Graphical Computation of Jupiter's Perturbation on Asteroids[1]

By

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Abstract

For the computation of Jupiter's perturbation on Asteroids, a graphical method is recommended. It promises the computer easiness and sufficient accuracy for the derivation of data referred to the geometrical relation between Jupiter and Asteroids. The scale of the graph is a moderate one such that the orbit of Jupiter just fits on a sheet of typewriter paper.

Subsequent computation may be promptly carried out with a slide rule.

The coefficients of orbit improvement are also derived by a graphical method.

The incessant increase of the number of asteroids has compelled the "Astronomischen Rechen-Institut" at Berlin, Germany, to ask voluntary collaborations for the computation of Jupiter's perturbation on Asteroids. And Dr. Stracke of the Rechen-Institut has published instructions[2] for the approximate calculation of special perturbation and for the improvement of orbital elements.

His method is that of the Variation of Elements which is generally treated in text books of theoretical astronomy, but Dr. Stracke has attained much simplification by using only three figure logarithms.

In view of the simplification, the computation also can be done with much easiness and moreover with sufficient accuracy by getting the necessary data for calculation with *graphical method* and by using a slide rule for the later computation. The scale of the graph may be a moderate one, such that the orbit of Jupiter can be graphed on a single sheet of typewriter paper.

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1. An abstract of a part of this paper was read at the Annual Meeting of the Mathematico-Physical Society of Japan, Kyoto, 1931 October.

2. G. Stracke: Veröffentlichungen des Astronomischen Rechen-Instituts zu Berlin, Nr. 44.

The weak point of the graphical method generally lies in its inaccuracy, but here the required accuracy is within its limit. And without this defect, it is much superior to numerical calculation in a few points, especially where the geometrical transformation of coordinates is concerned. Of all merits of the graphical method, the fact that it is carried out in parallel, so that accidental mistakes are to be detected easily, should receive special mention.

In the following treatment the same notations are used as those given in Dr. Stracke's paper.

Elements:

M	Mean Anomaly	E	Eccentric Anomaly
M_0	Mean Anomaly at the instant t_0	v	True Anomaly
ω	Argument of Perihelion	r	Radius vector
Ω	Longitude of Ascending Node	p	Parameter of Orbit
i	Inclination of Orbit to the Ecliptic	u	Argument of Latitude
		L	Mean Longitude
φ	Angle of Eccentricity	π	Longitude of Perihelion
μ	Mean daily motion	k	Gauss's constant in degree
a	Semi-major axis	w	Time interval

λ_1 Heliocentric longitude of Jupiter

β_1 ,, latitude ,, ,,

r_1 Radius vector of Jupiter

m_1 Mass of Jupiter in unit of Sun's mass

S, T, W the components of the perturbing force

Then we have,

1) $$M = M_0 + \mu(t - t_0) = L - \pi$$

2) $$E - \sin\varphi \sin E = M$$

3) $$\begin{cases} r \sin v = a\cos\varphi \sin E \\ r \cos v = a(\cos E - \sin\varphi) \end{cases}$$

4) $$a = \left(\frac{k}{\mu}\right)^{\frac{2}{3}}$$

5) $$u = v + \omega = v + \pi - \Omega$$

6) $$p = a \cos^2\varphi$$

7) $$\begin{cases} \cos B_1 \cos L_1 = \cos\beta_1 \cos(\lambda_1 - \Omega) \\ \cos B_1 \sin L_1 = \sin i \sin\beta_1 + \cos i \cos\beta_1 \sin(\lambda_1 - \Omega) \\ \sin B_1 \qquad = \cos i \sin\beta_1 - \sin i \cos\beta_1 \sin(\lambda_1 - \Omega) \end{cases}$$

8) $$\begin{cases} \xi_1 = r_1 \cos B_1 \cos(L_1 - u) \\ \eta_1 = r_1 \cos B_1 \sin(L_1 - u) \\ \zeta_1 = r_1 \sin B_1 \end{cases}$$

9) $$\rho_1^2 = r_1^2 + r^2 - 2r\xi_1$$

10) $$K = \frac{wkm_1}{\sqrt{p}}\left(\frac{1}{\rho_1^3} - \frac{1}{r_1^3}\right)$$

11) $$\begin{cases} S = K\xi_1 - \dfrac{wkm_1}{\sqrt{p}} \dfrac{r}{\rho_1^3} \\ T = K\eta_1 \\ W = K\zeta_1 \end{cases}$$

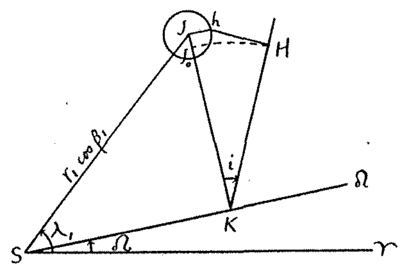
12) $$\begin{cases} w\dfrac{di}{dt} = r\cos u.\ W \\ w\dfrac{d\Omega}{dt} = \dfrac{r\sin u}{\sin i}.\ W \\ w^2\dfrac{d\mu}{dt} = -\dfrac{3kw}{\sqrt{a}}\sin\varphi \sin v.\ S - \dfrac{3kw}{\sqrt{a}}\dfrac{p}{r}.\ T \\ w\dfrac{d\pi}{dt} = -\dfrac{p}{\sin\varphi}\cos v.\ S + \dfrac{p+r}{\sin\varphi}\sin v.\ T + \tan\dfrac{i}{2} r \sin u.\ W \\ w\dfrac{dL}{dt} = -p\tan\dfrac{\varphi}{2}\cos v.\ S - 2\cos\varphi\ r.\ S \\ \qquad + \tan\dfrac{\varphi}{2}(p+r)\sin v.\ T + \tan\dfrac{i}{2} r\sin u.\ W \\ w\dfrac{d\varphi}{dt} = a\cos\varphi \sin v.\ S + a\cos\varphi\left[\sin\varphi + \cos v\left(1 + \dfrac{r}{a}\right)\right].\ T \end{cases}$$

Relative Positions of Jupiter and the Asteroid

We take the orbital plane of the asteroid as the fundamental plane of reference. Then the position of Jupiter can be represented by a point of orthogonal projection to the plane and the elevation above it.

For a while, considering the plane to be the ecliptic (in Fig. 1), let S be the position of the Sun, and a line $S\Upsilon$ be in the direction of the Vernal equinox. Draw a line SJ, making an angle λ_1 with the initial line $S\Upsilon$, and measure a curtate radius $r_1 \cos\beta_1$ from S to J, then J is the projected position of Jupiter in the ecliptic, and it should be understood that Jupiter lies just above that point as far as $r_1 \sin\beta_1$. If we describe a small circle h with a radius $r_1 \sin\beta_1$ about the point J, the elevation above the ecliptic (or the depression according to the case) may be clearly represented. For the sake of convenience of operation, $r_1 \cos\beta_1$ and $r_1 \sin\beta_1$ (in natural numbers) should be tabulated beforehand.

Fig. 1



Next, draw the nodal line $S\Omega$ passing through the Sun, making an angle Ω with $S\Upsilon$, then we notice that the part of Jupiter's orbit, or strictly speaking, the part of the ecliptic which lies on the side of this line into which the asteroid passes the Ascending node, is below the fundamental plane. If we draw a normal JK from the point J to $S\Omega$, intersecting at K with it, we have

$$13)\qquad \begin{cases} JK = r_1 \cos\beta_1 \sin(\lambda_1 - \Omega) \\ SK = r_1 \cos\beta_1 \cos(\lambda_1 - \Omega). \end{cases}$$

Now let Jh be a radius of the small circle above mentioned, drawn parallel and in the same or opposite direction with $S\Omega$, according to whether Jupiter lies above or below the ecliptic, i. e. $r_1 \sin\beta_1$ is positive or negative.

Make a base line KH through the foot K and at an angle i with the line JK in the clockwise direction, and then draw a line perpendicularly to the base line KH from the point h. If we denote the intersection point with H, then we will see from the figure 1 that the distance KH is equal to $JK \cos i + Jh \sin i$, or

$$14)\qquad KH = r_1 \cos i \cos\beta_1 \sin(\lambda_1 - \Omega) + r_1 \sin i \sin\beta_1$$

and also that the perpendicular distance hH is equal to $JK \sin i - Jh \cos i$. And it will easily be seen that this distance is the depression of Jupiter under the fundamental plane, but since we measure the height of Jupiter above the fundamental plane positively, we must change the sign and we have

$$15)\qquad hH = r_1 \cos i \sin\beta_1 - r_1 \sin i \cos\beta_1 \sin(\lambda_1 - \Omega).$$

In the graphical method, the elevation or depression can be easily visualized by measuring the height from H to h positively towards S.

Now, if we describe a small arc with a radius KH and its centre at K, cutting the line KJ at a point J_0, then the point J_0 represents the projected position of Jupiter on the fundamental plane. And join J_0 to S, then the angle $\Omega S J_0$ is nothing but the longitude in the fundamental plane. If we compare the equations (13), (14) and (15) with the equations (7), we will find easily that the angle has to be designated by L_1 and also that

$$SJ_0 = r_1 \cos B_1$$

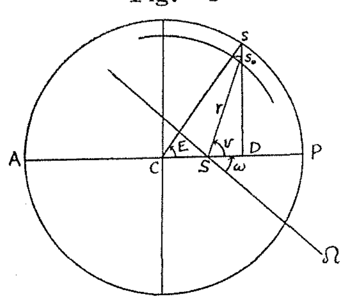
$$hH = r_1 \sin B_1.$$

The position of the asteroid also can be drawn graphically. For that purpose, first compute the mean anomaly M for an epoch t by the formula (1) and then require the eccentric anomaly E, where it is sufficient to get E as accurate as one-tenth of a degree.

This process may be carried out very conveniently by the use of the Keplerian which gives the mechanical solution of Kepler's equation[1].

Now draw the apside line ASP (Fig. 2) through the Sun, making an angle ω with the nodal line $S\Omega$, and denote the Perihelion point and the Aphelion point by P and A which are distant from S by $a(1-\sin\varphi)$, $a(1+\sin\varphi)$ respectively.

Fig. 2



Then if we describe a circle with AP as its diameter, we can get the position of the asteroid in the following manner.

Take a point s on the periphery of the circle PsA, subtending an angle PCs at its centre C equal to E, and draw a normal to the apside line ASP passing the point s, and denote the foot of the normal by D. Now take a point of s_0 in the line sD dividing the normal in the ratio such that $sD : s_0D = 1 : \cos\varphi$, then s_0 is the true position of the asteroid in the orbit. Consequently, the distance Ss_0 is the radius vector r, and the angle PSs_0 is the true anomaly v.

$$\begin{cases} s_0D = r \sin v \\ SD = r \cos v \end{cases}$$

1. M. N. Vol. **87** p. 207.

and the angle measured from $S\Omega$ should be denoted by u.

Necessary data for the computation

In the figure 3, if we get a line J_0Q passing through the point J_0 perpendicular to the line Ss_0, then we have

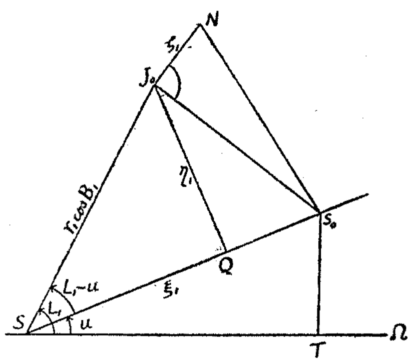
$$16)\qquad \begin{cases} SQ = SJ_0\cos(L_1-u) = r_1\cos B_1\cos(L_1-u) \\ J_0Q = SJ_0\sin(L_1-u) = r_1\cos B_1\sin(L_1-u). \end{cases}$$

And remembering the equations (8), we have the following relations:

$$\begin{cases} \xi_1 = SQ \\ \eta_1 = J_0Q \\ \zeta_1 = hH \end{cases}$$

where ξ_1 must be measured positively towards s_0; and η_1 is positive when J_0 proceeds s_0 in longitude i. e. J_0 lies in the left side of the direction Ss_0; about the sense of ζ_1, it has been already stated. To get the distance between Jupiter and the asteroid, first join the points J_0 and s_0, then the distance J_0s_0 expresses the projected distance in the fundamental plane. Now measure a distance J_0N equal to hH in (15) from J_0 perpendicularly to the line J_0s_0 and join the points N and s_0, then Ns_0 is the required distance ρ_1. Because it enters into the computation by the form of inverse cube, it is convenient to have a table of the inverse cube with the argument of ρ_1.

Fig. 3



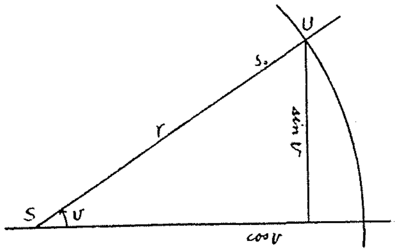
Other values necessary for the computation can be arrived at in the following manner:

Drawing a perpendicular s_0T from the point s_0 to the nodal line, measure the length of the perpendicular s_0T, and the distance from the Sun to the foot of the normal, ST; they give $r\sin u$ and $r\cos u$ respectively.

s_0T is positive when s_0 lies in the left side of the direction $S\Omega$, and ST must be measured positively along the direction of $S\Omega$.

Next, (Fig. 4) draw a circle around S with a unit radius, then it will cut the line Ss_0 or its elongation at U.

Fig. 4



Then referring to the apside line, measure the normal and its offset in exactly the same way. They are nothing but $\sin v$ and $\cos v$ respectively.

Formulae of Perturbation

The formulae of perturbation (12) are rearranged for the convenience of the computation as follows; the modified form will be more profitable for the use of the slide rule, provided that the value of φ is not especially small.

$$K=\frac{wkm_1}{\sqrt{p}}\left(\frac{1}{\rho_1^3}-\frac{1}{r_1^3}\right) \qquad wkm_1=75.28 \text{ for } w=80$$

$$\begin{cases} S=K\xi_1-\dfrac{wkm_1}{\sqrt{p}}\dfrac{r}{\rho_1^3} \\ T=K\eta_1 \\ W=K\zeta_1 \end{cases}$$

$$\begin{cases} \delta i=r\cos u.\ W \\ \delta\Omega=\dfrac{r\sin u}{\sin i}.\ W \end{cases}$$

$$k_2=41.29$$

$$\begin{cases} 80\delta\mu_1=-\dfrac{k_2\sin\varphi}{\sqrt{a}}\sin v.\ S \\ 80\delta\mu_2=-\dfrac{k_2 p}{\sqrt{a}}\dfrac{1}{r}.\ T \end{cases} \qquad \begin{cases} \delta\varphi_0=-\dfrac{1}{\tan\varphi}r.\ T \\ \delta\varphi_1=-\dfrac{a^{\frac{3}{2}}}{k_2\tan\varphi}.\ 80\delta\mu_1 \\ \delta\varphi_2=-\dfrac{a^{\frac{3}{2}}}{k_2\tan\varphi}.\ 80\delta\mu_2 \end{cases}$$

$$\begin{cases} \delta\pi_1=-\dfrac{p}{\sin\varphi}\cos v.\ S \\ \delta\pi_2=\dfrac{1}{\sin\varphi}(p+r)\sin v.\ T \\ \delta\pi_3=2\sin^2\dfrac{i}{2}.\ \delta\Omega \end{cases} \qquad \begin{cases} \delta L_0=-2\cos\varphi\ r.\ S \\ \delta L_1=2\sin^2\dfrac{\varphi}{2}.\ \delta\pi_1 \\ \delta L_2=2\sin^2\dfrac{\varphi}{2}.\ \delta\pi_2 \\ \delta L_3=\delta\pi_3 \end{cases}$$

Example

Here is given an example of computation. The scale of the graph was such that the astronomical unit was represented in 2 centimeters, so the orbit of Jupiter fits on to a sheet of typewriter paper. The angles were measured with a celluloid protractor of ordinary size of 7 centimeters radius, consequently E and λ_1 were plotted within one tenth of a degree of accuracy.

For the purpose of comparison with Dr. Stracke's method, the same example as given in the "Veröffentlichungen" was taken:

(694) Ekard

$t_0 = 1909$ Dec. 3.5 U. T.

$M_0 = 46^\circ.350$

$$\left.\begin{aligned} \omega &= 108.243 \\ \Omega &= 231.414 \\ i &= 15.756 \end{aligned}\right\} 1910.0$$

$\varphi = 18.867$

$\mu = 813''.30 = 0^\circ.22592$

$\log a = 0.42650$

and the perturbation of Jupiter was calculated for the same interval, from the initial epoch up to 1922 Dec. 15.5 U. T.

The computation scheme follows in the pages 56–60, where we find it convenient to arrange the data in columns instead of in rows. Similarly to the example given by Dr. Stracke, the starting elements were not used all through the whole interval, from 1910 Jan. 12.5 to 1922 Dec. 15.5, but the elements were improved twice, namely first on 1915 Jan. 26.5 and next on 1920 Feb. 9.5. The corrections of the elements are calculated in the following way:

$$\Delta M = \Delta L_\mu + \Delta L - \Delta\pi$$

$$\Delta\omega = \Delta\pi - \Delta\Omega$$

$$M = M_0 + \mu^0(t - t_0) + \Delta M$$

$$\Delta a = -\frac{a}{120}\,\frac{80\Delta\mu^0}{\mu^0}$$

If we can use the summation table instead of the integration, then the value of ΔL_μ, corresponding to the first date of computation, will be

$$\varDelta L_{\mu}(a) = {}^{II}F(a) + \frac{1}{12}F(a) - \frac{1}{240}F^{II}(a) + \ldots\ldots\ldots$$

and since $${}^{II}F(a) = \frac{1}{24}F(a-w) + \ldots\ldots\ldots$$

$$= \frac{1}{24}F(a) \text{ nearly,}$$

then $$\varDelta L_{\mu}(a) = \frac{1}{8}F(a).$$

Comparison with the Observations

For the comparison with the observations, we can get the ephemerides with a graphical method. But in this case the graphical method is a little short, and a larger scale should be used to obtain more accuracy. It may not be out of place here to show the graphical method of getting the Right Ascension and Declination. Moreover, for the subsequent computation of the coefficients of the improvement of orbit, we can again take advantage of the graphical computation.

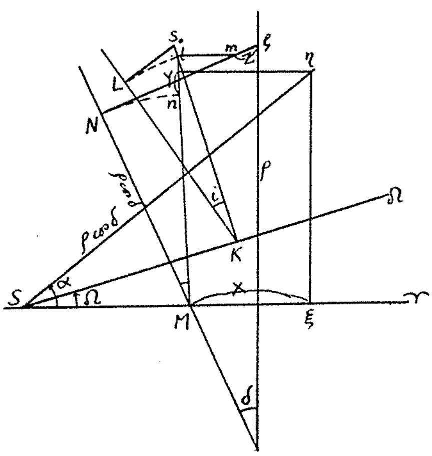
First, lay out a radius vector Ss_0 from the Sun S (Fig. 5) subtending an angle $u=v+\omega$ with the Nodal line $S\Omega$, and draw a normal s_0K to the line $S\Omega$ from the position of the asteroid s_0.

Then measure an angle i at K in the counter-clockwise direction and drop a normal from s_0 to the base line, let the foot be denoted by L. If we take a point l on the line Ks_0, such as

$$Kl = KL$$

then the point l is the projected position of the asteroid in the plane of the ecliptic. From l, draw a line lM perpendicularly to the equinoctial line $S\Upsilon$, and, in turn, another line lm perpendicularly to lM, and make $lm = Ls_0$.

Fig. 5



lm is to be drawn in the direction of $S\Upsilon$, when s_0 is in the side of Ω referred to LK, and if otherwise to the opposite.

Next, pass a line MN through M, making an angle ε in the counter-clockwise direction with Ml, then if we draw a normal from the point m to the line intersecting at N with it, we have

$$\begin{cases} x=SM \\ y=MN \\ z=Nm. \end{cases}$$

SM is measured as positive when M lies in the direction $S\Upsilon$, MN is positive when N lies in the lefthand side of the direction from S to Υ. Nm is positive when m is in the same side with Υ referred to MN.

When we add X, Y, and Z to these values of x, y and z respectively, we get $\rho\cos\delta\cos\alpha$, $\rho\cos\delta\sin\alpha$, $\rho\sin\delta$, and the subsequent process to get α, δ and ρ is easily seen from the figure.

Computation of the factors of "Bahnverbesserung".

When we are given with $t-t_0$, M, E and the orbital elements, we get graphically r, $\sin v$, so we can compute the coefficients l, m, n, and o with a slide rule by the following formulae:

$$\begin{cases} l=\dfrac{a\cos\varphi\dfrac{a}{r}(t-t_0)}{3600}\dfrac{1}{\rho}=\dfrac{t-t_0}{3600}o \\ m=\dfrac{(p+r)\sin v}{\cos\varphi}\dfrac{1}{\rho} \\ n=\dfrac{r}{\rho} \\ o=a\cos\varphi\dfrac{a}{r}\dfrac{1}{\rho} \end{cases}$$

$\sin F$ and $\cos F$ which are necessary for the computation of the absolute term can be derived graphically by the following process:

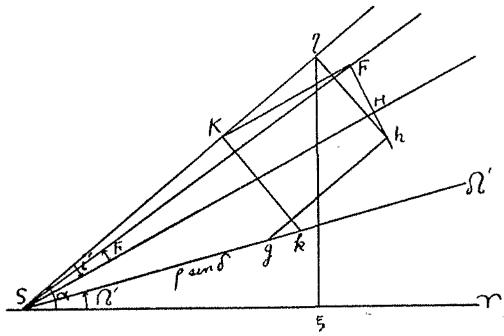
In Fig. 6, taking a point g on $S\Omega'$, distant from the Sun by $\rho\sin\delta$, close a rectangle with $g\eta$ as its diagonal and with a part of $S\eta$ as one side. Denote the angular point by h. Here we must be careful to measure the length Sg in the opposite direction when $\rho\sin\delta$ is negative. Next, pass a base line through S, making an angle i' with $S\eta$ in the clockwise direction, and project the point h to the base line, then the offset SH of the normal gives

$$\rho\cos i'\cos\delta+\rho\sin i'\sin\delta\sin(\alpha-\Omega')$$

which is expressed by $\rho\cos g\cos F$.

Again, taking a point k on $S\Omega'$, distant from the Sun by ρ, drop a normal kK to the line $S\eta$ through k. Subsequently, make a line parallel to SH through the point K and let it intersect with the line hH or its elongation. If we denote the intersection point with F, then the angle HSF is the required angle F. F is measured positively in the counter-clockwise direction from SH.

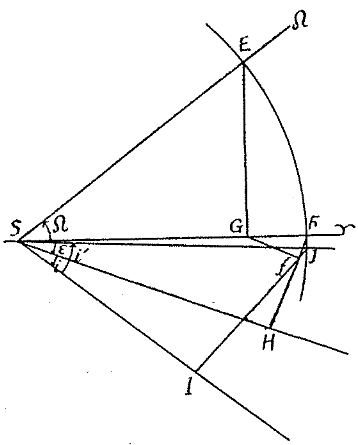
Fig. 6



The process may be carried out as the continuation of the preceding article. In that occasion, we must be careful to distinguish Ω', i' from Ω, i, the former set of which are the Right Ascension of the Ascending Node and Inclination of the orbit referred to the equator and the latter are those referred to the ecliptic.

To correct the position of the nodal line, the best way is first to find i', the inclination of the orbit to the equator.

Fig. 7



Describe an arc of circle of a unit radius, having its centre at S, and cut the nodal line $S\Omega$ and the Vernal line $S\Upsilon$ at E and F respectively. Next draw a normal through E to the Vernal line $S\Upsilon$, and let it be intersected with it at G.

Then subtend two lines SH, SI, making angles ε, $\varepsilon+i$ respectively with $S\Upsilon$ at S. Now drop a normal through F to the line SH intersecting at H, and drawing a line through G and parallel to SH, cut FH at f.

And subsequently, a normal to the line SI through f is to be drawn.

Then SI gives $\cos i'$, so if we cut the line If or its elongation at J with a unit radius from S, then the angle JSI is equal to i'.

Referring to the same diagram, but to Fig. 8, draw a line GK

through the point G making an angle ε with GE. Drop a normal from the point E to GK, intersecting at K. If we draw a line EN through E, making an angle $90-i'$ with the normal EK to the side of G, then it will cut the base line GK or its elongation at N. Then NK gives the equatorial distance (say) between the nodes of the equator and of the ecliptic. The alternative to get the length NK is to obtain the base line corresponding to a side, equal to EK, which subtends the angle JSI in Fig. 7.

The next step is thus: cut the normal GE at L with a radius equal to GK, and its centre at G. Then describe a circle, having SL as its diameter, and cut off a chord LM equal to KN, in the side nearer to ♈, then the line joining M and S gives the position of the nodal line in the equator. It should be noted here that in such a case where the normal from E to the equinoctial line does not meet with it in the branch of S♈, i. e. it falls on the Autumnal line S♈$'$, then substitute the diametrical line of the nodal line, and proceed in the same way as above, except that SI (in Fig. 7) is to be laid so as to make an angle $\varepsilon-i$, instead of $\varepsilon+i$.

Following remarks should be given here, to cover special cases.

a) Although it is not generally the case for asteroids to have a large value of i, there are sometimes cases where i is so large that the normal from the point f (Fig. 7) does not meet with the line SI, then substitute the diametrical line of SI.

b) And when the normal from the point f to the line SI or its substitute, is averted from SH line, then we should measure the chord LM in the semi-circle of the further side with respect to ♈.

c) With regard to the sense of the required nodal line, it should be noted that when Ω lies in the first two quadrants, then Ω' is to be taken either in the first or in the second quadrant, and when Ω lies in the last two quadrants, then Ω' is in the third or in the fourth quadrant.

d) When NK is equal to LS in length, the chord LM coincides with the diameter of the circle, and the position of the nodal line seems to be indeterminate, but it means that the nodal line is to be laid perpendicularly to the line LS. And when it is nearly the case, a small error in the length NK will affect the direction of Ω' very much, so it is recommended to use another method for an alternative. The process is thus: In Fig. 7 and 9, close a rectangle with F, f and G, and denote the new angular point by g, and let the line SH be elongated to intersect with the unit circle at h, then draw two normals gP, hQ to SI through the points g, h respectively.

If we cut fragments Sp in $S\Upsilon$, and Sq in $S\Omega$, respectively equal to gP and hQ, then the point o which is given by the intersection of the normal and the parallel to SF through p and q, will give the direction Ω', by joining S, o. Sq should be measured always towards Ω, but Sp is measured towards Υ for the value of i, as far as the line SI or its diametrical line passes the point g, starting from the line SH, and for the larger value of i, Sp is measured to the opposite.

Fig. 8

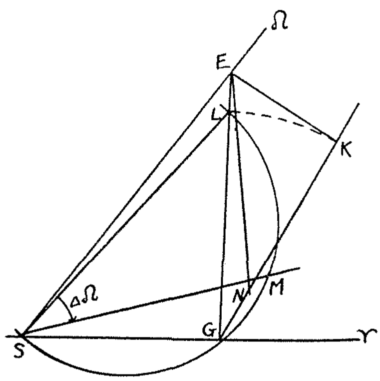
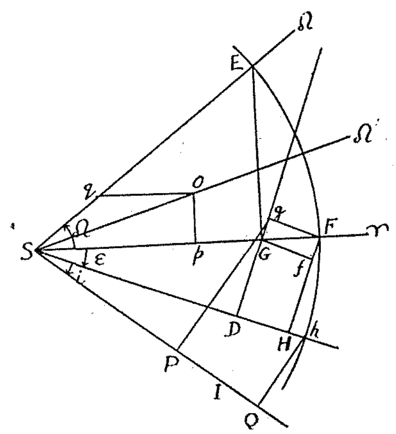


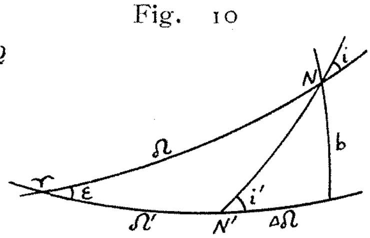
Fig. 9



The above process will be approved by geometrical consideration, but may be verified in the following way.

Referring to the figure 10, we have the trigonometrical relations between these spacial elements as follows:

Fig. 10



$$
17)\quad \begin{cases} \sin i' \sin\Omega' = \sin i \sin\Omega \\ \sin i' \cos\Omega' = \cos i \sin\varepsilon + \sin i \cos\varepsilon \cos\Omega \\ \cos i' = \cos i \cos\varepsilon - \sin i \sin\varepsilon \cos\Omega \\ \sin\Delta\Omega = \tan b \cot i' \\ \sin b = \sin\Omega \sin\varepsilon \end{cases}
$$

When we refer to Figure 7

$$\Upsilon S\Omega = \Omega$$

and as SE, SF were taken as unit length,

$$SG = \cos\Omega$$
$$fH = SG\sin\varepsilon = \cos\Omega\sin\varepsilon$$
$$SH = SF\cos\varepsilon = \cos\varepsilon$$
$$SI = SH\cos i - fH\sin i$$
$$= \cos i\cos\varepsilon - \sin i\sin\varepsilon\cos\Omega,$$

so, $SI = \cos i'$, and since $SJ = 1$

$$\angle JSI = i'.$$

And now we have, in Fig. 8

$$EG = \sin\Omega$$
$$EK = EG\sin\varepsilon = \sin\Omega\sin\varepsilon$$
$$KN = EK\cot i' = \frac{\sin\Omega\sin\varepsilon}{\tan i'} = LM$$
$$GK = GL = EG\cos\varepsilon = \sin\Omega\cos\varepsilon$$
$$SG = \cos\Omega$$

Consequently, $SL^2 = SG^2 + GL^2 = \cos^2\Omega + \sin^2\Omega\cos^2\varepsilon$
and

$$1 - SL^2 = 1 - \cos^2\Omega - \sin^2\Omega\cos^2\varepsilon = \sin^2\Omega\sin^2\varepsilon,$$

that is equal to $\sin^2 b$ from the trigonometrical relation (17).
Thence, we have $SL = \cos b$.

In the rectangular triangle LSM,

$$\sin LSM = \frac{LM}{SL} = \frac{\sin\Omega\sin\varepsilon}{\tan i'}\,\frac{1}{\cos b} = \frac{\tan b}{\tan i'}$$

or equal to $\sin\Delta\Omega$ from the same trigonometrical relation (17).

Consequently, we have

$$\angle LSM = \Delta\Omega$$

and SL is the projection of the nodal line $S\Omega$ in the ecliptic to the plane of the equator, so we can see ΥSM corresponds to Ω'.

In Fig. 9,

$$gD = FH = SF\sin\varepsilon = \sin\varepsilon$$
$$SD = SG\cos\varepsilon = \cos\Omega\cos\varepsilon$$
$$gP = gD\cos i + SD\sin i = \cos i\sin\varepsilon + \sin i\cos\varepsilon\cos\Omega,$$

so, $gP = \sin i'\cos\Omega'$ from the equation (17).

$$Sq = hQ = Sh\sin i = \sin i$$
$$op = Sq\sin\Omega = \sin i\sin\Omega = \sin i'\sin\Omega'$$

consequently, we have

$$\angle oSp = \Omega'.$$

Example for the Computation of the Coefficients

Thorough comparison of the computation for all the dates with that given by Dr. Stracke seems to be unnecessary. Here we show the comparison of both the methods for a single date,

1913 Aug. 2.0

for which,

$t-t_0$	$1337^d.5$	
M	$348°.78$	
ω	108.00	1913.0
Ω	231.44	1913.0
i	15.75	1913.0
φ	18.83	
a	2.67	
ε	$23°.45$	

are given.

We compute $b=2.53$ $p=2.39$ $ae=0.86$ $E=-16°.4$

and by the graphical construction, we read out the necessary data for computation as follows:

		Dr. Stracke's computation
r	1.84	1.845
$\sin v$	− .389	− .387
x	+1.284	+1.2857
y	−1.316	−1.3213
z	− .032	− .0294
α	$317°.3$	$317°.18$
δ	+18.0	+17.88
ρ	.920	.925

and the subsequent computation with a slide rule gives,

l	+1.48	+1.47
m	−1.89	−1.87
n	+2.00	+2.00
o	+3.99	+3.96

With regard to F, Dr. Stracke accidentally mistook Ω, i as the elements referred to the ecliptic instead of as those referred to the equator.

But for the purpose of comparison, the same values of Ω, i were used, resulting the following values

		Dr. Stracke's
$\sin F$	+ .020	+ .021
$\cos F$	+1.00	+1.000

Kyoto University Observatory, October, 1934

Tables

Data for the construction of the graph

Orbital constants		0.5 U. T.	M	E
			°	°
	1	1910 **1** 12	55.39	73.1
	2	**4** 2	73.46	92.0
	3	**6** 21	91.53	109.0
	4	**9** 9	109.61	124.8
a 2.670	5	**11** 28	127.68	139.7
$\sin\varphi$ .3234	6	1911 **2** 16	145.75	153.9
	7	**5** 7	163.82	167.8
$\cos\varphi$ .9463	8	**7** 26	181.90	−178.6
	9	**10** 14	199.97	−164.9
ae .863				
	10	1912 **1** 2	218.04	−151.0
b 2.53	11	**3** 22	236.12	−136.6
	12	**6** 10	254.19	−121.6
p 2.39	13	**8** 29	272.26	−105.6
	14	**11** 17	290.34	−88.2
	15	1913 **2** 5	308.41	−68.9
	16	**4** 26	326.48	−47.1
	17	**7** 15	344.56	−22.6
	18	**10** 3	2.63	3.9
	19	**12** 22	20.70	30.0
	20	1914 **3** 12	38.77	53.7
	21	**5** 31	56.85	74.7
	22	**8** 19	74.92	93.4
	23	**11** 17	92.99	110.4
	24	1915 **1** 26	111.07	126.0

Factors for the subsequent computation

$\frac{k_1}{\sqrt{p}}$	48.70	$\frac{k_2\sin\varphi}{\sqrt{a}}$	$\frac{1}{.1224}$
$\tan\varphi$	.3417	$\frac{k_2 p}{\sqrt{a}}$	60.40
$2\cos\varphi$	$\frac{1}{.5284}$	$\frac{a^{\frac{3}{2}}}{k_2\tan\varphi}$	.3092
$2\sin^2\frac{\varphi}{2}$	.0537	$\sin i$	.2715
$\frac{p}{\sin\varphi}$	$\frac{1}{.1353}$	$2\sin^2\frac{i}{2}$	.0375

	0.5 U.T.			ρ_1	ξ_1	η_1	ζ_1	r	$r\sin u$	$r\cos u$	$\sin v$	$\cos v$
1	1910	**1**	12	6.76	−2.11	4.87	1.20	2.42	−.84	−2.28	1.000	−.039
2		**4**	2	6.52	−1.01	5.25	1.10	2.70	−1.69	−2.10	.937	−.351
3		**6**	21	6.30	−.18	5.38	.97	2.95	−2.39	−1.73	.811	−.583
4		**9**	9	6.05	.50	5.36	.83	3.16	−2.91	−1.22	.659	−.752
5		**11**	28	5.84	.99	5.30	.69	3.34	−3.26	−.65	.499	−.870
6	1911	**2**	16	5.60	1.45	5.21	.54	3.45	−3.45	−.05	.330	−.947
7		**5**	7	5.37	1.84	5.10	.40	3.51	−3.46	+.58	+.158	−.989
8		**7**	26	5.13	2.20	4.95	.22	3.54	−3.33	1.18	−.018	−1.000
9		**10**	14	4.87	2.53	4.77	.06	3.51	−3.05	1.73	−.190	−.983
10	1912	**1**	2	4.59	2.86	4.56	−.10	3.44	−2.65	2.18	−.360	−.936
11		**3**	22	4.29	3.20	4.28	−.27	3.31	−2.12	2.54	−.529	−.852
12		**6**	10	3.96	3.60	3.91	−.43	3.14	−1.48	2.77	−.692	−.727
13		**8**	29	3.61	4.03	3.40	−.60	2.92	−.73	2.82	−.841	−.547
14		**11**	17	3.32	4.50	2.68	−.74	2.66	+.05	2.67	−.955	−.300
15	1913	**2**	5	3.12	4.95	1.54	−.88	2.36	.83	2.23	−1.000	+.044
16		**4**	26	3.21	5.13	−.10	−1.02	2.08	1.50	1.47	−.890	.460
17		**7**	15	3.65	4.59	−2.18	−1.14	1.87	1.83	+.44	−.526	.855
18		**10**	3	4.45	2.79	−4.16	−1.24	1.80	1.65	−.73	+.102	.996
19		**12**	22	5.31	.47	−4.94	−1.32	1.93	.98	−1.65	.660	.752
20	1914	**3**	12	6.07	−1.40	−4.72	−1.38	2.16	+.05	−2.16	.943	+.339
21		**5**	31	6.66	−2.63	−4.10	−1.40	2.45	−.92	−2.28	1.000	−.066
22		**8**	19	7.12	−3.31	−3.50	−1.42	2.73	−1.76	−2.08	.928	−.375
23		**11**	17	7.50	−3.75	−3.00	−1.42	2.97	−2.45	−1.68	.797	−.603
24	1915	**1**	26	7.79	−4.03	−2.58	−1.41	3.18	−2.95	−1.16	+.643	−.764

	$\frac{1}{\rho_1^3}$	$\frac{1}{r_1^3}$	$\frac{1}{\rho_1^3}-\frac{1}{r_1^3}$	K	$K\xi_1$	$T=K\eta_1$	$W=K\zeta_1$	$\frac{1}{.2053}\frac{k_1}{\sqrt{p}}\frac{r}{\rho_1^3}$	S
1	.00324	.00617	−.00293	−.1427	+.301	−.695	−.1712	.382	−.081
2	361	617	256	−.1246	.1258	−.654	−.1370	.476	−.350
3	400	617	217	−.1056	+.0190	−.568	−.1025	.575	−.556
4	452	618	166	−.0809	−.0405	−.434	−.0672	.696	−.7365
5	502	620	118	−.0575	−.0569	−.305	−.0397	.818	−.875
6	.00569	623	−.00054	−.0263	−.0372	−.137	−.0142	.9575	−.995
7	646	627	+.00019	+.0093	+.0171	+.0474	+.00372	1.106	−1.089
8	741	632	109	.0531	.1167	.2624	.01166	1.276	−1.159
9	866	638	228	.1110	.2809	.530	+.00667	1.479	−1.198
10	.01034	645	389	.1894	.542	.864	−.01894	1.735	−1.193
11	1266	653	613	.2984	.956	1.278	−.0807	2.044	−1.088
12	1610	662	948	.462	1.66	1.804	−.1984	2.466	−.806
13	2126	672	1454	.708	2.85	2.406	−.425	3.027	−.174
14	2733	682	2051	.998	4.49	2.673	−.738	3.543	+.947
15	.03293	693	2600	1.266	6.27	+1.950	−1.113	3.787	2.483
16	3023	705	2318	1.128	5.80	−.1128	−1.151	3.067	2.732
17	2056	716	1340	.653	3.00	−1.423	−.745	1.870	+1.130
18	1135	729	+.00406	+.1975	+.551	−.8215	−.245	.995	−.444
19	668	741	−.00073	−.0355	−.0170	+.1753	+.0468	.628	−.645
20	.00447	753	306	−.149	+.2088	.7038	.2056	.471	−.262
21	338	765	427	−.208	.5473	.8532	.2912	.404	+.143
22	277	776	499	−.2432	.805	.8518	.3454	.369	.436
23	237	787	550	−.268	1.005	.805	.3808	.343	.662
24	.00212	.00796	−.00584	−.2844	+1.147	+.735	+.4015	.328	+.819

			$-\frac{1}{.1224}$	-60.40		$-.3417$	$-.3092$	
	δi	$\delta\Omega$	$80\delta\mu_1$	$80\delta\mu_2$	$80\delta\mu$	$\delta\varphi_0$	$\delta(\varphi_1+\varphi_2)$	$\delta\varphi$
1	+.39	+.53	+.65	+17.35	+18.00	+4.92	−5.56	−.64
2	.29	.85	2.68	14.63	17.31	5.17	−5.35	−.18
3	.18	.90	3.68	11.63	15.31	4.91	−4.74	+.17
4	.08	.72	3.96	8.30	12.26	4.02	−3.79	.22
5	.03	.48	3.57	5.52	9.09	2.98	−2.81	+.17
6	.00	+.18	2.68	+2.40	5.08	+1.38	−1.57	−.19
7	.00	−.05	+1.41	−.82	+.59	−.49	−.18	−.67
8	.01	−.14	−.17	−4.48	−4.65	−2.72	+1.44	−1.28
9	+.01	−.08	−1.86	−9.12	−10.98	−5.44	3.40	−2.04
10	−.04	+.19	−3.51	−15.16	−18.67	−8.70	5.77	−2.93
11	−.25	.63	−4.70	−23.33	−28.03	−12.38	8.67	−3.71
12	−.55	1.08	−4.56	−34.7	−39.26	−16.58	12.13	−4.45
13	−1.34	+1.14	−1.19	−49.8	−50.99	−20.60	15.74	−4.86
14	−1.97	−.14	+7.49	−60.7	−53.21	−20.80	16.45	−4.35
15	−2.48	−3.40	20.28	−49.9	−29.62	−13.47	+9.16	−4.31
16	−1.69	−6.37	19.84	+3.28	+23.12	+.69	−7.15	−6.46
17	−.33	−5.02	4.86	46.0	50.86	7.80	−15.71	−7.91
18	−.18	−1.49	.37	+27.56	+27.93	+4.33	−8.64	−4.31
19	−.08	+.17	3.47	−5.48	−2.01	−.99	+.62	−.37
20	−.44	+.04	+2.02	−19.80	−17.78	−4.45	5.50	+1.05
21	−.66	−.99	−1.17	−21.02	−22.19	−6.12	6.86	.74
22	−.72	−2.24	−3.30	−18.84	−22.14	−6.80	6.85	+.05
23	−.64	−3.44	−4.32	−16.36	−20.68	−7.00	6.40	−.60
24	−.46	−4.36	−4.31	−13.95	−18.26	−6.84	+5.65	−1.20

	$\frac{1}{.1353}$			$.3234$		$.0375$		$\frac{1}{.5284}$	$.0537$	
	$\delta\pi_1$	$p+r$	$(p+r)\sin v$	$\delta\pi_2$	$\delta(\pi_1+\pi_2)$	$\delta\pi_3=\delta L_3$	$\delta\pi$	δL_0	$\delta(L_1+L_2)$	δL
1	−.02	4.81	+4.81	−10.35	−10.37	+.02	−10.35	+.37	−.56	−.17
2	−.91	5.09	4.77	−9.65	−10.56	3	−10.53	1.79	−.57	+1.25
3	−2.40	5.34	4.33	−7.61	−10.01	3	−9.98	3.11	−.54	2.60
4	−4.09	5.55	3.66	−4.91	−9.00	3	−8.97	4.41	−.48	3.96
5	−5.62	5.73	2.86	−2.70	−8.32	2	−8.31	5.53	−.45	5.10
6	−6.96	5.84	1.93	−.82	−7.78	+.01	−7.76	6.50	−.42	6.09
7	−7.96	5.90	+.933	+.14	−7.82	0	−7.82	7.24	−.42	6.82
8	−8.56	5.93	−.107	−.09	−8.65	−.01	−8.66	7.76	−.46	7.29
9	−8.71	5.90	−1.12	−1.84	−10.55	0	−10.55	7.96	−.57	7.39
10	−8.26	5.83	−2.10	−5.60	−13.86	+.01	−13.85	7.78	−.74	7.05
11	−6.85	5.70	−3.02	−11.92	−18.77	2	−18.75	6.82	−1.01	5.83
12	−4.33	5.53	−3.83	−21.38	−25.71	4	−25.67	4.80	−1.38	+3.46
13	−.70	5.31	−4.46	−33.20	−33.90	+.04	−33.86	+.96	−1.82	−.82
14	+2.10	5.05	−4.82	−39.80	−37.70	0	−37.70	−4.77	−2.02	−6.79
15	−.81	4.75	−4.75	−28.60	−29.41	−.13	−29.54	−11.10	−1.58	−12.81
16	−9.28	4.47	−3.98	+1.39	−7.89	−.24	−8.13	−10.76	−.42	−11.42
17	−7.15	4.26	−2.24	+9.86	+2.71	−.19	+2.52	−4.01	+.15	−4.05
18	+3.27	4.19	+.43	−1.09	2.18	−.06	2.12	+1.51	.12	+1.57
19	3.58	4.32	2.85	+1.35	5.13	+.01	5.14	2.36	.28	2.65
20	.66	4.55	4.29	9.35	10.01	0	10.01	+1.07	.54	+1.61
21	.07	4.84	4.84	12.76	12.83	−.04	12.79	−.66	.69	−.01
22	1.21	5.12	4.75	12.50	13.71	−.08	13.63	−2.25	.74	−1.59
23	2.95	5.36	4.27	10.63	13.60	−.13	13.47	−3.73	.73	−3.13
24	+4.62	5.57	+3.58	8.14	+12.76	−.16	+12.60	−4.93	+.69	−4.40

12h U.T.	80δμ	80Δμ	ΔL_μ	δL	ΔL	δπ	Δπ	δφ	Δφ	δΩ	ΔΩ	δi	Δi	12h U.T.
														1909
		0			°.000		°.000		°.000		°.000		°.000	12 3
1910														1910
1 12	18	18	* °.0009	0	.000	−10	−.010	−1	−.001	1	.001	0	.000	2 21
4 2	17	35	.0027	1	.001	−11	−.021	0	−.001	1	.002	0	.000	5 12
6 21	15	50	.0062	3	.004	−10	−.031	0	−.001	1	.003	0	.000	7 31
9 9	12	62	.0112	4	.008	−9	−.040	0	−.001	1	.004	0	.000	10 19
11 28	9		.0174	5		−8		0		0		0		
1911														1911
2 16	5	71	.0245	6	.013	−8	−.048	0	−.001	0	.004	0	.000	1 7
5 7	1	76	.0321	7	.019	−8	−.056	−1	−.001	0	.004	0	.000	3 28
7 26	−5	77	.0398	7	.026	−9	−.064	−1	−.002	0	.004	0	.000	6 16
10 14	−11	72	.0470	7	.033	−11	−.073	−2	−.003	0	.004	0	.000	9 4
		61			.040		−.084		−.005		.004		.000	11 23
1912														1912
1 2	−19	42	.0531	7	.047	−14	−.098	−3	−.008	0	.004	0	.000	2 11
3 22	−28	14	.0573	6	.053	−19	−.117	−4	−.012	1	.005	0	.000	5 1
6 10	−39	−25	.0587	3	.056	−26	−.143	−4	−.016	1	.006	−1	−.001	7 20
8 29	−51	−76	.0562	−1	.055	−34	−.177	−5	−.021	1	.007	−1	−.002	10 8
11 17	−53	−129	.0486	−7	.048	−38	−.215	−4	−.025	0	.007	−2	−.004	12 27
1913														1913
2 5	−30	−159	.0357	−13	.035	−30	−.245	−4	−.029	−3	.004	−2	−.006	3 17
4 26	23	−136	.0198	−11	.024	−8	−.253	−6	−.035	−6	−.002	−2	−.008	6 5
7 15	51	−85	.0062	−4	.020	3	−.250	−8	−.043	−5	−.007	0	−.008	8 24
10 3	28	−57	−.0023	2	.022	2	−.248	−4	−.047	−1	−.008	0	−.008	11 12
12 22	−2		−.0080	3		5		0		0		0		
1914														1914
3 12	−18	−59	−.0139	2	.025	10	−.243	1	−.047	0	−.008	0	−.008	1 31
5 31	−22	−77	−.0216	0	.027	13	−.233	1	−.046	−1	−.008	−1	−.008	4 21
8 19	−22	−99	−.0315	−2	.027	14	−.220	0	−.045	−2	−.009	−1	−.009	7 10
11 7	−21	−121	−.0436	−3	.025	13	−.206	−1	−.045	−3	−.011	−1	−.010	9 28
		−142			.022		−.193		−.046		−.014		−.011	12 17
1915														1915
1 26	−19	−161	−.0578	−4	.018	13	−.180	−1	−.047	−4	−0.18	0	−.011	3 7
4 16	−16	−177	−.0739	−5	.013	12	−.168	−2	−.049	−5	−.023	0	−.011	5 26
7 5	−14	−191	−.0916	−6	.007	10	−.158	−2	−.051	−5	−.028	0	−.011	8 14
9 23	−12	−203	−.1107	−7	.000	9	−.149	−3	−.054	−5	−.033	0	−.011	11 2
12 12	−9		−.1310	−7		8		−3		−4		0		
1916														1916
3 1	−7	−212	−.1522	−7	−.007	6	−.141	−3	−.057	−4	−.037	1	−.011	1 21
5 20	−4	−219	−.1741	−7	−.014	5	−.135	−3	−.060	−3	−.041	1	−.010	4 10
8 8	−1	−223	−.1964	−7	−.021	4	−.130	−3	−.063	−2	−.044	1	−.009	6 29
10 27	3	−224	−.2188	−7	−.028	4	−.126	−3	−.066	−1	−.046	0	−.008	9 17
		−221			−.035		−.122		−.069		−.047		−.008	12 6

* Refer to the page 49.

12h U.T.	80δμ	80Δμ	ΔL_μ	δL	ΔL	δπ	Δπ	δϖ	Δϖ	δΩ	ΔΩ	δi	Δi	12h U.T.
1917														1917
			°		°		°		°		°		°	
1 15	8		−.2409	−6		4		−3		0		0		
		−213			−.041		−.118		−.072		−.047		−.008	2 24
4 5	13		−.2622	−5		5		−3		0		0		
		−200			−.046		−.113		−.075		−.047		−.008	5 15
6 24	19		−.2822	−4		6		−3		0		0		
		−181			−.050		−.107		−.078		−.047		−.008	8 3
9 12	25		−.3003	−2		7		−3		0		0		
		−156			−.052		−.100		−.081		−.047		−.008	10 22
12 1	24		−.3159	0		−5		−3		0		0		
1918														1918
		−132			−.052		−.095		−.084		−.047		−.008	1 10
2 19	7		−.3291	2		4		−1		0		0		
		−125			−.050		−.091		−.085		−.047		−.008	3 31
5 10	−26		−.3416	3		12		3		0		0		
		−151			−.047		−.079		−.082		−.047		−.008	6 19
7 29	−59		−.3567	−1		25		6		0		−1		
		−210			−.048		−.054		−.076		−.047		−.009	9 7
10 17	−79		−.3777	−10		37		7		−5		−3		
		−289			−.058		−.017		−.069		−.052		−.012	11 26
1919														1919
1 5	−68		−.4066	−22		35		9		−17		−5		
		−357			−.080		.018		−.060		−.069		−.017	2 14
3 26	−29		−.4423	−32		20		12		−35		−6		
		−386			−.112		.038		−.048		−.104		−.023	5 5
6 14	15		−.4809	−34		5		16		−49		−5		
		−371			−.146		.043		−.032		−.153		−.028	7 24
9 2	46		−.5180	−30		−2		20		−56		−3		
		−325			−.176		.041		−.012		−.209		−.031	10 12
11 21	64		−.5505	−23		−1		22		−55		0		
		−261			−.199		.040		.010		−.264		−.031	12 31
1920														1920
2 9	70		−.5766	−15		4		21		−47		3		
		−191			−.214		.044		.031		−.311		−.028	3 20
4 29	69		−.5957	−10		12		20		−39		4		
		−122			−.224		.056		.051		−.350		−.024	6 8
7 18	64		−.6079	−5		19		17		−29		5		
		−58			−.229		.075		.068		−.379		−.019	8 27
10 6	56		−.6137	−1		23		14		−18		4		
		−2			−.230		.098		.082		−.397		−.015	11 15
12 25	49		−.6139	1		26		11		−11		4		
1921														1921
		47			−.229		.124		.093		−.408		−.011	2 3
3 15	39		−.6092	3		23		7		−5		3		
		85			−.226		.147		.100		−.413		−.008	4 24
6 3	26		−.6007	4		18		4		−1		2		
		112			−.222		.165		.104		−.414		−.006	7 13
8 22	15		−.5895	4		12		2		0		1		
		127			−.218		.177		.106		−.414		−.005	10 1
11 10	1		−.5768	3		4		2		0		0		
		128			−.215		.181		.108		−.414		−.005	12 20
1922														1922
1 29	−12		−.5640	2		−1		2		0		0		
		116			−.213		.180		.110		−.414		−.005	3 10
4 19	−19		−.5524	0		−3		3		−1		0		
		97			−.213		.177		.113		−.415		−.005	5 29
7 8	−16		−.5427	−2		−2		3		−1		0		
		81			−.215		.175		.116		−.416		−.005	8 17
9 26	−6		−.5346	−3		−3		2		0		0		
		75			−.218		.172		.118		−.416		−.005	11 5
12 15	3		−.5271	−3		−6		1		0		0		

Jupiter's Co-ordinates.

Date 12^h U.T.			λ_1	$r_1[\cos\beta_1]$	$r_1\sin\beta_1$	$\frac{10^5}{r_1^3}$
1910	1	12	183°.8	5.45	+.12	617
	4	2	189.8	5.45	.12	617
	6	21	195.9	5.45	.12	617
	9	9	201.9	5.45	.12	618
	11	28	208.0	5.44	.12	620
1911	2	16	214.1	5.43	+.11	623
	5	7	220.2	5.42	.11	627
	7	26	226.3	5.41	.10	632
	10	14	232.5	5.39	.09	638
1912	1	2	238.7	5.37	+.08	645
	3	22	244.9	5.35	.07	653
	6	10	251.2	5.33	.06	662
	8	29	257.6	5.30	.04	672
	11	17	264.0	5.27	.03	682
1913	2	5	270.5	5.24	+.02	693
	4	26	277.1	5.22	0	705
	7	15	283.8	5.19	−.01	717
	10	3	290.5	5.16	−.02	729
	12	22	297.3	5.13	−.04	741
1914	3	12	304.2	5.10	−.05	753
	5	31	311.1	5.08	.06	765
	8	19	318.1	5.05	.07	776
	11	7	325.2	5.03	.08	787
1915	1	26	332.3	5.01	−.09	796
	4	16	339.5	4.99	.10	805
	7	5	346.8	4.98	.10	812
	9	23	354.1	4.96	.11	817
	12	12	1.4	4.96	.11	821
1916	3	1	8.7	4.95	−.11	823
	5	20	16.0	4.95	.11	823
	8	8	23.3	4.96	.11	821
	10	27	30.6	4.96	.11	818
1917	1	15	37.9	4.97	−.10	813
	4	5	45.2	4.99	.09	806
	6	24	52.4	5.00	.08	798
	9	12	59.5	5.02	.07	789
	12	1	66.6	5.05	.06	778
1918	2	19	73.6	5.07	−.05	767
	5	10	80.6	5.10	.04	755
	7	29	87.5	5.12	.02	743
	10	17	94.3	5.15	−.01	731
1919	1	5	101.0	5.18	0	719
	3	26	107.7	5.21	+.02	707
	6	14	114.3	5.24	.03	696
	9	2	120.8	5.27	.04	684
	11	21	127.2	5.29	.06	674

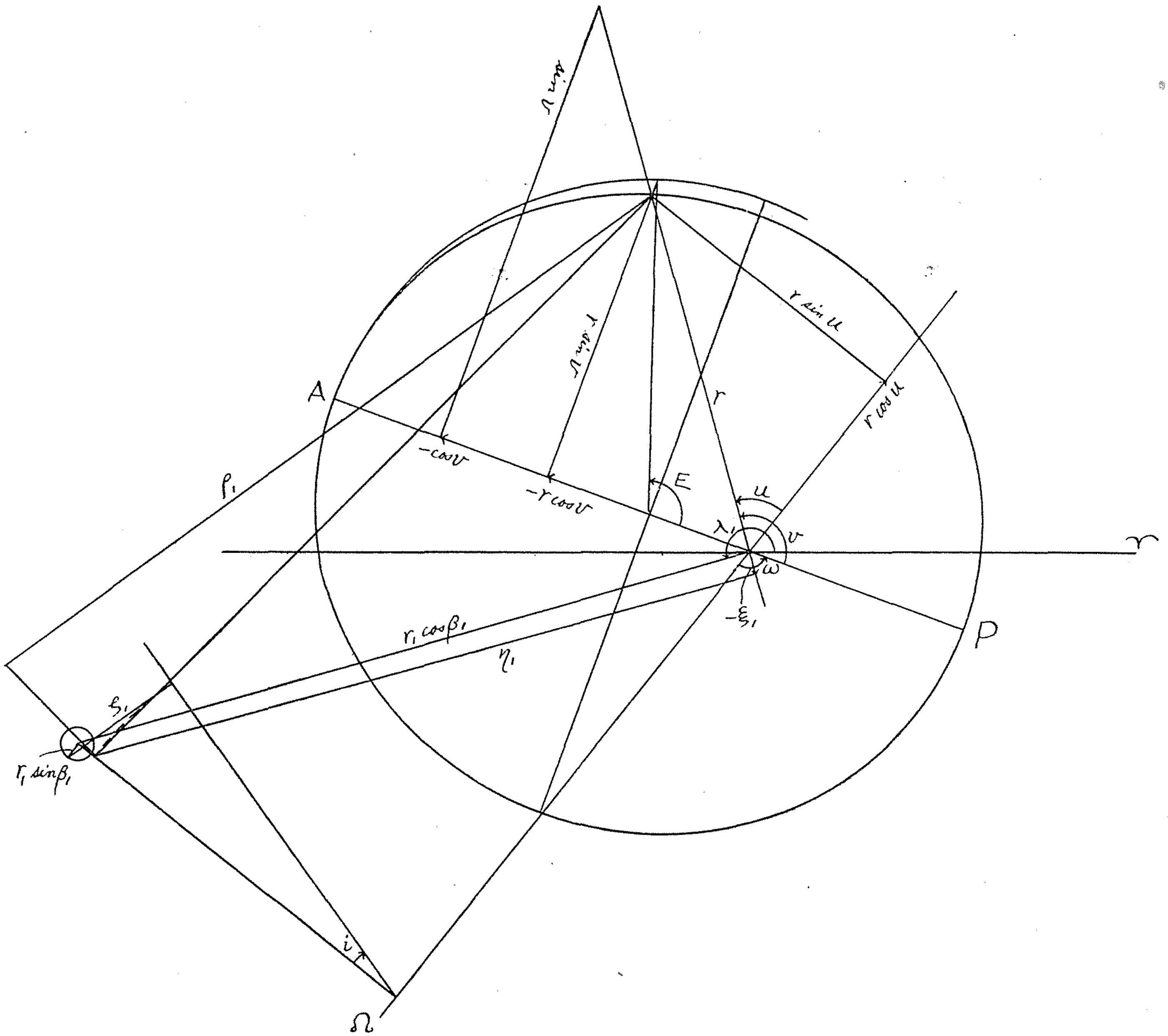
Date 12^h U.T.			λ_1	$r_1[\cos\beta_1]$	$r_1\sin\beta_1$	$\frac{10^5}{r_1^3}$
1920	2	9	133°.6	5.32	+.07	664
	4	29	140.0	5.34	.08	655
	7	18	146.2	5.37	.09	644
	10	6	152.4	5.39	.10	640
	12	25	158.6	5.40	.11	634
1921	3	15	164.8	5.42	+.11	628
	6	3	170.9	5.43	.12	624
	8	22	177.0	5.44	.12	620
	11	10	183.0	5.45	.12	618
1922	1	29	189.1	5.45	+.12	617
	4	19	195.1	5.45	.12	616
	7	8	201.2	5.45	.12	617
	9	26	207.2	5.45	.12	619
	12	15	213.3	5.44	.11	622
1923	3	5	219.4	5.43	+.11	625
	5	24	225.5	5.41	.10	630
	8	12	231.7	5.40	.09	636
	10	31	237.9	5.38	.08	643
1924	1	19	244.1	5.36	+.07	650
	4	8	250.4	5.33	.06	659
	6	27	256.7	5.31	.05	668
	9	15	263.1	5.28	.04	678
	12	4	269.6	5.26	.02	689
1925	2	22	276.2	5.23	+.01	700
	5	13	282.8	5.20	−.01	712
	8	1	289.5	5.17	−.02	724
	10	20	296.2	5.14	−.03	736
1926	1	8	303.1	5.11	−.05	749
	3	29	310.0	5.08	.06	761
	6	17	317.0	5.06	.07	772
	9	5	324.0	5.04	.08	783
	11	24	331.2	5.01	.09	793
1927	2	12	338.4	5.00	−.10	802
	5	3	345.6	4.98	.10	810
	7	22	352.8	4.97	.11	816
	10	10	0.1	4.96	.11	820
	12	29	7.5	4.95	.11	823
1928	3	18	14.8	4.95	−.11	824
	6	6	22.1	4.95	.11	823
	8	25	29.5	4.96	.11	820
	11	13	36.8	4.97	.10	815
1929	2	1	44.0	4.98	−.09	809
	4	22	51.2	5.00	.09	801
	7	11	58.4	5.02	.08	792
	9	29	65.5	5.04	.07	782
	12	18	72.6	5.06	.05	771

Jupiter's Co-ordinate. (continued)

Date 12h U.T.			λ_1	$r_1[\cos\beta_1]$	$r_1\sin\beta_1$	$\frac{10^5}{r_1^3}$
1930	**3**	8	79.6°	5.09	−.04	760
	5	27	86.5	5.11	−.03	748
	8	15	93.3	5.14	−.01	735
	11	3	100.1	5.17	.00	723
1931	**1**	22	106.8	5.20	+.01	711
	4	12	113.4	5.23	.03	699
	7	1	119.9	5.26	.04	688
	9	19	126.4	5.29	.05	677
	12	8	132.8	5.31	.07	667
1932	**2**	26	139.1	5.34	+.08	658
	5	16	145.4	5.36	.09	649
	8	4	151.6	5.38	.10	642
	10	23	157.8	5.40	.10	635
1933	**1**	11	164.0	5.42	+.11	629
	4	1	170.1	5.43	.12	625
	6	20	176.2	5.44	.12	621
	9	8	182.3	5.45	.12	618
	11	27	188.3	5.45	.12	617
1934	**2**	15	194.3	5.46	+.12	616
	5	6	200.4	5.45	.12	616
	7	25	206.4	5.45	.12	618
	10	13	212.5	5.44	.12	621
1935	**1**	1	218.6°	5.43	+.11	624
	3	22	224.7	5.42	.10	629
	6	10	230.8	5.40	.09	634
	8	29	237.0	5.38	.08	641
	11	17	243.3	5.36	.07	648
1936	**2**	5	249.6	5.34	+.06	657
	4	25	255.9	5.31	.05	666
	7	14	262.3	5.29	.04	676
	10	2	268.8	5.26	.02	687
	12	21	275.3	5.23	.01	698
1937	**3**	11	281.9	5.20	.00	710
	5	30	288.6	5.17	−.02	722
	8	18	295.3	5.14	−.03	735
	11	6	302.1	5.12	−.04	747
1938	**1**	25	309.0	5.09	−.06	759
	4	15	316.0	5.06	.07	771
	7	4	323.1	5.04	.08	782
	9	22	330.2	5.02	.09	792
	12	11	337.4	5.00	.10	802
1939	**3**	1	344.6	4.98	−.10	809
	5	20	351.9	4.97	.11	816
	8	8	359.2	4.96	.11	821
	10	27	6.5	4.95	.11	824

1910 6 21.5

Fig. 11



(in the original size)