Surface Fluctuations of Lake Biwa caused by the Muroto Typhoon

By Takaharu Nomitsu

(Received July 26, 1935)

Abstract

The extraordinary fluctuations of the water of Lake Biwa due to the typhoon on Sept. 21 last year are investigated, and the writer's theory of meteorological tunamis and seiches is applied to them. For the southern shallow portion of the lake, 3.4m deep on average, the fluctuation can almost be explained as that found in the case of "no bottom-friction", so that the bottom-current is not much less than at the surface, while in the northern deep portion of 36.4m depth, the phenomena more nearly resemble the case of "no bottom-current". The coefficient of bottom-friction (assumed linearly proportional to the slip velocity) and the eddy viscosity during the storm are estimated as follows:

	Coef. of friction	Eddy viscosity
Southern part	0.16 c.g.s.	33 c.g.s.
Northern part	3.35	1020

The height of wind-waves on the north coast of the lake are also determined from various kinds of traces left after the storm, and utilised to test Stevenson's and Boergen's experimental formulae. Stevenson's formula seems to include the effect of both wind wave at the coast and the seiche motion, while Boergen's represents merely the wind-wave of progressive nature and consequently half the height of coastal waves when used for the coast of a lake.

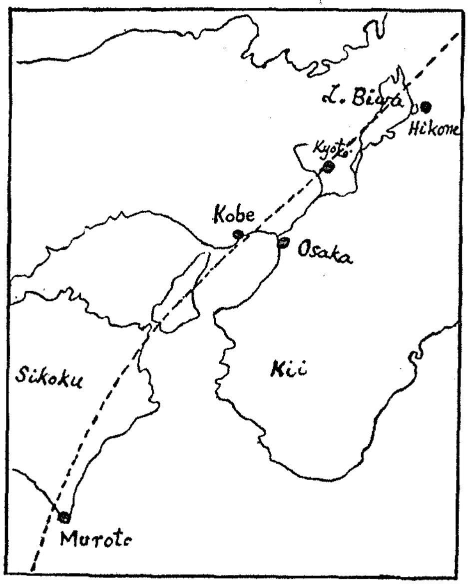
§ 1. Limnograms and other observation-data

The typhoon on Sept. 21 last year, which established a 684 mm world record for deepest central pressure at Cape Muroto in Sikoku, passed close to Lake Biwa between half past 8 and 9 a.m. as shown in Fig. 1. The wind and pressure observed at Hikone Meteorological Observatory, the nearest to the lake, are as shown in Fig. 2.

As a consequence, the surface of the lake water fluctuated abnormally and the oscillations were beautifully recorded on the limnograms at nine stations, six of which belong to the Siga Prefectural office, two to the Osaka Branch Office of Public Works of the Home Office, and one to the Drainage Office of Kyoto. The positions of the stations are indicated in Fig. 3, and the limnograms are reproduced in Fig. 4

Fig. 1

Path of the Muroto Typhoon.



with the permission of the respective offices, for which the writer here expresses his best thanks.

Besides collecting these records, the writer made a week's tour round the lake immediately after the typhoon, inspecting the condition of the limnological stations and the natural features of the coast, and estimating the maximum heights of water reached during the typhoon by such marks as the dust and sand washed up on the dry land, the shore grass swept down in one direction, damaged parts of the coastal road and bank, etc. Special care was taken to distinguish between the height of the rise of the mean water-level and that of the wind-waves in each locality. The maximum height of the water surface plus wave crest, was investigated at places directly facing the strongest wind in the typhoon, i. e. south; while for the estimation of the rise of mean level only, excluding the wave effect, the traces left by the water were sought on the east- or

Fig. 2

Wind and pressure at Hikone.

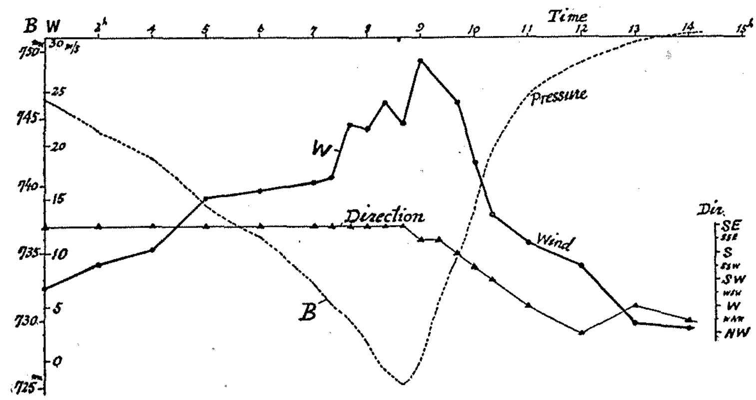


Fig. 3

Isobaths of Lake Biwa and limnological stations.

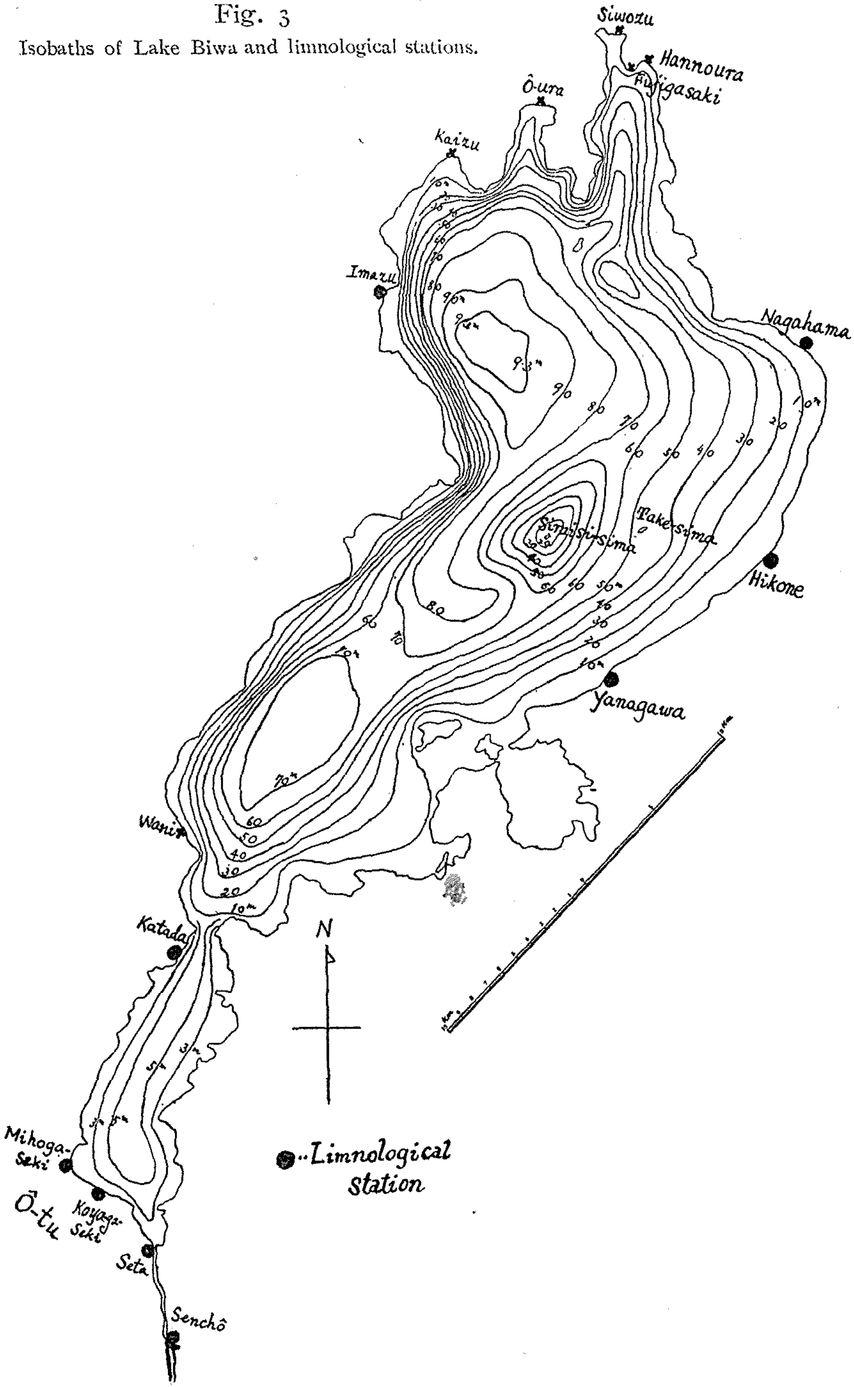
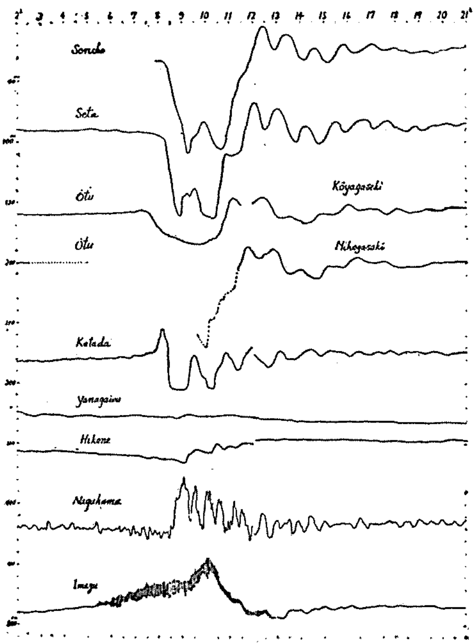


Fig. 4

Limnograms at 9 stations.



west-ward creeks and ditches or on the northern side of a headland, etc. Two of the groups of water-heights thus obtained showed a definite difference from each other, though within each group there was a considerable degree of conformity, proving that the rise of mean level and the wave-height had been satisfactorily separated. Table 1 shows the results of estimation in the northern part of the lake.

The rise of mean level did not differ much at any of these places, but the wave heights were greatest at Kaizu and Fujigasaki. People say that at the latter places the

Table 1 Water rise at the typhoon in m.

	Imazu	Kaizu	O-ura	Siwozu	Fujigasaki	Hanno-ura
Greatest rise	1.8	2.1—2.4	1.2—1.5	1.5—1.8	2.0	1.9
Level rise	0.5	0.6—0.9	0.5	0.3	0.3	0.3
Wave height	1.3	1.8	1.0	1.5	1.7	1.6

spray went over the roofs of the houses, and the writer witnessed two steamers wrecked at Kaizu, which shows the violence of the wind. In the southern portion of the lake, the change in the water-surface due to the typhoon was mainly fall rather than rise, and, of course, no traces other than limnograms could be found, but many persons told me that they had seen, at Ôtu and Seta, a large patch of the lake bottom exposed, a phenomenon unprecedented in history!

The main object of this paper is to explain the above fluctuations of the lake surface in relation to my theory[1] of meteorological tunamis

1. These Memoirs, A, **18**, 201 (1935).

and seiches. As a secondary subject, a test of Stevenson's and Boergen's formulae for wave-heights will also be attempted.

§ 2. Wave heights and the empirical formulae of Stevenson and of Boergen.

Stevenson gives the following empirical formula for wave height, based upon the observations at Scotish lakes, $H = 1.5\sqrt{D} + (2.5 - \sqrt[4]{D})$, where H denotes the height of wave in feet, and D the fetch windward in nautical miles. On the other hand Boergen's formula for sea waves is

$$H = \frac{Q}{3} \Big/ \left(1 + \frac{1.86}{t}\right)\left(1 + 1.86 \times 1.94 \frac{Q}{D}\right),$$

where Q denotes the wind-velocity in m/sec., t the time in hours after the wind begins to blow, and D is the fetch in sea miles as before, but H is the wave height in metres. In the present case of lake Biwa, measuring the fetch D from a map issued by the Land Survey and taking Q and t from Fig. 2, we get the following table:

Table 2. Calculated wave-heights

		Imazu	Kaizu	O-ura	Siwozu	Fujigasaki	Hanno-ura
D (km)		23	28	18 (30)	31	29	30
H (m)	S	1.83	1.93	1.32 (1.97)	1.99	1.96	1.97
	B	0.57	0.66	0.36 (0.57)	0.58	0.56	0.57

Comparing this with the observed values in Table 1, we see that the value given by Stevenson's formula almost coincides with the observed maximum height of wind-wave plus the rise of mean level which includes seiche effect in turn. That the formula of Stevenson will correspond to water-height of this nature may be easily understood from the original data taken as the basis of its reduction. Such a close coincidence as the above, however, was beyond my expectation, since in the formula both intensity and duration of the wind are left out of consideration. This coincidence perhaps means that the severest storm at any place might be of the same order in its intensity and duration.

Boergen's formula, on the contrary, though it seems very rational in form, is not easy to use because of the difficulty of estimating the duration of the strongest wind of constant value. At any rate, the values in Table 2 are calculated, taking $Q = 23$ m/s and $t = 2.5$ hours for SE wind, and $Q = 25$ m/s and $t = 1.5$h for S wind. The figures obtained appear unexpectedly small at first sight, being only about one half the observed wave-height separated from the rise of mean level or seiche effect. Reconsideration, however, shows that in reality the coincidence is not so bad. Probably Boergen's formula is suitable for progressive waves in a sea far away from the land, but not for waves of stationary character on a leeward coast and consequently doubly amplified by reflection.

§ 3. Several periods and damping of the surface-fluctuations

Now apart from the wind waves considered above, let us investigate the surface-oscillations for longer periods recorded on the limnograms.

Various periods :—From Fig. 4, we observe that oscillations with periods of about 4.1 and 1.14 hours are most noticeable.

(1) Oscillation of period 4.1 h=250 min. developed conspicuously in the southmost region only, where the first fall of level reached 70 cms or more; but at Katada it was very slight, and in the northern main body of the lake only a trace, if any, could be detected. Its phase is obviously the same everywhere in the oscillating portion south of Katada, and its amplitude increases as it moves towards Seta. These facts, together with the topography and the isobaths of the lake, show that this undulation of four hour period may be a fundamental oscillation of the southern shallow part as a *bay*, whose mouth, and consequently node, lies a little north of Katada.

(2) Oscillations of 1.14h=68.4min. are next conspicuous both in the southern and northern parts of the lake. The amplitude is greatest at Seta and Nagahama, and falls to about half at Katada and Ôtu and becomes very small at Hikoné and Imazu. At Yanagawa, almost half-way between Nagahama and Katada, the oscillation almost vanishes. The phase is the same at Katada and Seta on the one hand, and at Nagahama, Hikoné and Mihogasaki on the other. The two groups are in opposite phase. Thus it seems to me that the oscillation of about 70 minutes is the primary oscillation of the long elliptic deep basin between Nagahama and Katada, and that the oscillation in the southern shallow portion must be a secondary one excited by the action of the main body.

Besides the above widely distributed oscillations, the following local oscillations may be noticed.

(3) Oscillations of 20 to 30 and 12 min. occur, strongly marked, at Nagahama, also at Imazu though here much feebler in intensity.

The oscillating area is perhaps the rectangular basin between Nagahama, Imazu, Siraisi islet and Takesima.

(4) An oscillation of about 5 min. period is conspicuous at Imazu. The corresponding oscillating area seems to be the "shelf" of lake, but the decision will be reserved for future investigation.

For later informations, let us here add briefly the habit of fluctuation at the places under considerations, examining the limnograms for the past ten years which we owe to the courtesy of the Siga Prefectural Office.

(a) On the Ôtu side of the lake, once or twice a month oscillations of 4 hour and also of 70 min. period occur with amplitude up to 5 cms.

(b) At Hikone and Yanagawa, especially at the latter, the limnograms is very monotone, seiches of amplitude greater than 10 cms being very rare.

(c) Nagahama is very sensitive to seiches, and the amplitude of the seiche attains 20 to 30 cms for a few days every month.

(d) At Imazu, notwithstanding its situation at the head of the lake, the limnogram is, contrary to expectation, usually the most monotone in the lake. The amplitude has never exceeded 4 cms for 10 years except in the Muroto typhoon, so that the limnograph of 1/20 reduction draws only a straight line every day, merely widening the line occasionally. This fact will be very important in the discussion of the fundamental oscillation of the main body of the lake.

Damping factors:—We next turn to the damping of the level fluctuations caused by the Muroto typhoon.

Measuring the successive crests and furrows for the two principal oscillations of 4.1 and 1.14 hour period, we get the following table of damping factor i. e. the ratio of the successive amplitudes.

Table 3. *Damping factor and index*

(A) Damping per half period of the 4.1 hour oscillations

Station	Senchô	Seta	Kôyazeki	Mihogasaki	mean	Index $\frac{1}{2}\nu\beta_0^2$
						sec.$^{-1}$
After storm	0.566	0.524	0.511	0.520	0.530	8.85×10^{-5}
During storm	0.125	0.149	—	0.100	0.134	2.73×10^{-4}

(B) Damping per one period of the 70 min. oscillations

i. Southern part of the lake

Station	Senchô	Seta	Kôyazeki	Mihogasaki	Katada	mean	Index $\frac{1}{2}\nu\beta_0^2$
Damp. factor	0.762	0.747	0.822	0.741	0.707	0.756	6.82×10^{-5}

ii. Northern main body of the lake

Station	Nagahama	Hikone	Katada	mean	Index $\frac{1}{2}\nu\beta_0^2$
Damping factor	0.750	0.702	0.707	0.720	8.05×10^{-5}sec.$^{-1}$

The damping index is also calculated. This term signifies the index when the damping factor is considered as an exponential function of time, $e^{-\frac{1}{2}\nu\beta_0^2 t}$, as was explained in our theoretical paper[1] on meteorological tunamis and seiches. From the table we notice that:

(1) The damping of the oscillation of 70 min. period is much less in the southern shallow part than in the northern deep portion. This will suggest the origin of the oscillation in the southern portion, as will be explained later.

(2) The damping index in the southern part is three times greater during the storm than after. This agrees roughly with the fact that the eddy viscosity ν varies with the velocity of current. For, the largest change of level till noon is 70+20=90 cms in half a period, while in the afternoon a level-change of 30 cms maximum occurs in half a period. Thus the current of water, being assumed proportional to the level change in the same interval of time, will bo in the ratio 90 : 30=3 : 1 during and after the storm.

§ 4. The nature of the oscillation of 4.1 h period and its development

Since the oscillation of 4.1 hour period occurs only in the southmost part of the lake, it must clearly be a vibration of the southern part of the lake. Now, what is the nature of this vibration? To answer this question, let us refer to the formulae of the proper period of a closed canal and a bay.

For a rectangular *lake* of length L and of depth H, the period of its free vibration of mth order is usually given by

$$\text{period}=2\pi/\sigma=2\pi/\sqrt{gH(m\pi/L)^2}, \qquad \ldots\ldots\ldots\ldots\ldots(1)$$

which is derived for an ideal fluid with "no bottom-friction" and disregarding the earth's rotation. According to Proudman[2] and also to the writer[3], the action of the earth's rotation $\bar{\omega}$ will change the formula as below:

$$\text{speed}=\sigma=\sqrt{gH(m\pi/L)^2+4\bar{\omega}^2}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

1. loc. cit. 2. M. N. R. A. S. Geophys. Suppl. **2**, 197 (1929), eq. (3.42).
3. These Memoirs, A, **17**, 249 (1934), eq. (17).

Numerical test will show, however, that the rotation-effect can be neglected for an ordinary lake or bay. In the case of a viscous fluid with bottom-friction, the writer's formula[1] is

$$\left.\begin{aligned}\sigma&=\sqrt{B_0\frac{g\sin\beta_0 H}{\beta_0}\left(\frac{m\pi}{L}\right)^2-\left(\frac{\nu\beta_0^2}{2}\right)^2},\\ \text{where } B_0&=\frac{2(\beta_0^2+h^2)\sin\beta_0 H}{H(\beta_0^2+h^2)+h}\cdot\frac{1}{\beta_0},\quad \beta_0\tan\beta_0 H=\frac{f'}{\nu}\equiv h,\quad \nu\equiv\frac{\mu}{\rho}\end{aligned}\right\}\qquad(3)$$

and μ denotes the viscosity, ρ the density and f' the coefficient of friction at the water-bottom. The viscosity term in the radical will commonly be negligible for small oscillations, but if the disturbance of water is considerable as on a stormy day, the eddy viscosity becomes so marked as to lengthen the period and in extreme cases will even make the motion non-periodic.

For a *bay* of length L', we can use the above formula, only putting $L=2L'$ and applying the so-called "mouth correction".

Now let us return to the problem of oscillation of the southern part of lake Biwa. Seeing that the 4.1 h oscillation is most conspicuous at Seta while almost vanishing at Katada and that both the depth and breadth of the water basin abruptly change at a point slightly north of Katada, it is obvious that the oscillation is of the nature of *bay*-oscillation.

As to the mouth and head of the bay, we may consider the following four cases from Fig. 3, of which isobaths are drawn after a report[2] of the Kobe Marine Observatory on a map issued by the Military Land Survey.

Table 4. Possible areas of oscillation as a bay

	Bay mouth	Bay head
I	Narrowest point north of Katada	Top of River Seta
II	,,	Obliquely along Ôtu city
III	Line through Wani	Top of River Seta
IV	,,	Obliquely along Ôtu city

Measuring the length L', breadth B and mean depth H for each oscillating area, we get Table 5.

1. These Memoirs, A, **18**, 201 (1935), eq. (39). 2. 海洋氣象台彙報 第八號 大正十五年。

Table 5. Data of the supposed areas of oscillation

	L'	B	H	L'/B	Mouth correction	Period	$L'/H \propto \zeta$
	km	km	m			h	
I	17	3.3	3.4	5.15	1.16	3.78	5.0
II	15	"	"	"	"	3.34	4.41
III	20	3.7	3.98, 3.83	5.34	1.155	4.08	5.25
IV	18	"	4.05, 3.88	"	"	3.65	4.65

The basin south of Katada is nearly of uniform depth and hence the simple arithmetical mean can be taken as the mean depth for all ordinary purposes; but since north of Katada the depth increases abruptly, in cases III and IV we should take the inverse-square-root-mean for the period-calculation and the inverse-mean for the calculation of level rise. The double values of H (III and IV) in the table have this significance. "Period" is calculated by eq. (1) with mouth correction. All these modes are probably possible in reality, depending on circumstances; for, on the limnograms during 10 years the actual period is found to change within a range of 30 minutes according to days. To which of the four modes does the extraordinary oscillations at the recent typhoon then belong?

By trials of the period and the level rise, the first mode appears as the most probable for the recent great fluctuations.

Now, in order to estimate the effect of the viscosity upon the period of oscillation, let us first compare the observed speed σ_{obs} with the ideal speed σ_0 of an ideal fluid.

$$\sigma_{obs}=2\pi/(4.1\times 3600)=4.25\times 10^{-4}, \qquad \sigma_0=2\pi/(3.78\times 3600)=4.6\times 10^{-4},$$

$$B_0\frac{\sin\beta_0 H}{\beta_0 H}=\frac{\sigma_{obs}^2+[\nu\beta_0^2/2]^2}{\sigma_0^2}=\frac{4.25^2+0.88^2}{4.6^2}=0.89.$$

Progress of the level-fluctuations:—The intensity of wind or barometric gradient may be represented by the steady value of level rise, $\bar{\zeta}$, which would be attained in a very long time if the motive force remained constant. For the wind of velocity Q, the tracting force is

$T=3.2\times 10^{-6}\,Q^2$ in c. g. s. units.

$$\left.\begin{aligned} &\bar{\zeta}=\int n\frac{T.dL}{g\rho(H+\zeta)}, \qquad n=\frac{1+\frac{1}{2}f'H/\nu}{1+\frac{1}{3}f'H/\nu} \qquad \text{for wind,} \\ &\text{and} \\ &\bar{\zeta}=\int \gamma_0 dL \qquad \text{for barometric action,} \end{aligned}\right\} \qquad (4)$$

where n is a coefficient of value 1 (for no bottom-friction) to 3/2 (no bottom-slip), and γ_0 denotes the barometric gradient in water column. When the motive force, and hence $\bar{\zeta}$, varies with time, the level rise ζ at any time t after the commencement of the action will be given by our formula[1]

$$\zeta_m(t)=\int_0^t \frac{\partial}{\partial \tau}\bar{\zeta}_m(\tau)\left[1-\frac{\sqrt{4\sigma_m^2+(\nu\beta_0^2)^2}}{2\sigma_m}\cos\{\sigma_m(t-\tau)-\varepsilon\}\right]d\tau, \qquad (5)$$

where the suffix m means the order of component fluctuation.

Now since we have wind-records along the lake only at Hikone, we must necessarily assume that the wind of the recent typhoon over the part of the lake under consideration is everywhere the same as at Hikone, though acting 20 min. earlier. Using this approximation we calculate first the component of T along the longitudinal axis (N 20° E) of the basin, and then obtain $\bar{\zeta}$ at Seta for the wind (Fig. 5). Add to this a correction for the barometric depression. For a lake of small dimension compared with the range of low pressure, the barometric effect upon the lake-level will naturally be far smaller than in an open sea where the water-supply is unlimited, and it will be sufficient to estimate roughly from the weather charts drawn at each hour or half-hour. In the present case the barometric effect does not exceed 5% of the wind effect. Substituting the total $\bar{\zeta}$ in eq. (5), we calculate $\zeta(t)$ by mechanical integration. Table 6 shows a part of the calculations for the period from 6 a. m. to 2 p. m.

Table 6. Calculation of level change at Seta, taking $n=1$.

t	6^h	7	8^h			9			10		11	12	13	14^h
			0^m	20	40^m	0	20	40	0	20				
Q (m/s)	16.0	16.5	21.5	23.0	22.0	28.0	—	24.0	18.0	13.5	11.1	8.4	3.5	3.0
Direction	SE	SE	SE	SE	SE	SSE	SSE	S	SSW	SW	W	NW	W	WNW
$\bar{\zeta}_1$ for Wind	7.3	7.8	13.2	17.0	15.7	76.4	65.4	95.7	60.3	25.6	2.4	1.9	0.2	1.5
$\bar{\zeta}_1$ „ Press.	−1.5	−3.2	−2.8	−2.9	0	4.0	4.1	4.2	4.4	3.5	1.8	0	0	—
Total $\bar{\zeta}_1$	5.8	4.7	10.4	14.1	15.7	80.4	69.5	99.9	64.7	29.1	4.2	1.9	0.2	1.5
ζ_1 (cm)	**0**	**0.8**	**1.4**	**3.9**	**4.4**	**10.6**	**31.4**	**56.5**	**72.7**	**72.5**	**28.5**	**−10.1**	**−12.6**	
$\bar{\zeta}_3$	0.6	0.5	1.2	1.4	1.6	9.2	7.8	11.0	6.8	3.2	0.5	0.2		
ζ_3 (cm)	**0**	**0.3**	**0.9**	**0.6**	**0.4**	**3.4**	**14.5**	**10.3**	**4.9**	**4.3**	**−2.9**	**−1.5**	**0**	

In Fig. 6 we plot the values of ζ_1 thus obtained and see that they agree with the actual fluctuations in general run. The superposition

1. These Memoirs, A, **18**, 201 (1935).

of ζ_3 on ζ_1 makes the first fall of level a little earlier, but does not modify the general features of the limnographic progress.

Fig. 5

Wind at Hikone and corresponding ζ for several places.

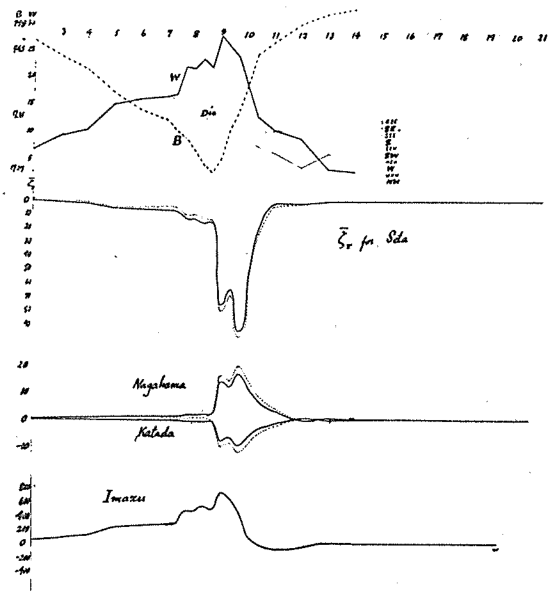
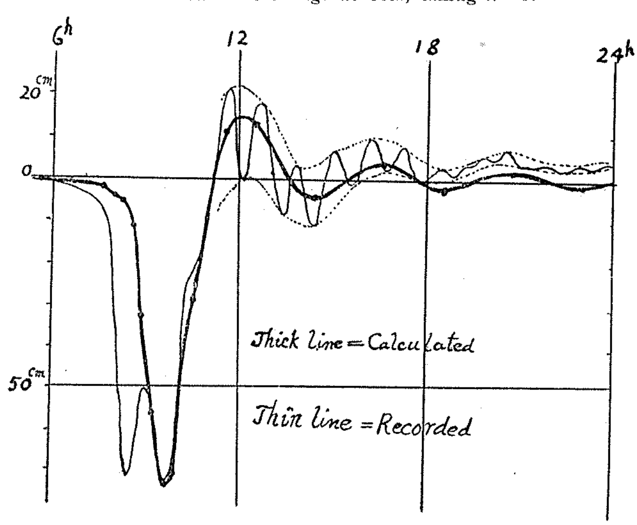


Fig. 6

Calculated level change at Seta, taking $n=1$.



We here attach a similar calculation using the wind-data at Senchô situated in the valley of Seta gorge. For topographic reasons, the latter cannot be extended to the later period of the typhoon, but it is very suggestive for the initial stage of the fall of level.

Table 7. Initial stage calculated from the wind-data at Senchô

Time	7^h		8						9^h		
	30^m	40	0	10	20	30	40	50	0	20	40^m
Comp. Q^2	19.7	19.0	26.5	69.5	181.0	489.0	431.0	175.0	89.1	36.7	2.3
ζ_1 (cm)	0		2.1			12.8			72.2		40.7

§ 5. 70 min. period oscillation.

This oscillation appeared not only in the southern part but also in the main body of the lake.

Nature of the oscillation:—In the northern section, we notice (a) the greatest amplitude, though in opposite phase, at Katada and Nagahama, (b) no oscillation at Yanagawa, (c) a small oscillation at Hikone and Imazu. These facts show that the corresponding oscillating area is the elliptical basin between Katada and Nagahama. By actually measuring the depth at each tenth of the longitudinal axis and calculating the various means as in table 8, we get the period 71.4 min.

Table 8. Depth of the basin from Katada to Nagahama

No.	Depth H	$1/\sqrt{H}$	$1/H$	$\Sigma(\Delta L/H)$
1	7.32^m	0.2770	0.0770	0.0385
2	24.6	0.2015	0.0406	0.0794
3	33.3	0.173	0.0300	0.1091
4	41.2	0.156	0.0243	0.1334
5	39.1	0.160	0.0256	0.1590
6	48.8	0.143	0.0205	0.1795
7	47.8	0.145	0.0209	0.2004
8	53.6	0.137	0.0186	0.2190
9	42.1	0.154	0.0237	0.2427
10	29.6	0.184	0.0336	0.2763
11	0			0.447 (half remained)
Mean depth	36.37^m	29.1^m	22.4^m	

The bent area extending to Imazu also has a period of the same order,[1] but the oscillation here appears irregular, as explained in § 3.

1. Mr. Suda gives the period=71.8 min. for between Wani and Imazu.

Then what will be the nature of the oscillation in the *southern* part of the lake? Some will take it to be a harmonic oscillation of the southern basin itself, and sometimes such a harmonic oscillation may really take place, but the writer considers it to be forced oscillation excited by the primary oscillation of the northern main body and on this particular occasion at least it can not be harmonic.

The reasons are as follows:

(1) The period of 70 min. is numerically 1/3.5 of 4.1 hours, but not 1/3.

(2) The 70 min. oscillation is very large at Katada where the fundamental vibration of 4.1 h period has its node.

(3) As shown before, the third harmonic oscillation calculated from the wind of the typhoon is decidedly small, while the real amplitude of 70 min. oscillation in the afternoon is very large, rather greater than that of the 4.1 h oscillation.

(4) The phase is the same at Seta and Katada but opposite at Mihogasaki.

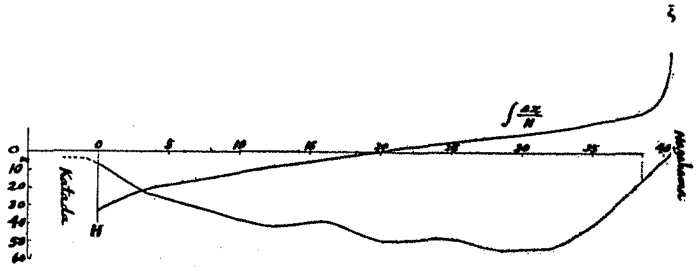
(5) Damping is less for the 70 min. oscillation in the southern portion than in any other oscillation, as already explained.

Thus the 70 min. oscillation in the southern part resembles the derived tide in a bay excited by the outside ocean-tide, or the joint oscillation in a two-step basin dealt with by Proudman.[1]

Amplitude-ratio:—This oscillation has the largest amplitude of 30 cm at Nagahama, but 20 cm at Katada. The difference is due to the distribution of water-depth. The change of depth from Katada to Nagahama is given in Table 8, and the last column of the table corresponds to the values of ζ above the level of Katada, taking $L. T/g\rho$ as unity. Fig. 7 shows the profile of the basin and the amplitude-ratio

Fig. 7

Profile of the section from Nagahama to Katada.



1. loc. cit.

of the oscillation there. The difference of levels at Nagahama and at Katada being 44.7 units, we see that:

Level fall at Katada : Level rise at Nagahama = 16.5 : 28.2,

which nearly coincides with the actual ratio.

Time-relation of the level fluctuation:—In the same manner as in § 4, we calculate the change of level in the oscillation area under consideration due to the Muroto Typhoon. Assuming that the wind all over the region is the same as at Hikone, and taking $n=3/2$ and the axial direction of the oscillation area = N 50° E, Table 9 and Fig. 8 are obtained.

Table 9. Level fluctuation in the northern main body

(A) At Nagahama

t	6^h	7	8			9			10		11	12	13^h
			0^m	20	40	0	20	40	0	20^m			
Wind Q	16.0	16.5	21.5	23.0	22.0	28.0	—	24.0	18.0	13.5	11.0	8.4	3.5
Dir.	SE	SE	SE	SE	SE	SSE	SSE	S	SSW	SW	W	NW	W
ζ for Wind	1.2	1.4	2.3	2.5	2.4	20.2	17.6	24.7	17.7	10.0	4.5	−0.3	0.4
ζ for Press.	−1.2	−1.6	−1.3	−1.1	0	2.0	2.3	2.7	3.0	2.0	1.0	0	0
Total ζ (cm)	0	−0.2	1.0	1.4	2.4	22.2	19.9	27.4	20.7	12.0	5.0	−0.3	0.4
ζ (cm)	0	−0.6	1.2	1.1	1.3	10.2	41.5	30.8	16.3	21.0	−4.5	−6.1	

(B) At Katada

t	6	7	8			9			10		11	12	13
			0	20	40	0	20	40	0	20			
ζ f. Wind	0.75	0.8	1.4	1.5	1.4	11.8	10.2	14.4	10.4	5.8	2.5	−0.2	0.3
ζ f. Press.	−0.8	−1.6	−1.4	−1.5	0	2.0	2.0	2.1	2.2	1.8	0.9	0	0
Total ζ (cm)	0	−0.8	0	0	1.4	13.8	12.2	16.5	12.6	7.6	3.4	−0.2	0.3
ζ (cm)	0	−1.5	0.7	1.1	2.2	9.6	29.4	19.3	11.5	16.2	−8.4	−4.2	−1.9

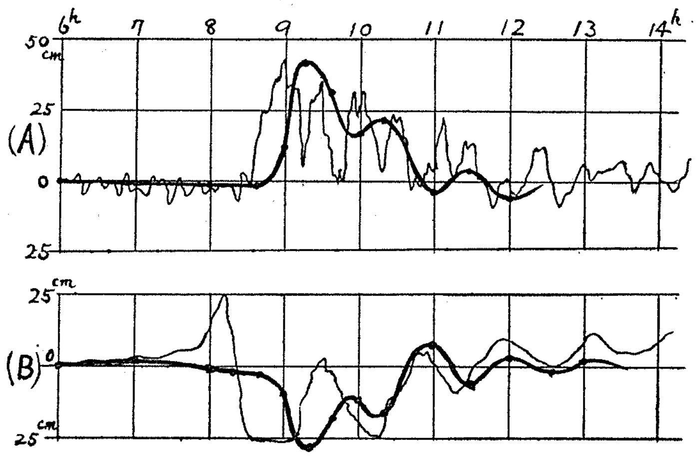
The writer believes that he has thus explained the essential features of the mechanism of surface fluctuation in the northern portion.

Next, as to the oscillation of the same kind in the southern basin, it is sufficient to consider it relatively to that at Katada. Now, the depth of the basin south of Katada is only about one tenth of that in the northern main part, and the whole lake forms a two-step basin. According to Proudman[1], the co-oscillation of such a two-step basin will be given by

$$\zeta = Z\cdot\frac{\cos k'(x-L')}{\cos k'L'}\cdot\sin(\sigma t+\alpha), \qquad k'=\frac{\sigma}{\sqrt{gH'}} \tag{6}$$

1. loc. cit.

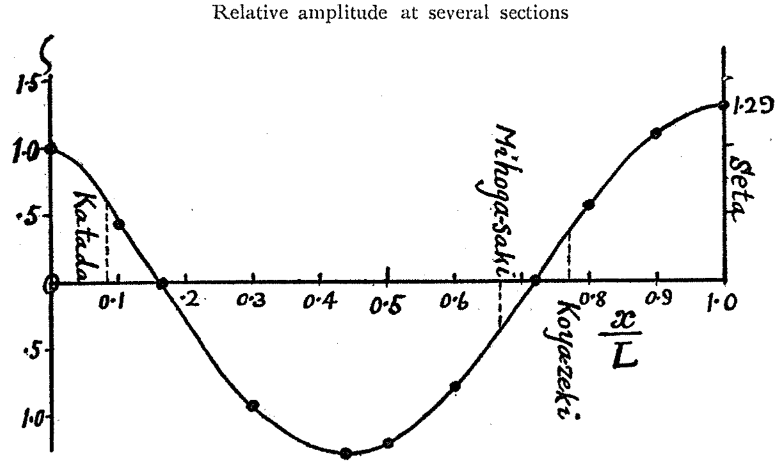
Fig. 8
Level change calculated and recorded on limnogram
(A) Nagahama



(B) Katada

in the shallower portion of length L' and depth H', if the main part is very deep compared with the portion considered. This equation exactly coincides with the formula for derived tide in a bay given by Harris[1] or Defant[2]. In the eq. the origin of coordinates is taken at the place of abrupt change of depth or the bay-mouth, and x is measured horizontally towards the other end of the region. In the

Fig. 9
Relative amplitude at several sections



1. Manual of Tides, IV (1900), p. 681. 2. Ozeanographie (1929), p. 193.

present case, the above formula gives the amplitude-ratio at ten sections as shown in Fig. 9.

This diagram will explain the phase relation between Katada, Mihogasaki and Seta. The great amplitude at Seta is partly due to this action, though mainly accelerated by the contraction of the section near the bay head.

§ 6. Characteristic general rise of level at Imazu, etc.

We shall here describe some notable points left untouched as yet.

(1) At Imazu, besides the short-period oscillations, a characteristic general rise of non-periodic nature is conspicuous. Comparing this with the universal fall at Nagahama, Hikone and Yanagawa in the early morning up to 8 a. m., we may conclude that the general rise at Imazu is due to the accumulation of water from the opposite eastern coast of far wider extent, and also to the passing of the center of the typhoon almost directly over Imazu. (cf. the last curve in Fig. 5).

(2) At Katada, the very first stage of level-change was elevation, notwithstanding its situation near the mouth of the " bay ". This fact shows how severe the drift of the wind was during the typhoon, and how great a hindrance was formed even by the slight contraction of section north of Katada.

(3) After the storm, the whole lake shows a gradual increase in level amounting to a few cms. This is of course due to the precipitation accompanying to the typhoon.

§ 7. Estimation of μ and f', etc.

From the actual values of the three quantities, the damping factor, the oscillation period and the maximum ζ, we can estimate three other quantities, ν, f' and β_0.

(A). *In the southern portion of the lake* ($H=3.4$ m) :—Since the max. fall of level in the southern basin nearly corresponds to

$$n \equiv \left(1+\frac{1}{2}\frac{f'\rho H}{\mu}\right) \Big/ \left(1+\frac{1}{3}\frac{f'\rho H}{\mu}\right) \doteqdot 1,$$

$f'\rho H/\mu$ must be very small, and the condition for β_0 in eq. (3) gives $f' \doteqdot 2H\cdot(\frac{1}{2}\nu\beta_0^2)$.

Putting the actual value of the damping index $\frac{1}{2}\nu\beta_0^2$, we get

$f'=2\times 340\times 2.73\times 10^{-4}=0.16$ c. g. s. during the storm

$\;\;=2\times 340\times 0.885\times 10^{-4}=0.06$ after the storm.

Next, from the actual and the ideal period we have

$$0.89 = B_0 \frac{\sin \beta_0 H}{\beta_0 H} = \frac{2(1+\tan^2\beta_0 H)}{1+\tan^2\beta_0 H + \tan\beta_0 H/\beta_0 H}\left(\frac{\sin\beta_0 H}{\beta_0 H}\right)^2,$$

which gives $\beta_0 H = 1.38$, or $\beta_0^2 = 1.65 \times 10^{-5}$.

Now since obviously $\nu = \frac{2}{\beta_0^2} \times$ damping index (i. e., $\frac{1}{2}\nu\beta_0^2$), we have

$\nu = 2.73 \times 10^{-4} \times 2/1.65 \times 10^{-5} = 33$ c. g. s. during the storm
$= 0.885$,, ,, $= 11$ after the storm.

(B) *In the northern main body of the lake* ($H = 36.4^{m}$) :—Here the most appropriate value of n is 1.4, which corresponds to $f'\rho H/\mu = 12$. From the relation $\beta_0 H \tan \beta_0 H = f'\rho H/\mu = 12$, there results

$\beta_0 H = 1.45$, $\beta_0 = 1.45/3640 = 3.98 \times 10^{-4}\ \mathrm{cm}^{-1}$,

which again gives

$$\nu = \frac{2}{\beta_0^2} \times \text{damping index} = 2 \times 8.05 \times 10^{-5}/(3.98 \times 10^{-4})^2 = 1020,$$

and $$f' = 12 \frac{\nu}{H} = 3.35 \text{ c. g. s.}$$

The writer here expresses his cordial thanks to the Hattori Hôkô-kwai for the grant with which the present research has been carried out.