Combined Mirror and Lever Extensometer

By

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(Received Jan. 20, 1937)

Abstract

This instrument is a combined form of Martens' lever extensometer[1] and his mirror one.[2] *It consists of two clips carrying two small rotating mirrors, being held* on opposite sides of the specimen by a spring. Two double knife edges to which the levers are attached, are pivoted in shallow seats formed in the clips. The lower ends of the levers rotate the mirrors whose angles of rotation are measured by the telescopes and the scales. The two mirrors rotate in opposite directions when the specimen is elongated by applying a certain load. When adapted to determine the least stress at which Hooke's Law ceases to be exact the instrument attains a great magnification. The magnification obtained with the present instrument is about 7000.

Arrangement

This instrument, shown in Fig. 1, is so arranged that it can easily measure mechanically a small elongation of a solid rod. The two clips (CC) are held on opposite sides of the test-rod (R) by a spring (S) which rests in grooves (GG). Each clip is pointed at the lower end which is gripped directly on the test-rod, while the upper end has the double knife edge (N) interposed between it and the rod. Each mirror (M) is mounted on a frame which is fixed to each clip by means of the pivots (HH) centred in small holes drilled in the glass. The frame has an adjusting screw (K) which balances the mirror on the side opposite the clip. An arm (A) which is firmly fixed to the centre of each double knife edge, has a lever (L) clamped perpendicularly to the arm at one end, while the other end carries an adjusting screw (F) which balances the lever. A pivot (J) attached to the back side of each mirror is parrallel to the axis of rotation of the mirror, and is loosely inserted in the U-shaped end (U) of each lever.

Fig. 2a is a schematic sketch of a clip combined with a lever. The instrument is arranged in the following way: The upper part (AB) of the lever is perpendicular to the line IA joining both ends of the double knife edge. The central line along the part (AB) of

1) Batson and Hyde, Mechanical Testing, Vol. I, § 114.
2) " " " § 115.

Fig. 1

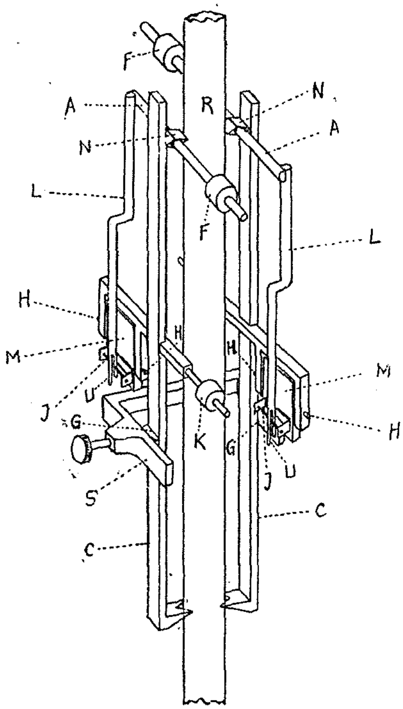
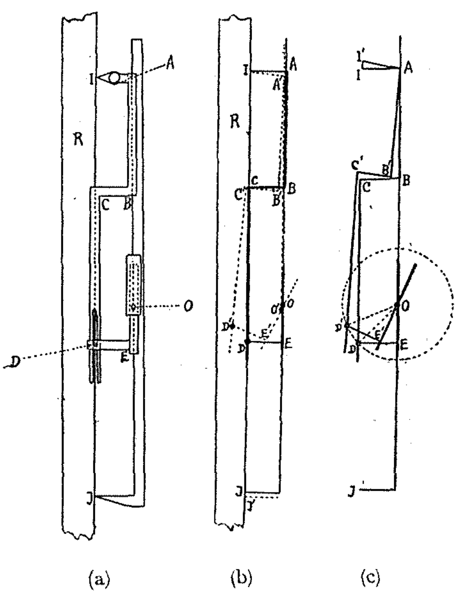


Fig. 2



the lever and the rotating axis (O) of the mirror lie in the same plane as the inner side of the clip. The middle part (BC) of the lever is perpendicular to both the upper part (AB) and the lower part (CD) of the lever; accordingly AB is parallel to CD. The main parts of the instrument have approximately the following sizes.

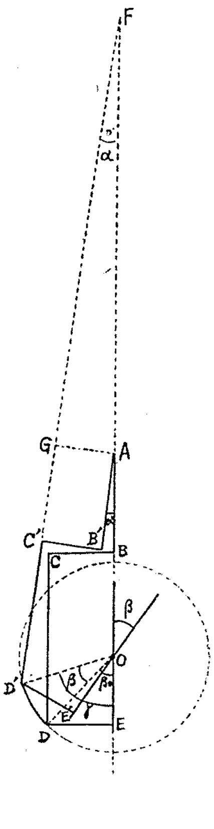
The double knife edge: IA = 5 mm.
The upper part of the lever: AB = 15 mm.
The middle part of the lever: BC = 5 mm.
The lower part of the lever: CD = 35 mm.
The gauge length: IJ = 100 mm.
The distance between the axis of rotation (O) of the mirror and the shallow seat formed in the clip to which the double knife edge is set: AO = 45 mm.
The distance from the pivot (D) inserted in the U-shaped end of the lever to the plane OE parallel to the mirror surface: DE = 5 mm.
The distance between the axis of rotation (O) of the mirror and the projection E of the pivot to the plane (OE): OE = 5 mm.

Mechanism

At first the position of the mirror is adjusted in such a way as to be parallel to the inner surface of the clip as is shown in Fig. 2a. The motion of the test-rod under load is transmitted through the double knife edge to the lever which rotates the mirror. The angle of rotation of the mirror is measured by a telescope and a scale as in Martens' mirror extensometer.

In Fig. 2b the lines IA, ABCD, OE, and AOJ show the initial positions of the double knife edge, the lever, the mirror and the clip respectively, and the dotted lines IA′, A′B′C′D′, O′E′ and A′O′J′ show the final positions corresponding to the above-mentioned parts when the test-rod is loaded. The axis of rotation (O) of the mirror is considered to move with almost the same amount as the contact point of the double knife edge with the clip, that is AA′=OO′. In order to obtain the angle of rotation of the mirror, we may consider that the points J, O and A are fixed in their positions, and the lever is rotated around the point A by elongating the test-rod, as is shown in Fig. 2c. Thus the displacement AA′ of one end of the double knife edge as shown in Fig. 2b is now represented by the displacement I′I of the other end of the double knife edge in Fig. 2c. By the rotation of the mirror the pivot D fixed on the back side of the mirror moves to a point D′ on the circle described with the radius OD around the fixed centre O. Fig. 3 is drawn, for the sake of clearness, in calculating the amount of elongation of the test-rod under load.

Fig. 3



In the following, the friction at such contact points as the rotating axis of the mirror, the pivot on the back side of the mirror, and both the ends of the double knife edge, is neglected as a minor factor.

If DE$=b$, OE$=c$, OD$=e$ in Figs. 2c and 3, then $e=\sqrt{b^2+c^2}$. When the double knife edge rotates in a small angle α for the elongation δl of the test-rod, the lever also rotates in the same angle in Fig. 2c, so that $\angle$IAI′$=\angle$BAB′$=\alpha$. If IA$=a$, we have approximately $\delta l=a\alpha$. In Fig. 3

let β be the angle of rotation of the mirror which corresponds to the angle of rotation α of the lever, then $\angle DOD'=\beta$. Denote $\angle DOE$ by β_0, and $\angle D'OE$ by γ, then $\gamma=\beta_0+\beta$. Produce BA and D'C' so as to meet at a point F, then $\angle AFD'=\alpha$. Draw AG perpendicular to FD' from A and denote AG by p, then $BC=B'C'=AG=p$.

From ΔFAG we get $AF=p \operatorname{cosec} \alpha$.

From $\Delta FD'O$ we have $\dfrac{\sin\alpha}{OD'}=\dfrac{\sin(\gamma-\alpha)}{OF}$,

that is $\dfrac{\sin\alpha}{e}=\dfrac{\sin(\gamma-\alpha)}{p\operatorname{cosec}\alpha+f}$,

where AO is denoted by f.

Therefore $\sin\alpha=\dfrac{e}{f}\sin(\gamma-\alpha)-\dfrac{p}{f}$.

For small value of α, as is actually the case, the above equation becomes $\alpha=\dfrac{e}{f}\sin(\gamma-\alpha)-\dfrac{p}{f}$.

In the present case $b=5$ mm., $c=5$ mm., $p=5$ mm., and $f=45$ mm., accordingly $e=\sqrt{50}$, $\beta_0=45^\circ$, and $\gamma-\alpha=45^\circ+(\beta-\alpha)$.

Therefore $\alpha=\dfrac{e}{f}\dfrac{1}{\sqrt{2}}\left\{\cos(\beta-\alpha)+\sin(\beta-\alpha)\right\}-\dfrac{p}{f}$.

But $\dfrac{e}{\sqrt{2}}=5$, so that $\dfrac{e}{\sqrt{2}}=p$.

Therefore $\alpha=\dfrac{p}{f}\left\{\cos(\beta-\alpha)+\sin(\beta-\alpha)-1\right\}$.

As $\beta-\alpha$ is small in the experiment with this instrument, by expanding the sine and the cosine of $\beta-\alpha$ in the above equation in its power series, we have

$$\alpha=\frac{p}{f}\left\{\left[1-\frac{(\beta-\alpha)^2}{\underline{|2}}+\frac{(\beta-\alpha)^4}{\underline{|4}}-\cdots\cdots\right]+\left[(\beta-\alpha)-\frac{(\beta-\alpha)^3}{\underline{|3}}+\frac{(\beta-\alpha)^5}{\underline{|5}}-\cdots\cdots\right]-1\right\}$$

$$=\frac{p}{f}(\beta-\alpha)\left\{1-\frac{\beta-\alpha}{\underline{|2}}-\frac{(\beta-\alpha)^2}{\underline{|3}}+\frac{(\beta-\alpha)^3}{\underline{|4}}+\frac{(\beta-\alpha)^4}{\underline{|5}}-\cdots\cdots\right\}.$$

If we neglect the terms of the order higher than $\beta-\alpha$ as first approximation, we have

$$\alpha=\frac{p}{f}(\beta-\alpha),$$

that is $\alpha = \frac{p}{f+p}\beta.$

Substituting $p=5$, and $f=45$, we get

$$\alpha = \frac{1}{10}\beta.$$

Accordingly $\beta - \alpha = \frac{9}{10}\beta.$

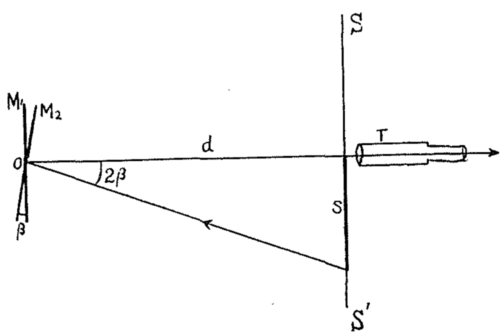
If we neglect the terms of the order higher than $(\beta-\alpha)^2$, and then by substituting $\beta-\alpha=\frac{9}{10}\beta$ for $\beta-\alpha$ on the right hand side of the above equation, we have as the second approximation,

$$\alpha = \frac{1}{10}\beta\left(1 - \frac{81}{200}\beta\right).$$

Further neglecting the terms of the order higher than $(\beta-\alpha)^3$, and substituting the above value of α obtained as the second approximation, we have, by neglecting the terms of the order higher than β^4, as the third approximation for the terms of the second and the third power of $\beta-\alpha$

$$\alpha = \frac{1}{10}\beta\left(1 - \frac{81}{200}\beta - \frac{3159}{20000}\beta^2\right).$$

Fig. 4



The angle of rotation of the mirror is observed by a telescope and a scale apart from the mirror. This arrangement is shown diagrammatically in Fig. 4. In this figure M_1 represents the initial position of the mirror, M_2 its final position, T the telescope and SS′ a scale whose distance from the mirror is denoted by d. If s denotes the scale reading corresponding to the angle of rotation β of the mirror, then

$$s = d\tan 2\beta.$$

Now let us consider the case of a very large elongation, and if we take $d=1650$ mm., and $s=330$ mm., then

$$\tan 2\beta = 0.2.$$

Therefore $\beta \doteq 0.1$ radian,

which is small, and in such a case we have

$$s=2d\beta.$$

In the case of such a small value of β as that mentioned above, the straight scale may be used. Because the amount to be corrected is negligibly small. In the case of a relatively large value of β, however, it is preferable to use a circular scale whose centre of curvature lies at the mirror. Because with such a scale the value of β is obtained directly from the scale reading without any correction.

Substituting the above value 0.1 of β in the correction terms $\frac{81}{200}\beta$ and $\frac{3159}{20000}\beta^2$ in the equation obtained as the third approximation, the correction terms take the values 0.04050 and 0.001 58 respectively. Even for such a large value of β, the correction terms take such small values. As the value of β is smaller than the above in practice, the values of the correction terms become smaller, so that we may take only the first term. Hence we get as the relation between α and β the following equation,

$$\alpha=\frac{1}{10}\beta\left(1-\frac{81}{200}\beta\right).$$

Similarly for the mirror which is mounted on the opposite clip, we get a similar relation such as

$$\alpha=\frac{1}{10}\beta\left(1+\frac{81}{200}\beta\right).$$

As is shown in Fig. 1, two clips are held on opposite sides of the specimen by a spring, and the two mirrors rotate in opposite directions as the specimen is elongated. Thus if we take the mean value of the two scale readings reflected from the mirrors, the difference of two scale readings resulting from a slight bending of the specimen and the correction terms in the above two equations cancel. Hence the proportionality between α and β is obtained.

Calculation of the constant

If we denote the mean values of each pair of α, β and s relating to two knife edges, two mirrors and two scales by $\bar{\alpha}$, $\bar{\beta}$ and $\bar{s}$ respectively, we have

$$\delta l=a\bar{\alpha},\quad \bar{\alpha}=\frac{1}{10}\bar{\beta},\quad \text{and}\quad \bar{\beta}=\frac{\bar{s}}{2d}.$$

Therefore $\delta l=\frac{1}{20}\frac{a}{d}\bar{s}.$

Further, if we denote the gauge length IJ by l in Fig. 2, the elongation per unit length of the specimen is represented by

$$\frac{\delta l}{l}=\frac{1}{20}\frac{a}{ld}\xi .$$

If we take $a=5$ mm., $l=100$ mm., $d=1650$ mm., as is actually the case, then $\frac{\delta l}{l}=0.000001515\,\xi$

where ξ is to be measured in mm. unit.

Testing

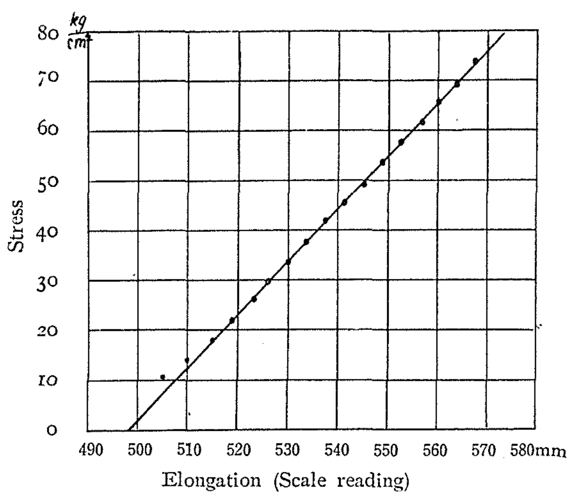
Within the elastic limit of a specimen the stress per unit area P is linearly proportional to the elongation per unit length $\frac{\delta l}{l}$ according to Hooke's Law, that is $P=E\frac{\delta l}{l}$

where E is the Young's modulus of the specimen. Hence

$$P=k\xi$$

where k is a certain constant.

Fig. 5



It is shown by the above equation, that the stress per unit area and the scale reading are proportional. In order to test the above relation the writer employed a few aluminium rods of about 20 cm. in length and about 6 mm. in diameter. The upper end of each rod was clamped to a support and a load was applied to the lower end by a spring balance. He observed the scale readings corresponding to the gradually increased loads, and plotted a curve showing the relation between the stress per unit area and the scale reading. One such curve is shown in Fig. 5. From this curve it is clearly seen that the stress per unit area increases linearly with the scale reading except at the start. The deviation from the linear relation at a few points at the start, is of course due to a loose contact somewhere in the instrument.

Determination of the constant

The constant 0.000001515 as previously mentioned is obtained by assuming the following conditions: (1) The friction at the contact points in several parts is negligible, (2) Rough values of the sizes of various parts of the instrument are granted, (3) The plane containing the rotating axis of the mirror and the central axis of the upper part of the lever is supposed to coincide completely with the inner side of the clip. Hence we must admit that the above-mentioned constant may deviate more or less from the true constant of the instrument. Thus the writer experimentally determined the true constant by the following two methods.

Method I: He attached the present instrument and Martens' mirror extensometer of known constant to the same test-rod, and determined the constant for various elongations of the rod by comparing the two scale readings obtained with the two instruments.

Method II: He determined the constant by calculation from the stress-scale reading curve given in Fig. 5 which is obtained with a cold-worked aluminium rod, by adopting the average value of Young's modulus of aluminium set forth in his former research, "Young's Modulus of Aluminium Rod Composed of Large Crystal Grains."[1) The results obtained by the above two methods are almost the same, and the value 0.000001448 was obtained as the required constant.

With this value for the constant of the present instrument its magnification is 6906, when the telescope is 165 cm. away from the mirror, a relatively high magnification.

In conclusion the writer wishes to express his sincere thanks to Professor U. Yoshida for his kind advice and invaluable suggestions in constructing this instrument.

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1) These Memoirs, A, **17**, 389, (1934)