known that, when a *free-free* rod of the length l vibrates laterally with two nodes, the positions of the nodes are approximately at a distance of $0.224 \times l$[1] from the ends. So the rod under examination was suspended horizontally with two strong strings at these nodes, and the middle part of the rod was tapped horizontally and normally to its length-wise direction by a padded hammer. The sound set up by this method was recorded on a photographic film by an audiometer made by the Adam Hilger Company, and then the frequency of the lateral vibration of the rod was measured from the records. One of

1. Rayleigh, *The Theory of Sound*, 2nd Ed., Vol. 1, § 178.

the records is shown in Fig. 1 in the annexed Plate. For the rectangular rod, the writer took the precaution to place one pair of the side surfaces of the rod vertically, so that its vibration took place in the direction perpendicular to them.

The rod was placed again in an electric vacuum furnace, and heated for about 80 hours at the temperature of about 630°C, which is a little lower than the melting point of aluminium. After the furnace was gradually cooled, the rod was taken out, and the frequency of its lateral vibration was again measured as before.

In order to compare the elastic properties of the rod composed of large crystal grains with those of the rod composed of fine crystal grains, another piece of cylindrical rod of required size was prepared, without any heat treatment, from a commercial aluminium rod, and the frequency of its lateral vibration was also measured by the same method as described above.

It is known that, in the case of the lateral vibration of a free-free rod, its frequency N is represented approximately by

$$N=\frac{\varkappa m^2}{2\pi l^2}\sqrt{\frac{E_0}{\rho}}\,.$$ [1]

From this follows

$$E_0=\frac{4\pi^2 l^4 N^2}{\varkappa^2 m^4}\rho.$$

In the above equations E_0 is Young's modulus of the rod, ρ the volume density, l the length, and $\varkappa$ the radius of gyration of a cross-section perpendicular to the axis of the rod, about the axis passing through the centre of the cross-section and perpendicular to the plane of vibration. The values of $\varkappa$ are $\frac{r}{2}$ for a circular rod of the radius r, and $\frac{a}{\sqrt{12}}$ for a rectangular rod of the thickness a; and m takes the value 4.730^2 in the present case.

Young's modulus E_0 was first calculated by the above equation, and the values of E_0 for various rods are shown in the 4th column in Table I. However, Young's modulus, represented by the above equation gives only an approximate value obtained by neglecting the effects of the rotation and the shear of the rod, so that it needs a

1. Rayleigh, *The Theory of Sound*, 2nd Ed., Vol. 1, § 170.
2. Rayleigh, *The Theory of Sound*, 2nd Ed., Vol. 1, § 174.

certain correction resulting from these effects. According to E. Goens[1], if T represents Timoshenco's correction factor, E_0 Young's modulus uncorrected, and E the corrected value corresponding to E_0, then the relation between E and E_0 is given by the following equations :—

$$E=E_0T=E_0\left\{1+\frac{r^2}{l^2}\left(12.37+3.42\frac{E_0}{G}\right)\right\} \quad \text{for a circular rod,}$$

and $$E=E_0T=E_0\left\{1+\frac{a^2}{l^2}\left(4.124+1.231\frac{E_0}{G}\right)\right\} \quad \text{for a rectangular rod,}$$

where G is the elasticity of torsion. In the present research, the writer used 2.69×10^{11} for G, taken as the mean from the values obtained by E. Goens.[2] The values of E thus obtained for various rods are shown in the 5th column in Table I.

After finishing the experiment mentioned above, the specimens were etched slightly in dilute hydrochloric acid, and the average sizes of the crystal grains were estimated. The results are shown in the 3rd column in Table I. For example, the largest crystal grain in the specimens examined in the present experiment had the areal size of about 22×1 sq. cm. The appearances of the etched surfaces of some specimens are reproduced in Fig. 2, in the annexed Plate, in order to show the sizes of the crystal grains, which are different in each specimen.

In order to compare the value of Young's modulus obtained by the method already mentioned above with that obtained by the ordinary elongation method for all the specimens examined before, and to test their limits of elasticity, Martens' extensometer[3] was used. Young's modulus E measured by this apparatus is given, of course, within the limit of elasticity, by the following simple equation :—

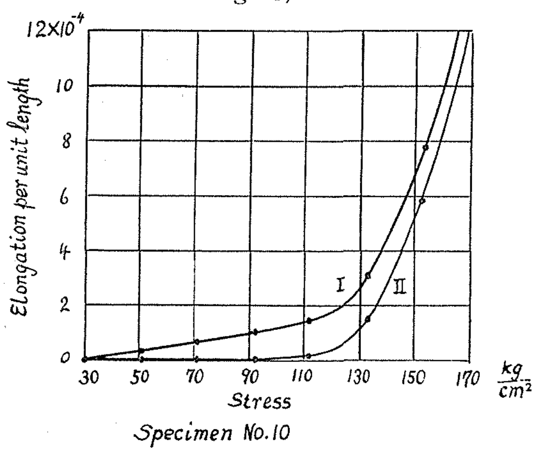
$$E=\frac{W}{A}\Big/\frac{\Delta l}{l},$$

where W is the load applied to the rod, A the cross-sectional area of the rod, and $\frac{\Delta l}{l}$ the elongation per unit length of it.

The following process was made to find out the limit of elasticity and to obtain the stress strain diagram of the rod. In measuring the elongation, when a very small load was initially applied to the rod of known cross-sectional area, the reading of the scale of this apparatus

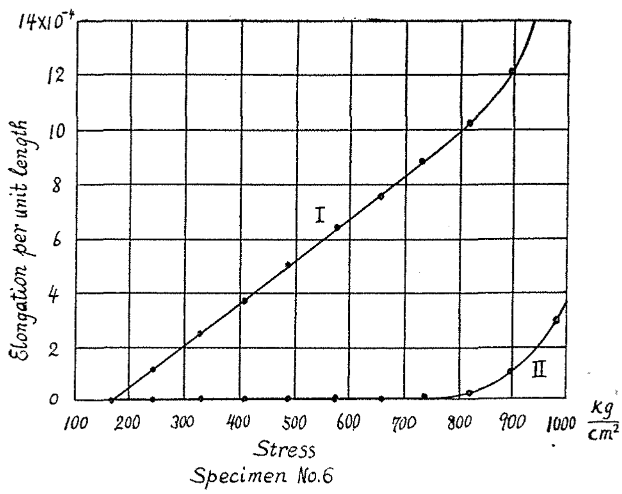
1. Annalen der Physik **11**, 1931
2. Annalen der Physik **17**, 1933
3. *Handbuch der Experimental Physik*, Bd. V, 176, 1930

Fig. 1, a.



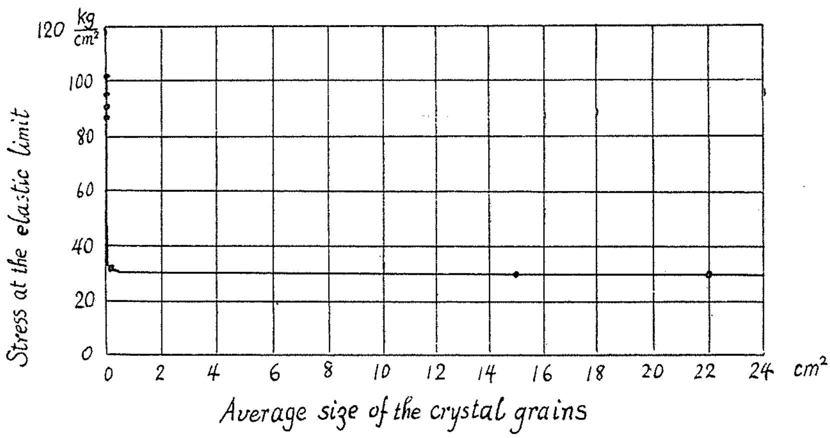
was taken as zero point. Then we took the scale reading by adding a certain load, and again took the scale reading after this adding load was removed. When this process was repeated, by increasing the load more and more, the elongation per unit length increased correspondingly. The relations, thus obtained, between the stress per unit area and the elongation per unit length, were plotted in the curves shown in Fig. 1, by taking the former and the latter as abscissa and ordinate respectively. For the specimen composed of large crystal grains the relation is represented in Fig. 1a, and that of fine ones in Fig. 1b. Curves I in these two figures represent the relation between the applied stress per unit area and the corresponding elongation per unit length, and Curves II the relation between the applied stress per unit area and the permanent elongation per unit length. In Fig. 1a Curve II lies almost on the axis of abscissa up to the point about 90 kg. per sq. cm., which must be regarded as the elastic limit. Similarly Curve II in Fig. 1b shows that about 700 kg. per sq. cm. is the limit of elasticity of the specimen composed of fine grains. As already mentioned, Young's modulus was calculated within these limits of elasticity.

Fig. 1, b.

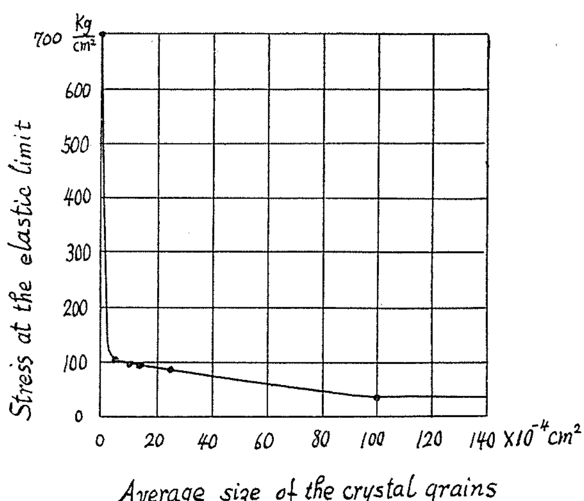


In order to show the relation between the average size of the crystal grain and the elastic limit, curves are roughly plotted in Figs. 2a and

Fig. 2, a.



b.



2b by taking the former as abscissa and the latter as ordinate. The curve shown in Fig. 2b is to show this relation more clearly when the crystal grain is small; and the elastic limit of the fine grained specimen, which is not annealed, is also taken into consideration by assuming the size of its crystal grain to be vanishingly small.

After the measurement of the longitudinal elongation for the various rods was finished, the breaking stress per unit of the initial cross-sectional area for them was measured, the values of which are shown in the last column in the table.

The specimens examined in this study were four aluminium rods composed of fine crystal grains obtained by the method of cold working, and nine aluminium rods composed of large crystal grains which were prepared by the stress-annealing method. The sizes of specimens were between 1.38 cm.–1.10 cm. in diameter, 35.15 cm.–29.25 cm. in length for circular rods, and 1.17 cm.-0.90 cm. in thickness, 1.49 cm.-1.47 cm. in width, 35.10 cm.–34.75 cm. in length for rectangular rods.

Experimental Results

The results obtained by the above two methods are tabulated in Table I.

It may be stated, from the 5th and 6th columns in the table, that the values of Young's modulus obtained by the acoustical vibration coincide with those obtained by the elongation method within the limit of experimental errors. It is found, as seen from the 7th column in the table, that the elastic limit of the specimen composed of fine crystal grains, which was obtained by the cold working method, is greater than that of the specimen composed of large ones, and also that it decreases gradually as the crystal grains become greater. So far as the present experiment is concerned, the minimum value of the elastic limit of aluminium of large crystal grains was about 30 kg. per sq. cm. H. J. Gough and D. Hanson[1] obtained a smaller value of about 10 kg. per sq. cm. As is seen from the curve in Fig. 2a the elastic limit decreases rapidly from the size of about 0.02×0.02 sq. cm. to that of about 0.1×0.1 sq. cm., and for large crystal grains it is almost constant independent of their sizes. It is also found clearly, from the curve in Fig. 2b, that the elastic limit decreases very rapidly when fine crystal grains grow larger, even to a minute degree. It is also evident, from the last column in the table, that the breaking stress per unit initial area of the cross-section of the rod composed of large crystal grains is considerably smaller than that of the fine-grained specimens.

So far it may be said clearly, from the present research, that the

1 *Phil. Trans. of the Royal Society*, **226**, 1926.

Table I

Specimen	State	Average size of the grains sq. cm.	Young's modulus by acoustical vibration uncorr. $E_0 \times 10^{-11}$	Young's modulus by acoustical vibration corrected $E \times 10^{-11}$	Young's modulus by extension $E \times 10^{-11}$	Limit of elasticity. kg. per sq. cm.	Breaking stress per unit initial cross-sectional area. kg. per sq. cm.
Circular rod No. 6	Cold worked		6.72	6.77	6.65	700	1949.2
" No. 15	"		6.78	6.84	6.74	720	2333.5
" No. 16	"		6.74	6.80	6.78	800	2340.7
" No. 17	"		6.69	6.76	6.69	800	2343.4
Circular rod No. 13	Extended by about 1.5% after annealing		6.80	6.86			
"	Recrystallized	about 0.02×0.02	6.75	6.81	6.77	100	772.5
" No. 7	Extended by about 1.5% after annealing		6.75	6.77			
"	Recrystallized	about 0.02×0.02	6.73	6.78	6.62	100	801.5
Rectangular rod No. 6	Extended by about 1.5% after annealing		6.65	6.70			
"	Recrystallized	about 0.03×0.03	6.61	6.66	6.69	100	722.6
" No. 7	Extended by about 1.5% after annealing		6.69	6.74			
"	Recrystallized	about 0.03×0.03	6.65	6.70	6.64	90	691.4
Circular rod No. 8	Extended by about 1.5% after annealing		6.74	6.78			
"	Recrystallized	about 0.04×0.04	6.69	6.74	6.73	90	782.9
" No. 10	Extended by about 1.5% after annealing		6.63	6.67			
"	Recrystallized	about 0.05×0.05	6.68	6.73	6.68	90	756.7
Rectangular rod No. 5	Extended by about 1.5% after annealing		6.66	6.69			
"	Recrystallized	about 0.1×0.1	6.85	6.88	6.66	30	584.4
Circular rod No. 9	Extended by about 1.5% after annealing		6.57	6.62			
"	Recrystallized	about 15×1	6.63	6.68	6.62	30	693 3
" No. 11	Extended by about 1.5% after annealing		6.77	6.82			
"	Recrystallized	about 22×1	6.72	6.76	6.73	30	709.2

value of Young's modulus of aluminium rod composed of fine crystal grains is the same as that of the rod composed of large crystal grains, which are prepared by the stress-annealing method.

Lastly in the case of the free-free rod the maximum amplitude in the lateral vibration with two nodes was measured by a microscope, and from this value the maximum longitudinal elongation per unit length of the rod was found to be of the order of 10^{-7} by a simple calculation. The longitudinal elongation per unit length of the rod corresponding to the stress at the elastic limit 30 kg. per sq. cm., which is the smallest value obtained in the present experiment for the specimens composed of large crystal grains, is very much larger than the value obtained above, and takes the value of the order of 10^{-5}. Thus it is evident that maximum longitudinal elongation of the rod in the case of lateral vibration, was very far below the elastic limit. Consequently it is found that the acoustical vibration method is very much preferable to the elongation method in measuring Young's modulus of the metal rod composed of large crystal grains, as it least disturbs the perfectness of the metal crystal.

In conclusion the writer wishes to express his sincere thanks to Professors K. Tamaki and U. Yoshida for their kind advice and invaluable suggestions, and to Messrs. K. Yamashita, I. Aoki, and T. Watanabe for their generous aid, without which this research could not have been accomplished.

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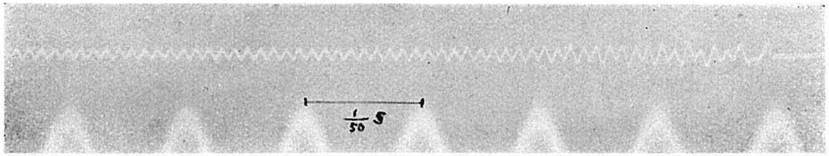
Plate

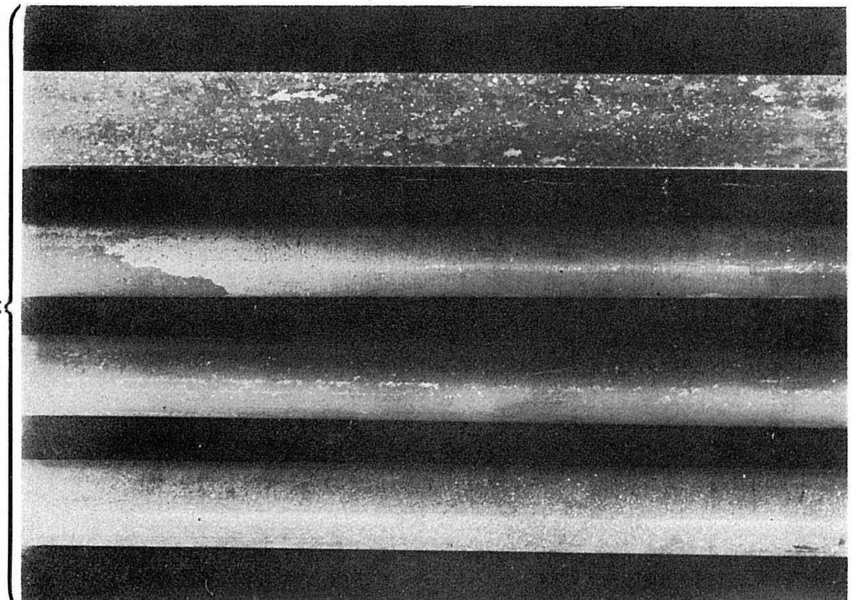
Fig. 1.



Specimen

Circular rod No. 8

Fig. 2.



0.77 ×

Specimens

Rectangular rod No. 5
Average size of crystal grains
about 0.1 × 0.1 cm.²

Circular rod No. 9
Average size of crystal grains
about 15 × 1 cm.²

Circular rod No. 11
Average size of crystal grains
about 22 × 1 cm.²

Circular rod No. 13
Average size of crystal grains
about 0.02 × 0.02 cm.²