A Study of the Minor Fluctuation of the Atmospheric Pressure (IV)

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Part III. Seiches of Lakes or Bays due to Microbaric Disturbances[1]

1. Introduction

Earthquakes are counted among the causes of seiches, but all observers agree that the majority of seiches are of meteorological origin. Prof. Chrystal and E. M. Wedderburn[2] enumerate 8 different causes for the development of seiches, the most important of which, they say, is microbaric disturbances, except in cases of shallow lakes. This paper, as part of the author's microbarographic study, is devoted to seiches due to microbaric disturbances, apart from a general discussion of the development of seiches. The microbarograms adopted and compared here are those recorded at Kyoto. The mareograms are those recorded in the Bay of Ôsaki, Wakayama Prefecture. They were taken under the direction of Dr. Nomitsu, Professor of Oceanography of our Geophysical Institute. In regard to the seiches of the bay, Mr. Y. Toyohara[3] has already published a report. His report, therefore, is to be referred to for general descriptions, etc. But, unfortunately, the bay is situated at some distance from Kyoto (Ca. 80 km.) And so, limnograms recorded in the Lake Biwa, which is a large lake lying east of Kyoto, are adopted in addition. Unfortunately again, complete limnograms can be obtained nowhere else except at Ôtsu, which is on the southern part of the lake, a part which must be considered an arm of the lake stretching far into the land. Therefore, the limnograms at Ôtsu belong to what Prof. Chrystal regards as exceptional cases, where the influence of wind is remarkable. Wind, indeed, is observed to have a striking influence on the

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[1] An outline of this part of the paper was read at the Annual Meeting of the Physico-Mathematical Society of Japan in Oct. 1931. [2] Trans. Roy. Soc. Edinb. **41**; Q. J. Roy. Met. Soc. **68** 211–217 (1922). [3] These Memoirs. A. **15** No. 3. (1932).

development of seiches in the southern part of the lake. Concerning the general causes of the development of seiches there, a paper will be made public by Mr. T. Takahashi, the authors collaborator in the present study, so that nothing will here be said about them. Although conditions were not very satisfactory, as has just been stated, for the comparison between microbarograms and limnograms, the author was able to get some fairly good examples, and he hopes that an examination of these examples may contribute something to the study of the phenomenon in question. For these reasons, the author has come to the decision to lay this paper before the public.

2. Examinations of the Mareograms at the Bay of Ôsaki

The dates of the development of seiches in the bay, their mean periods, the meteorological data at Wakayama and the results of microbarographic observations at Kyoto,—these since July, 1926, when a tide-gauge was first set up in the Bay of Ôsaki, up to May, 1930 are compared in Table I. It may be mentioned in this connection that Mr. Y. Taguchi has reported on the relation between the secondary undulations in the Bay of Shimotsu, which is adjacent to that of Ôsaki, and on the meteorological factors. In Table I, microbaric disturbances are proved to be the most important cause of the seiche-development, as Prof. Chrystal and others believe. In my discussion, henceforth, a line will be drawn between secondary undulations due to periodic fluctuations of pressure, i. e., atmospheric waves, and those due to sudden non-periodic disturbances.

Table I

Limnographic Osc.	Microbarographic Note	P.T.	Date	Wind and Press. at Wakayama		
				Direction	m/s	mm(700+)
			1926			
15^h29 – 9^h30 VII (8)	11^h29 – 10^h30 (6,27)	C	29a	WSW	2	51.8
18^h18 – 10^h19 VIII (9,19)	14^h – 21^h18 (13)	C	18p	—	0	55.5
$3^h\ 3$ – $12^h\ 4$ IX (19,7)		T	3a	N	2	57.0
0^h – 23^h17 IX (9,19)	0^h – 23^h17 (15,13)	T	17a	NNE	4	51.0
$0^h\ 7$ – $15^h\ 8$ X (7,19)	$23^h\ 6$ – $24^h\ 8$ (9,24)	B	7a	NE	3	64.0
16^h22 – 9^h23 XI (20,7)	18^h22 – 8^h23 (9)	C	22p	NE	3	64.2
$9^h\ 7$ – $19^h\ 8$ XII (17,7)	$6^h\ 7$ – $19^h\ 8$*(7,9)	D	7a	ESE	2	59.8
3^h – 24^h20 XII (17,8)	21^h19 – 10^h20 (6)	D	20a	NE	2	70.8

Limnographic Osc.	Microbarographic Note	P.T.	Date	Wind and Press. at Wakayama		
				Direction	m/s	mm(700+)
1927						
9^h-18^h7 I (18,9)	6^h-21^h7 (7)	C	7a	E	5	64.0
15^h24-9^h25 II (21,28)	16^h24-7^h25*	D	24p	—	0	67.0
15^h8-19^h9 III (19,6)	9^h-17^h8 (8)	D	8p	S S E	7	64.9
15^h-24^h10 III (8)	(Squall at 22^h10^m)	C	10p	—	0	55.5
6^h12-9^h14 III (9)	2^h13-22^h14 (10,20)	C	12a	—	0	66.1
0^h-12^h20 III (19)		C	20a	N N E	6	54.0
18^h23-20^h24 V (19,8)	15^h23-11^h24* (14,6)	D	23p	W S W	3	58.7
15^h-21^h8 X (8)	15^h-17^h8 (5)	D	8p	—	0	57.3
4^h8-21^h9 XII (21,8)		C	8p	E	4	65.2
1928						
0^h-12^h10 I (9,21)	1^h-8^h10* (11)	D	10a	S	3	66.2
0^h-24^h12 V (8)	14^h12-6^h13 (16,5)	T	12a	—	0	49.6
0^h13-15^h14 VI (19,7)	13^h13-10^h14* (7,13)	C	13a	S E	4	53.0
18^h21-18^h22 VI (18,7)	8^h-14^h21 (12)	D	21p	W	2	54.2
0^h24-9^h25 VI (9,19)	16^h24-6^h25 (8)	C	24a	—	0	58.9
0^h-24^h25 IX (9)	6^h-12^h25 (7)	D	25a	—	0	57.5
0^h-15^h30 IX (21,9)	16^h29-9^h30 (10)	B	30a	E N E	2	59.0
0^h-24^h5 X (8,19)	14^h4-22^h5 (14)	B	5a	—	0	63.2
0^h18-24^h19 XI(19,11)	23^h17-24^h19 (7,11)	C	18a	E N E	2	68.5
1929						
3^h-8^h7 IV (7,17)	7^h-9^h7*	D	7a	S S E	5	51.8
21^h19-6^h21 IV (9)	13^h20-5^h21	C	19p	—	0	60.6
18^h26-9^h27 IV (7)	19^h26-4^h27* (13,4)	D	26p	S S W	10	53.9
4^h-16^h18 V (19)	0^h-14^h18 (9)	B	18a	E N E	3	58.6
15^h-21^h9 IX (25)	16^h9-11^h10 (32)	T	9p	N E	4	51.4
0^h21-9^h22 IX (19,21)	14^h19-7^h22*(38,15)	T	21a	—	1	59.9
6^h28-6^h30 IX (19,9)	13^h28-7^h30 (31)	T	28a	E N E	2	61.3
0^h-9^h4 X (19,7)		T	4a	N E	2	60.8
15^h6-6^h9 XI (18,9)	19^h5-24^h9* (14,8)	D	6p	—	1	69.0
0^h10-9^h11 XI (19,7)	16^h9-7^h11 (12)	B	10a	E N E	3	68.1
0^h20-18^h21 XII (7)	10^h19-22^h21* (14)	C	20a	E N E	2	59.5
1930						
0^h26-15^h27 II (21,7)	12^h26-14^h28* (6,12)	C	26a	E N E	3	60.5
0^h-15^h4 III (19)	6^h-20^h4 (16)	D	4a	N N E	4	61.4
0^h-12^h5 V (9)	17^h4-11^h5 (7)	B	5a	N N E	5	58.0

Remarks. The first column shows the Duration-Times of the occurrences of secondary undulations of Ôsaki Bay.

The figures denote the time and date, Roman numerals the month, and the figures in bracket the periods of predominant undulations. The second column shows the durations of the corresponding disturbances of the microbarogram at Kyoto.

* indicates a remarkable disturbance. The figures in brackets are the periods of the predominant waves of pressure. The third column shows the types of pressure-distribution on the weather charts, and the adopted notations are as follows:

B: when border of anticyclone passes near our observatory. C: when cyclones are near. D: when discontinuous lines pass near. T: when typhoon situates far south region or passes near.

The fourth column shows the time corresponding to the third and fifth columns. The figures denote the date; (a) and (p) indicate 6^h and 18^h respectively.

The fifth column shows wind and pressure at Wakayama Meteorological Observatory.

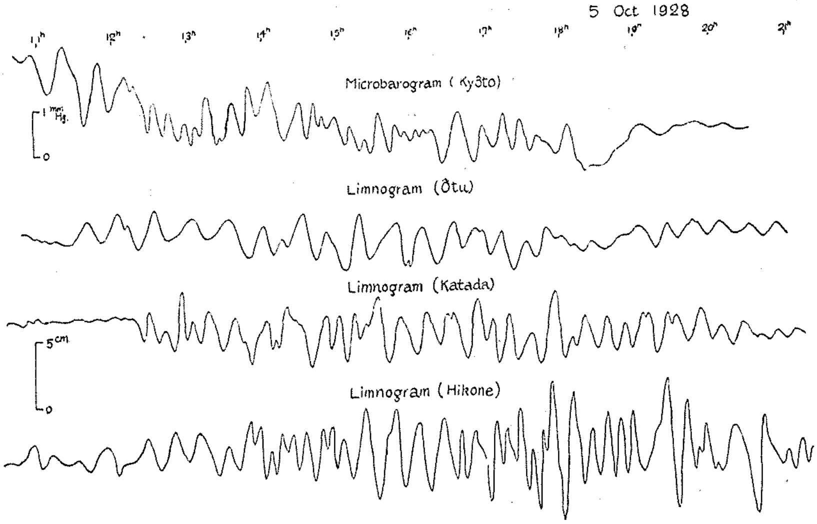
3. Seiches of Lakes or Bays due to Travelling Waves of Pressure

Although it is well known that periodic fluctuation of pressure is one of the important causes of the development of seiches, a good example is rarely found. A typical example, therefore, is given first of all.

Ex. Developed Seiches of Oct. 5th, 1928. The description of the microbaric disturbances of the day is given in the foregoing chapter treating the classification of disturbances types. Fig. 1 shows the microbarogram at Kyoto and the limnograms at Lake Biwa. Fig. 2 is a reproduction of mareograms at Ôsaki and Shimotsu. In these figures, the period of the waves of pressure is 7, 10, and 19 min., and the period of the seiches of Lake Biwa is 20–25 min. at Otsu; and 12, 21 min. at Katada; 7, 11, and 19 min. at Hikone; 19.0, 9.5, and 7.0 min. at Ôsaki; 19.0 and 7.0 min. at Shimotsu. Each of these observed periods is approximate to one of the theoretical periods and also to one of the periods of microbaric fluctuations. Moreover, on that day, the wind was almost calm and no trace of violent phenomena such as squalls was experienced in the vicinity of Japan, so that there is no other way of accounting for the seiche-development but by supposing that it depends on microbaric disturbances. The microbaric disturbance of the day, in view of both its great amplitude and of its long duration, is a very rare phenomenon. And from the fact that the seiches and secondary undulations are unusually remarkable in other places, it may safely be asserted that this is one of the best examples of its kind.

It is a well-known fact, as shown in this example, that the seiche-development is observed, when the period of fluctuations of pressure

Fig. 1



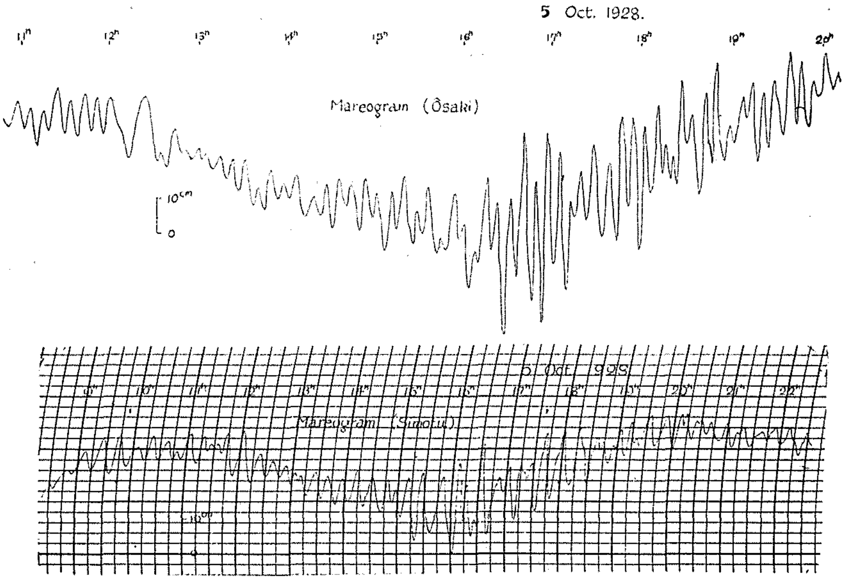


Fig. 2

coincides with that of the seiches. But fluctuations of pressure seem, hitherto, to have been considered to be stationary. The author had already noticed, as recorded in the foregoing part of this paper, that waves of pressure travel. Now, travelling waves of pressure can cause resonance, which may simply be explained in the following way. In this discussion, the author neglects the turbulance of the sea or a lake, which, must actually modify the character of the seiches. But the essence of the author's argument is in no way altered by the omission. To make matters simpler, the forced wave of pressure is assumed: $P_0 = P\cos k(x-Vt)$. (1) The equation of the motion of a long wave and the equation of the continuity are written down in the usual notation:—

$$\frac{\partial^2\xi}{\partial t^2} = gh\frac{\partial^2\xi}{\partial x^2} + \frac{kP}{\rho}\sin k(x-Vt), \quad (2) \qquad \eta = -h\frac{\partial\xi}{\partial x}. \quad (3)$$

(i) Solutions for a lake. The boundary conditions, in this case, are:— $\xi = 0$ at $x = \pm l$ where $2l$ stands for the length of the lake. A simple appropriate solution for this case is:—

$$\xi = -\frac{P}{k\rho c^2(m^2-1)}\Big[\sin k(x-Vt) - \frac{1}{\sin 2klm}\cdot \Big\{\sin k(l+Vt)\sin km(x-l) + \sin(l-Vt)\sin km(x+l)\Big\}\Big]. \quad (5)$$

$$\eta = \frac{hP}{\rho c^2(m^2-1)}\Big[\cos k(x-Vt) - \frac{m}{\sin 2klm}\cdot \Big\{\sin k(l+Vt)\cos km(x-l) + \sin k(l-Vt)\cos km(x+l)\Big\}\Big]. \quad (6)$$

where $m = V/c$ and $c = \sqrt{gh}$. This solution is obtained immediately by referring to the theory of the forced tide of canals.[1]

(ii) Solutions for a bay. The boundary conditions, in this case, are:— $\xi = 0$ at $x = l$, $\eta =$ *only forced wave i. e. free wave* $= 0$ *at* $x = 0$, where $x = 0$ shows the position of the entrance to the bay, and l shows the length of the bay. The solutions, in this case, are

$$\xi = -\frac{P}{k\rho c^2(m^2-1)}\Big[\sin k(x-Vt) - \frac{\sin k(l-Vt)}{\sin 2klm}\cdot \Big\{\sin km(x+l) - \sin km(x-l)\Big\}\Big], \quad (7)$$

$$\eta = \frac{hP}{\rho c^2(m^2-1)}\Big[\cos k(x-Vt) - \frac{m\sin k(l-Vt)}{\sin 2klm}\cdot \Big\{\cos km(x+l) - \cos km(x-l)\Big\}\Big]. \quad (8)$$

In (6) and (7), if $V = c$ and consequently $m = 1$, then $\eta \to \infty$ apparently,

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[1] Lamb, Hydrodynamics. 5 ed. 252.

but, on the contrary, η converges to a finite value in this case. But when $\sin 2klm \to 0$, $\eta \to \infty$ This corresponds to the cases of resonance. In other words, $2klm = s\pi$ (s : integer) (9). Now suppose that T_i is the calculated seiche-period of a lake or bay, namely,

$T_i = 2l/ic$ for a lake (10), $T_i = 4l/(2i-1)c$ for a bay (11),
($i = 1, 2, 3, \ldots\ldots\ldots\ldots$),

conditions in (9) prove to be, $T_i/T = s/2i$ for a lake (12), $T_i/T = s/2i-1$ for a bay (13). The foregoing is a treatment of a simple ideal case, but it leads to comprehension of the relations in more complicated cases.

4. Seiches due to Rapid Fluctuations of Barometric Pressure

Fig. 3

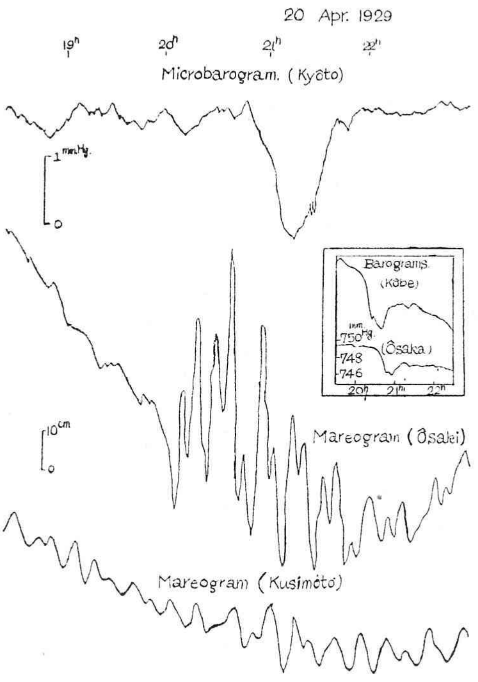
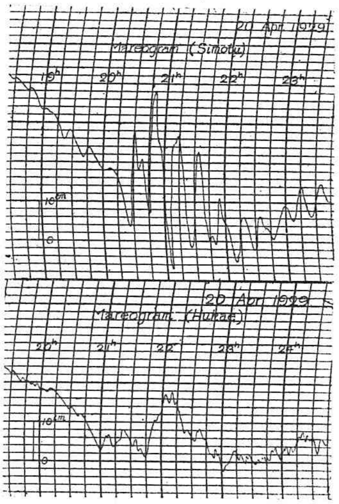


Fig. 4



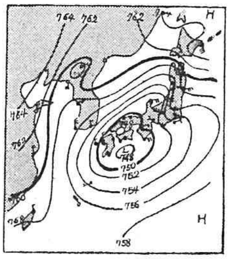
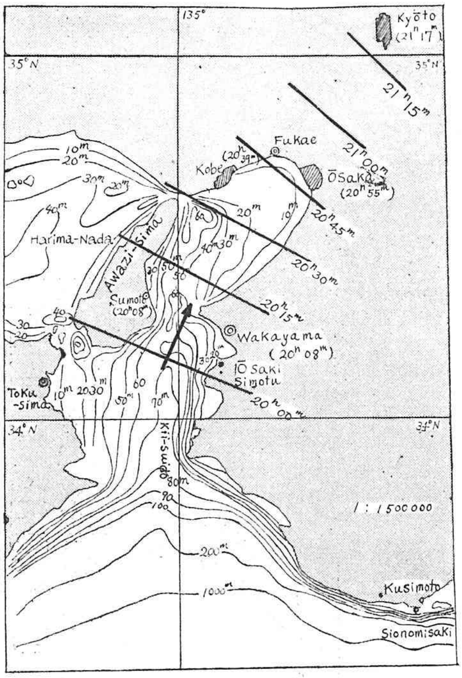


Fig. 5

Weather Chart.
18^h, 20th, Apr. 1929.

Ex. 20th Apr. 1929. A well-developed seiche was recorded by the mareograph at about 20^h in the Bay of Ôsaki. (See Figs. 3 and 4) On the weather chart of the time (Fig. 5), a typhoon is seen in the northern part of the Pacific Ocean near the southern coast of Japan, but it is not directly attacking the bay. Pressure distributions of similar kind have several times been witnessed, but such a seiche has rarely been recorded. On the other hand, microbarograms at Kyoto show the passage of a microbarographic depression at about 21^h. The state of its propagation can be determined by the barograms of neighbouring observatories (Fig. 6). Thus it is inferred that the depression passed the Bay of Ôsaki at about 20^h, from which time the seiche developed in the bay. This striking coincidence necessarily leads to the assumption that the developed seiche was due to the passage of the microbarographic depression.

Fig. 6



But the physical explanation of this phenomenon is not simple. The depth of fluctuation of pressure being 4mm Hg. at most, its statical effect alone cannot account for a variation of water surface amounting to 5.4 cm. Reports of researches of this kind are very rare. So far as the author knows, Mr. Marmer[1] is the only investigator, who has given, in "The Tide," an example of this phenomenon illustrated by a diagram. Mr. Marmer says, in his report, that there were no peculiar features about the wind, only there was a kick in the barogram, in correspondence to which, a seiche occurred. Although he gives no further description nor discussion, his report has been a great help to the present author, who has endeavoured to complete the physical explanation of this problem, but has not as yet been successful. Below is stated the author's opinion, which may be somewhat speculative. According to Prof. J.

Proudman, an elevation of the water-surface, with a closed end at $x=0$, and with an infinite extension in the positive direction of x, generated by pressure-disturbance on a sea-surface of uniform depth, can be shown by the following equation[2];

$$\zeta=\frac{1}{1-V^2/c^2}\left\{F(t-x/V)-\frac{V}{c}F(t-x/c)\right\}$$

where $\xi=F(t-x/V)\quad c=\sqrt{gh}\,.$

Accordingly, at $x=0$, $\zeta=\dfrac{1}{1+V/c}F(t)$

and if $V/c\rightarrow-1,\quad \zeta\rightarrow\infty$

which shows the occurrence of resonance. Actual cases are, of course, different from this simple, ideal case in various points. But, qualitatively considered, where a coast lies on one side, (where the depth (h) extends over a fairly large portion of the sea) and where the condition,

$$\frac{\textit{Velocity of propagation of barometric fluctuation}}{\textit{Velocity of free wave}}\fallingdotseq-1$$

is satisfied, it is reasonable to suppose that the fluctuation of the water surface at the coast may become several times as great as its statical value. Of course, this magnifying power depends on the length of the domain in which the condition is satisfied. Looking at the matter from another point of view, in the actual example just presented, the microbaric disturbance is propagating toward NNE at the rate of 25.5 m/s. If we assume $h=50$ m, 60 m and 70 m, then $c=22$, 24 and 26 m/s respectively. Therefore, the domain where $V/c\fallingdotseq-1$, must be a sea where $h=60$–70 m. This value cannot occur in the interior of the Bay of Ôsaki, but in the Kii Channel outside the bay, may occur over a considerable area. (See Fig. 6) From these facts, it is to be supposed that a fairly great rise and fall of the water-surface occurred outside the bay owing to the resonance effect which has just been described. C. K. M. Douglas[3] gave a similar example, when he treated a disturbance of the sea-surface occasioned by a thunderstorm occurring in the English Channel. After having simplified the records at the Bay of Ôsaki, and eliminated the seiche from them, there still remains the variation of the water-level. This variation is about five times as great as the statical value due to pressure. Magnifications of this degree can be regarded as possible by the application of the principle of resonance

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[1] Marmer "The Tide" 151 (1926). [2] M. N. R. A. Soc. Geophys. Suppl. **II 4** 197 (1929). [3] Met. Mag. **64** 187 (1929).

to this example. It is easily to be understood that, when there is a variation of the sea-level at the mouth of the bay, a secondary undulation develops in the interior. Now, at the mouth of the bay ($x=0$), if we put $\eta=f(t)$, the rise and fall of the sea-surface which it causes in the bay are shown by the equation :—

$$\eta=\frac{2c}{l}\sum_{s=0}^{\infty}\sin\frac{(2s+1)\pi x}{2l}\int_0^t F(\mu)\sin\sigma_s(t-\mu)d\mu.$$

Now, if $F(t)=a\sin nt$ for $0<t<t_1$ and $F(t)=0$ for $t>t_1$, then

$$\eta=\frac{2ca}{l}\sum_{s=0}^{\infty}\frac{1}{\sigma_s^2-n^2}[\sigma_s\sin nt-n\sin\sigma_s t]\sin\frac{(2s+1)\pi x}{2l} \qquad (0<t<t_1),$$

$$\eta=\frac{2ca}{l}\sum_{s=0}^{\infty}\frac{1}{\sigma_s^2-n^2}[n\sin\sigma_s(t-t_1)\cos nt_1+\sigma_s\cos\sigma_s(t-t_1)\sin nt_1 - n\sin\sigma_s t]\sin\frac{(2s+1)\pi x}{2l} \qquad (t>t_1),$$

and if $\sigma_m/n\rightarrow 1$ then

$$\eta_m=\frac{2ca}{l}\,\frac{\sin\sigma_m t-\sigma_m t\cos\sigma_m t}{2\sigma_m}\times\sin\frac{(2m+1)\pi x}{2l} \qquad (0<t<t_1).$$

Therefore, when the period of forced vibration of the bay agrees with that of free vibration, the seiche develops to a remarkable extent. But in the example now under consideration, this effect is not great. To sum up ; in this example, conditions in the outer sea being favourable for the variation of pressure causing resonance, a fairly great disturbance of the sea-surface occurs, which, in its turn, develops the seiche in the bay as a secondary phenomenon.

5. Remarks and Conclusion

(i) The reproduction of the records of 15^h 17th Sept. 1926, when a typhoon passing near the Bay of Ôsaki, caused a very great seiche to occur is given in Y. Toyohara's report. But in this case also, a microbarographic depression is observed to have accompanied the seiche. (See Fig. 7) Moreover, the beginning of the seiche synchronizes, not with the passage of the centre of the typhoon, but with the passage of the microbarographic depression. As this minor depression propagated NNE at the rate of 27 m/s, there was, as in the foregoing example, an area of greater or smaller extent near the mouth of the bay, where $V/c \fallingdotseq -1$.

Fig. 7.

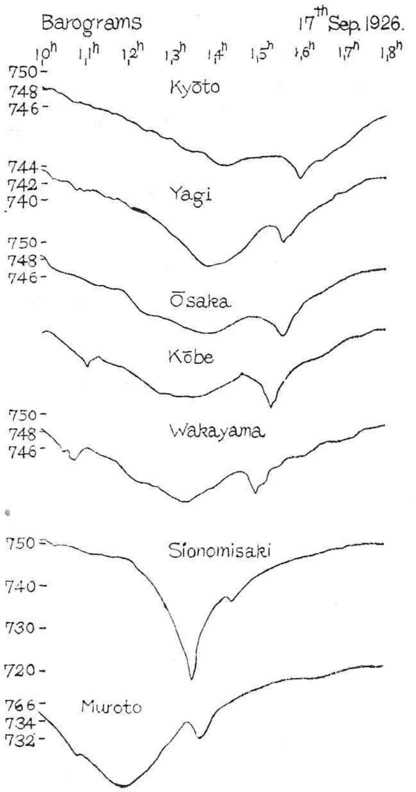


Fig. 8.

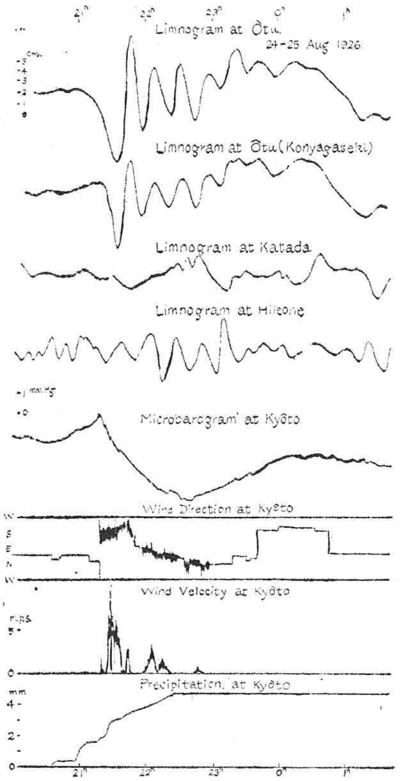
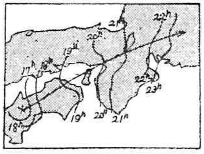
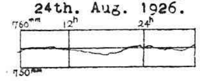


Fig. 8b.

Isochronous Chart of Thunder. 24. Aug. 1926.



Barogram at Kyoto 24th. Aug. 1926.



(ii) The microbaric disturbances with a propagating velocity of 60 m/s, described in § 1, Section V, Part II, are not accompanied by any remarkable seiche. The reason is perhaps that there is no domain near by where $V/c \fallingdotseq -1$.

(iii) Examples of seiches corresponding to squalls are not unfrequently met with in the southern portion of Lake Biwa (which must be regarded as a shallow bay). Fig. 8 shows one such example. In cases of squalls, pressure changes are always accompanied by wind changes. As this example occurred in shallow water, the variation of the wind must have been its chief cause, but a pressure variation must be of more importance in the devolopment of seiches in a deep lake.

Conclusion

(i) By comparing microbarograms and limnograms, the author came to the same conclusion as Prof. Chrystal, that the most important cause of seiches, except in shallow lakes, is microbaric disturbances.

(ii) By citing a typical example of seiches due to the periodic variation of pressure, and basing his argument on the fact that the variation is progressive, the author has modified somewhat the view hitherto taken of the matter.

(iii) By applying J. Proudman's theory to a typical example of seiches occasioned by non-periodic but rapid variation of pressure, the author showed that when the area of the sea which satisfies the condition $V/c \fallingdotseq -1$ is extensive, in the proximity of the mouth of a bay, there is a remarkable development of the seiche.